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# OPTIMIZATION OF WATER DISTRIBUTION NETWORKS

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The paper approaches the optimization of water distribution networks supplied from one or more node sources, according to demand variation. Traditionally, in pipe optimization, the objective function is always focused on the cost criteria of network components. In this study an improved linear model is developed, which has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, operating expenses etc. The paper treats looped networks wich have concentrated outflows or uniform outflow along the length of each pipe. An improved model is developed for optimal desing of new or partially extended water distribution networks, which operate either by means of gravity or a pump system.The model is based on the method of linear programming and allows the determination of an optimal distribution of commercial diameters for each pipe in the network and the length of the pipes which correspond to these diameters. Also, it is possible to take into account the various functional situations characteristic found during operation. This paper compares linear optimization model to the some others, such as the classic model of average economical velocities and Moshnin optimization model. This shows the good performance of the new model. For different analyzed networks, the saving of electrical energy, due to diminishing pressure losses and operation costs when applying the developed model, represents about 10…35 %.

*Key words*: Water supply, Distribution, Looped networks, Optimal design, Linear optimization model.

## 1. INTRODUCTION

Distribution networks are an essential part of all water supply systems. The reliability of supply is much greater in the case of looped networks. Distribution system costs within any water supply scheme may be equal to or greater than 60 % of the entire cost of the project. Also, the energy consumed in a distribution network supplied by pumping may exceed 60 % of the total energy consumption of the system.

Attempts should be made to reduce the cost and energy consumption of the distribution system through optimization in analysis and design. A water distribution network that includes bosster pumps mounted in the pipes, pressure reducing valves, and check-valves can be analyzed by several common methods such as Hardy-Cross, linear theory, and Newton-Raphson [20].

Traditionally, pipe diameters are chosen according to the average economical velocities (Hardy-Cross method) [5]. This procedure is cumbersome, uneconomical, and requires trials, seldom leading to an economical and technical optimum.

This paper develops a linear model for optimal design of new and partially extended distribution systems supplied by pumping or gravitation. It is based on linear programming and allows for the determination of optimal distribution of commercial diameters along the length of each pipe and the length of pipe sectors corresponding to these diameters. It is possible to take into account various functional situations characteristic found during operation and uniform outflows along the length of each pipe. This model can serve as guidelines to supplement existing procedures of network design.

## 2. BASIS OF HYDRAULIC CALCULATION

A distribution network may be represented by orientation comprising a finite number of arcs (pipes, pumps, fittings) and a set of nodes as well as reservoirs and pumps or pipe intersections.

In the case of a complex topology, with reservoirs and pumps at the nodes, the number of open–loops (pseudoloops) $N_{RP}$–1 is added to the number of closed-loops, so that the total number $M$ of independent loops is determined from the equation:

$$M = T - N + N_{RP}\,, \tag{1}$$

where: $T$ is the number of pipes; $N$ – number of nodes; $N_{RP}$ – number of pressure generating facilities.

The hydraulic calculation of a distribution networks involves in determining the diameters, discharges and head losses in pipes, in order to guarantee at each node the necessary discharge and pressure.

When performing the hydraulic calculation of a distribution network, the laws of water flow in all the pipes must be respected:

– discharge continuity at nodes:

$$f_j = \sum_{\substack{i=1\\ i\neq j}}^{N} Q_{ij} + q_j = 0 \;\; (j = 1,\ldots, N - N_{RP})\,, \tag{2}$$

in which: $f_j$ is the residual discharge at the node $j$; $Q_{ij}$ – discharge through pipe $ij$, with the sign (+) when entering node $j$ and (–) when leaving it; $q_j$ – consumption discharge (demand) at node $j$ with the sign (+) for node inflow and (–) for node outflow.

- energy conservation in loops:

$$\Delta h_m = \sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m = 0 \;\; (m = 1,\ldots, M)\,, \tag{3}$$

in which: $\Delta h_m$ is the residual head loss (divergence) in the loop $m$; $h_{ij}$ – head loss of the pipe $ij$; $\varepsilon_{ij}$ – orientation of flow through the pipe, having the values (+1) or (–1) as the water flow sense is the same or oposite to the path sense of the loop $m$, and (0) value if $ij \notin m$; $f_m$ – pressure head introduced by the potential elements of the loop $m$, given by the relations:

- simple closed-loops:

$$f_m = 0 \tag{4}$$

- closed-loops containing bosster pmps installed in the pipes:

$$f_m = \sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij} H_{p,ij}\;; \tag{5}$$

- open-loops with pumps and/or reservoirs at nodes:

$$f_m = Z_I - Z_E\,, \tag{6}$$

where: $Z_I$, $Z_E$ are piezometric heads at pressure devices at the entrance or exit from the loop; $H_{p,ij}$ – pumping head of the bosster pump integrated on the pipe $ij$, for the discharge $Q_{ij}$, approximated by parabolic interpolation on the pump curve given by points [13].

## 3. NETWORK DESIGN OPTIMIZATION CRITERIA

Optimization of distribution network diameters considers a mono- or multicriterial objective function. Cost or energy criteria may be used, simple or complex, which considers the network cost, pumping energy cost, operating expenses, included energy, consumed energy, total expenses etc. These criteria can be expressed in a complex objective multicriterial function [16], with the general form:

$$F_c = \xi_1 \sum_{ij=1}^{T} (a + bD_{ij}^{\alpha}) L_{ij} + \psi \sum_{j=1}^{NP} Q_{p,j} \left(\sum h_{ij} + H_0\right)_j\,, \tag{7}$$

in which:

$$r_a = \frac{(1+\beta_0)^t - 1}{\beta_0 (1+\beta_0)^t}, \tag{8}$$

$$\xi_1 = r_a p_1 + \frac{t}{T_r}; \quad \xi_2 = r_a p_2 + \frac{t}{T_r}, \tag{9}$$

$$\psi = \frac{9.81}{\eta}\left(f\,\sigma\xi_2 + 730 r_a e\,\tau \sum_1^{12} \Phi_k\right), \tag{10}$$

where: $T$ is the number of pipes in a network; $a, b, \alpha$ – cost/energy parameters depending on pipe material [14]; $D_{ij}$, $L_{ij}$ – diameter and length of pipe $ij$; $NP$ – number of pump stations; $Q_{p,j}$ – pumped discharge of pump station $j$; $\Sigma h_{ij}$ – sum of head losses along a path between the pump station and the critical node; $H_0$ – geodezic and utilization component of the pumping total dynamic head; $\beta_0 = 1/T_r$ – amortization part for the operation period $T_r$; $p_1$, $p_2$ – repair, maintenance and periodic testing part for network pipes and pump stations, respectively; $t$ – period for which the optimization criterion expressed by the objective function is applied, having the value 1 or $T_r$; $\eta$ – efficiency of pump station; $f$ – installation cost of unit power; $\sigma$ – a factor greater than one which takes into account the installed reserve power; $e$ – cost of electrical energy; $\tau = T_p/8{,}760$ – pumping coefficient, which takes into account the effective number $T_p$ of pumping hours per year; $\Phi_\kappa$ – ratio between the average monthly discharge and the pumped discharge [13].

For networks supplied by pumping, the literature [1, 4, 8, 21] suggests the use of *minimum an-nual total expenses criterion* (CAN), but choosing the optimal diameters obtained in this way, the networks become uneconomical at some time after construction, due to inflation.

Therefore, it is recommended the fore-mentioned criterion be subject to dynamization by using the *criterion of total updated minimum expenses* (CTA), the former being in fact a specific case of the latter when the investment is realised within a year; the operating expenses are the same from one year to another and the expected lifetime of the distribution system is high. In particular, the use of energetical criteria different from cost criteria is recommendable. Thus, another way to approach the problem, with has a better validity in time and the homogenization of the objective function is network design according to *minimum energetic consumption* (WT).

The general function (7) enables us to obtain a particular objective function by particularization of the time parameter $t$ and of the other economic and energetical parameters, characteristic of the distribution system. For example, from $t = 1$, $r_a = 1$, $e = 1$, $f = 0$ the minimum energetic consumption criterion is obtained.

## 4. DEVELOPMENT OF LINEAR OPTIMIZATION MODEL

For the evaluation of the energy disipated in pipes with variable discharge, a complex computational relation has been established by specialised studies [7]. Using following nondimensional parameters:

$$-\textit{for outflow}: \quad \theta_{ij} = \frac{Q_c}{Q_0}, \tag{11}$$

$$-\textit{for pipe}: \quad \omega_{ij} = \frac{\alpha_0 D_{ij}}{\lambda_{ij} L_{ij}}, \tag{12}$$

the expression of head loss $h_{ij}^*$, for pipes with uniform outflow along their length, take the form:

$$h_{ij}^* = h_{ij}\left(\Theta_{ij} - \frac{\Omega_{ij}}{L_{ij}}\right), \tag{13}$$

where:

$$\Theta_{ij} = \frac{4}{3}\frac{\theta_{ij}^2 - 3\theta_{ij} + 3}{(2-\theta_{ij})^2}, \tag{14}$$

$$\Omega_{k,ij} = \frac{4\alpha_0 D_{k,ij}}{\lambda_{k,ij}(2-\theta_{ij})}, \tag{15}$$

in which: $h_{ij}$ is the head loss for pipe $ij$ with concentrated outflow; $Q_c$ – outflow along the length of the pipe $ij$; $Q_0$ – inflow in the initial section of the pipe $ij$; $\alpha_0$ – nonuniformity coefficient of velocity distribution in the cross–sections of the pipe; $\lambda_{ij}$ – friction factor of pipe $ij$.

The total length of a pipe $ij$, with the discharge $Q_{ij}$, may be divided into $s_{ij}$ partial length (sector), $k$, of $D_{k,ij}$ diameters and $X_{k,ij}$ lengths. Taking into account the Darcy-Weisbach's functional equation, the friction slope $J_{k,ij}$ for each sector $k$ of the pipe $ij$ can be calculated, in the hypothesis of concentrated outflow, with the equation:

$$J_{k,ij} = \frac{h_{k,ij}}{X_{k,ij}} = \frac{8}{\pi^2 g}\lambda_{k,ij}\frac{Q_{ij}^2}{D_{k,ij}^r}, \tag{16}$$

where: $r$ is an exponent having the value 5.0; $g$ – gravitational acceleration; $\lambda_{k,ij}$ – friction factor of sector $k$ in pipe $ij$, can be calculated using the Colebrook-White formula, or the explicit equation proposed in [2] for the transitory turbulence flow.

Since in real conditions the discharge decreases from one cross–section to another in the sense of outflow, an increase of pressure is accomplished at the outlet of the pipe, by a phenomenon similar to rebound, which has as the effect of diminishing the head loss.

The expression of friction slope in each sector $k$ of the pipe $ij$, for the uniform outflow along the length of the pipe, is rewritten as:

$$J_{k,ij}^* = \frac{h_{k,ij}^*}{X_{k,ij}} = J_{k,ij}\left(\Theta_{ij} - \frac{\Omega_{k,ij}}{X_{k,ij}}\right). \tag{17}$$

For $\Theta_{ij}=1$ and $\Omega_{k,ij}=0$ the general equation (17) takes the particular form (16), valid for pipes with constant discharge. Specific consumption of energy for distribution of water $w_{sd}$, in kWh/m$^3$, is obtained by referring the hydraulic power dissipated in pipes to the sum of discharges:

$$w_{sd} = 0.00272\frac{\sum_{ij=1}^{T} R_{ij}\left|Q_{ij}\right|^{\beta+1}}{\sum_{\substack{j=1\\q<0}}^{N}\left|q'_j\right|}, \tag{18}$$

where $q'_j$ is the outflow at the node $j$.

Computation of the optimal design of looped networks must be performed in two stages:

– establishment of optimal distribution for discharges through pipes, $Q_{ij}$, according to the minimum bulk transport criterion [17], which takes into account the network reliability;

– computation of optimal pipes diameters, $D_{k,ij}$, taking into account the optimized discharges.

The series of commercial diameters which can be used $D_{k,ij} \in [D_{max,ij}, D_{min,ij}]$ for each pipe $ij$ are established using the limit values of optimal diameters $D_{max,ij}$ and $D_{min,ij}$, computed by optimization relation (19) for pumping operation networks or relation (20) for gravity networks:

$$D_{max(min),ij} = E_{max(min)}^{\frac{1}{\alpha+r}} Q_p^{\frac{1}{\alpha+r}} Q_{ij}^{\frac{\beta}{\alpha+r}}, \tag{19}$$

$$D_{max(min),ij} = \sqrt{\frac{4Q_{ij}}{\pi V_{min(max),ij}}}, \tag{20}$$

in which:

$$E = \frac{10.33\, n'^2 r\, \psi}{a b \xi_1}, \tag{21}$$

where: $Q_{ij}$ is the discharge of the pipe $ij$; $Q_p = \Sigma Q_{p,j}$ – pumped discharge; $V_{min}$, $V_{max}$ – limits of the economic velocities; $n'$ – Manning roughness coefficient of the pipes; $E$ – economy-energy factor of the pipes, which has a maximum value and a minimum value [14], corresponding to the limit values of the variation of economy-energy parameters ($p_1$, $p_2$, $\eta$, $f$, $\sigma$, $e$, $\tau$, $\Sigma\Phi_k$) for the distribution system, included in $\psi$ and $\xi_1$.

A penalty coefficient $p_{ij}$ is used when optimizing diameters in the case of extending a network, which has the value equal to the value of corresponding imposed diameter, for pipes with fixed diameters, resulting in $D_{k,ij} = p_{ij}$.

Admitting that a pipe $ij$ of length $L_{ij}$ of a pumping operation network made up of $T$ pipes, can be divided into $s_{ij}$ sectors $k$ of diameters $D_{k,ij}$ and lengths $X_{k,ij}$ and taking into account the notations:

$$c^*_{k,ij} = \xi_1 (a + b D^{\alpha}_{k,ij}), \tag{22}$$

$$Z_{IPP,j} = (\sum h_{ij} + H_0)_j, \tag{23}$$

the objective function (7) takes the form:

$$F_c = \sum_{ij=1}^{T} \sum_{k=1}^{s_{ij}} c^*_{k,ij} X_{k,ij} + \psi \sum_{j=1}^{NP} Q_{p,j} Z_{IPP,j} \to \min . \tag{24}$$

The unknowns of the objective function are variables $X_{k,ij}$ and $Z_{IPP,j}$, being $NP + \sum_{ij=1}^{T} s_{ij}$ in number.

When the pressure device is comprised of one or more reservoirs ($\psi = 0$), the expresion (24) of the objective function becomes:

$$F_c = \sum_{ij=1}^{T} \sum_{k=1}^{s_{ij}} c^*_{k,ij} X_{k,ij} \to \min, \tag{25}$$

minimizing the included energy or the network cost and having as unknowns the variables $X_{k,ij}$.

Hence, the values of the variables must be determined in order to minimize the objective function $F_c$, provided the following constraints are satisfied:

– *constructive constraints:*

$$\sum_{k=1}^{s_{ij}} X_{k,ij} = L_{ij} \quad (ij = 1,...,T); \tag{26}$$

– *functional constraints* which are written for each operating situation, and which must provide the necessary pressure $HN_o$ at the critical nodes, starting on different path from the pressure devices $IPP_j$ (Fig. 1):

$$Z_{IPP,j} - \sum_{ij=1}^{NT_j} \sum_{k=1}^{s_{ij}} \varepsilon_{ij} \Theta_{ij} J_{k,ij} X_{k,ij} \geq ZT_o + HN_o - \sum_{ij=1}^{NT_j} (\sum_{k=1}^{s_{ij}} \varepsilon_{ij} \Omega_{k,ij} J_{k,ij} + H_{p,ij}), \tag{27}$$

where: $NT_j$ is the pipes number of a path $IPP_j$ – O; $ZT_o$ – elevation head at the critical node O; $Z_{IPP,j}$ – available piezometric head at the pressure device $j$; $H_{p,ij}$ – pumping head of the bosster pump mounted in the pipe $ij$;

– *hydraulic constraints* characteristic only for looped networks, expressing the energy conserva-tion in loops:

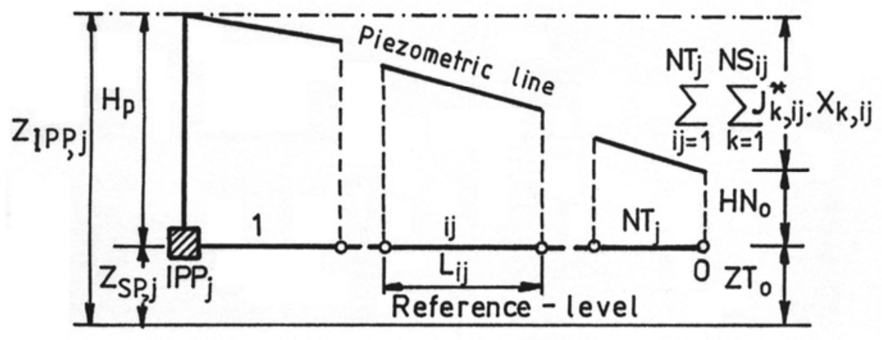


Fig. 1 – Scheme of a path $IPP_j$ – critical node O.

$$\sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij}\,\Theta_{ij}\,J_{k,ij}\,X_{k,ij} = \sum_{\substack{ij\in m\\ m=1}}^{T} \varepsilon_{ij}\,\Omega_{k,ij}\,J_{k,ij} + f_m \qquad (m=1,\ldots,M), \tag{28}$$

in which $\varepsilon_{ij}$ is the orientation of the pipes and the pressure head $f_m$ is given by the relations (4), (5) and (6).

In the case that the available piezometric heads $Z_{IPP,j}$ are known, and it being unnecessary to determine them by optimization, the objective function (24) takes the form (25), while values $Z_{IPP,j}$ are contained in the free term of constraints (27) and (28). As the objective function (24) or (25) and constraints (26), (27), (28) are linear with respect to the unknowns of system the optimal solution is determined according to the linear programming method, using the Simplex algorythm.

Computing the unknowns $Z_{IPP,j}$ by optimization, for pumping operation networks results in the corresponding pumping head:

$$H_{p,j} = Z_{IPP,j} - Z_{SP,j}, \tag{29}$$

where $Z_{SP,j}$ is the water level in the suction basin of $IPP_j$.

Taking into account head loss $H_{IPP,j-n}$ on the path $IPP_j - n$:

$$H_{IPP,j-n} = \sum_{ij=1}^{NT_j}\sum_{k=1}^{s_{ij}} \Theta_{ij}\,J_{k,ij}\,X_{k,ij} - \sum_{ij=1}^{NT_j}\left(\sum_{k=1}^{s_{ij}} \Omega_{k,ij}\,J_{k,ij} + H_{p,ij}\right), \tag{30}$$

the piezometric head $Z_n$ and the residual pressure head $H_n$ at the node $n$ are determined from the relations:

$$Z_n = Z_{IPP,j} - H_{IPP,j-n}, \tag{31}$$

$$H_n = Z_n - ZT_n, \tag{32}$$

where $ZT_n$ is the elevation head at the node $n$.

For an optimal design, the piezometric line of a path of $NT_j$ pipes, situated in the same pressure zone, must represent a polygonal line which resemble as closely as possible the optimal form expressed by the equation:

$$Z_n = Z_{IPP,j} - \left[1 - \left(1 - \frac{d}{\sum_{ij=1}^{NT_j} L_{ij}}\right)^{\frac{\beta\,\alpha}{\alpha+r}+1}\right] \sum_{ij=1}^{NT_j} h_{ij}, \tag{33}$$

in which: $Z_n$ is the piezometric head at the node $n$; $d$ – distance between node $n$ and the pressure device $j$.

The computer program OPLIRA has been elaborated based on the linear optimization model, in the FORTRAN programming language for IBM-PC compatible microsystems.

## 5. NUMERICAL APPLICATION

The looped distribution network with the topology from Fig. 2 is considered. It is made of cast iron and is supplied by pumping with a discharge of 0.23 $m^3/s$. The following data is known: pipes length $L_{ij}$, in m, elevation head $ZT_j$, in m, and necessary pressure $HN_j$ = 24 m $H_2O$.

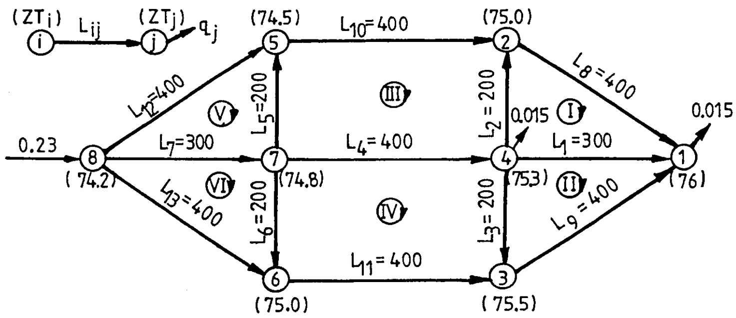

Fig. 2 – Scheme of the designed distribution network.

A comparative study of network dimensioning is performed using the classic model of average economical velocities (MVE), Moshnin optimization model (MOM) [1] and the linear optimization model (MOL) developed above, the last being applied in the hypothesis of concentrated outflow (MOL–N), as well as of uniform outflow along the lenght of the pipes (MOL–D).

Calculus was performed considering a transi-tory turbulence regime of water flow and the optimization criterion used was that of minimum energetic consumption. Results of the numerical solution performed by means of an IBM–PC computer, referring to the hydraulic characteristics of the pipes are presented in Tables 1 and 2.

*Table 1*

Hydraulic characteristics of the pipes determined with the models MVE and MOM

| Pipe *i - j* | *L* [m] | MVE | | | | MOM | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $Q_{ij}$ [$m^3/s$] | $D_{ij}$ [mm] | $h_{ij}$ [m] | $V_{ij}$ [m/s] | $Q_{ij}$ [$m^3/s$] | $D_{ij}$ [mm] | $h_{ij}$ [m] | $V_{ij}$ [m/s] |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4-1 | 300 | 0.00782 | 100 | 4.009 | 1.00 | 0.00786 | 150 | 0.510 | 0.45 |
| 4-2 | 200 | 0.00512 | 100 | 1.177 | 0.65 | -0.00174 | 100 | -0.154 | 0.22 |
| 4-3 | 200 | 0.00512 | 100 | 1.177 | 0.65 | -0.00174 | 100 | -0.154 | 0.22 |
| 7-4 | 400 | 0.05924 | 300 | 0.963 | 0.84 | 0.04557 | 250 | 1.473 | 0.93 |
| 7-5 | 200 | 0.00517 | 100 | 1.199 | 0.66 | 0.00097 | 100 | 0.052 | 0.15 |
| 7-6 | 200 | 0.00517 | 100 | 1.199 | 0.66 | 0.00097 | 100 | 0.052 | 0.15 |
| 8-7 | 300 | 0.09576 | 350 | 0.833 | 1.00 | 0.07370 | 300 | 1.104 | 1.04 |
| 2-1 | 400 | 0.01669 | 150 | 2.886 | 0.94 | 0.01666 | 200 | 0.662 | 0.53 |
| 3-1 | 400 | 0.01669 | 150 | 2.886 | 0.94 | 0.01666 | 200 | 0.662 | 0.53 |
| 5-2 | 400 | 0.03538 | 250 | 0.902 | 0.72 | 0.04221 | 250 | 1.270 | 0.86 |
| 6-3 | 400 | 0.03538 | 250 | 0.902 | 0.72 | 0.04221 | 250 | 1.270 | 0.86 |
| 8-5 | 400 | 0.05403 | 250 | 2.051 | 1.10 | 0.06506 | 300 | 1.155 | 0.92 |
| 8-6 | 400 | 0.05403 | 250 | 2.051 | 1.10 | 0.06506 | 300 | 1.155 | 0.92 |

*Table 2*

Hydraulic characteristics of the pipes determined with the models MOL–N and MOL–D

| Pipe | MOL-N | | | | | | MOL-D | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *i - j* | $Q_{ij}$ [m$^3$/s] | *k* | $X_{k,ij}$ [m] | $D_{k,ij}$ [mm] | $h_{k,ij}$ [m] | $V_{k,ij}$ [m/s] | $Q_{ij}$ [m$^3$/s] | *k* | $X_{k,ij}$ [m] | $D_{k,ij}$ [mm] | $h_{k,ij}$ [m] | $V_{k,ij}$ [m/s] |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4-1 | 0.01462 | 1 | 107 | 250 | 0.045 | 0.30 | 0.01458 | 1 | 254 | 250 | 0.226 | 0.30 |
| | | 2 | 193 | 200 | 0.249 | 0.47 | | 2 | 46 | 200 | 0.070 | 0.46 |
| 4-2 | -0.00484 | 1 | 200 | 150 | -0.136 | 0.30 | -0.00501 | 1 | 200 | 150 | -0.176 | 0.28 |
| 4-3 | -0.00484 | 1 | 200 | 150 | -0.136 | 0.30 | -0.00500 | 1 | 200 | 150 | -0.176 | 0.28 |
| 7-4 | 0.04612 | 1 | 400 | 250 | 1.508 | 0.94 | 0.04575 | 1 | 400 | 250 | 1.442 | 0.93 |
| 7-5 | 0.00380 | 1 | 200 | 125 | 0.224 | 0.32 | 0.00383 | 1 | 200 | 150 | 0.108 | 0.25 |
| 7-6 | 0.00380 | 1 | 200 | 125 | 0.224 | 0.32 | 0,00383 | 1 | 200 | 150 | 0.108 | 0.25 |
| 8-7 | 0.08007 | 1 | 200 | 350 | 0.394 | 0.83 | 0.07962 | 1 | 82 | 350 | 0.075 | 0.82 |
| | | 2 | 100 | 300 | 0.430 | 1.13 | | 2 | 218 | 300 | 0.782 | 1.12 |
| 2-1 | 0.01329 | 1 | 400 | 200 | 0.429 | 0.42 | 0.01331 | 1 | 400 | 200 | 0.470 | 0.42 |
| 3-1 | 0.01329 | 1 | 400 | 200 | 0.429 | 0.42 | 0.01331 | 1 | 400 | 200 | 0.470 | 0.42 |
| 5-2 | 0.04194 | 1 | 56 | 300 | 0.069 | 0.59 | 0.04212 | 1 | 400 | 250 | 1.159 | 0.86 |
| | | 2 | 344 | 250 | 1.078 | 0.85 | | | | | | |
| 6-3 | 0.04194 | 1 | 56 | 300 | 0.069 | 0.59 | 0.04212 | 1 | 400 | 250 | 1.159 | 0.86 |
| | | 2 | 344 | 250 | 1.078 | 0.85 | | | | | | |
| 8-5 | 0.06187 | 1 | 400 | 300 | 1.048 | 0.88 | 0.06210 | 1 | 400 | 300 | 0.964 | 0.88 |
| 8-6 | 0.06187 | 1 | 400 | 300 | 1.048 | 0.88 | 0.06210 | 1 | 400 | 300 | 0.964 | 0.88 |

The significance of (–) sign of discharges and head losses in Tables 1 and 2 is the change of flow sense in the respective pipes with respect to the initial sense considered in the Fig. 2.

In Fig. 3 there is a graphic representation, starting from the node source 8 to the control node 1, on the path 8–5–2–1, the piezometric lines being obtained using the three mentioned models of computation, evidencing their deviation from the optimal theoretical form. Figure 3 also includes the corresponding values of the objective function $F_c$, the network included energy $W_c$, pumping energy $W_e$, as well as specific energy consumption for water distribution $w_{sd}$.

According to the performed study it was established that:

– all the pipes of the network are operating in a transitory turbulence regime of water flow;

– there is a general increase of pipes diameters obtained by optimization models (MOM, MOL) with respect to MVE, because the classical model does not take into account the minimum consumption of energy and the diversity of economical parameters;

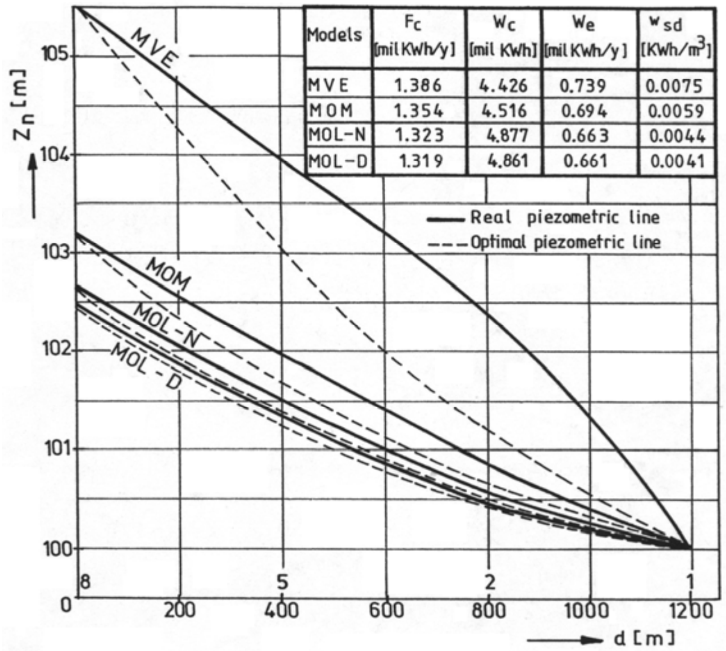


Fig. 3 – Piezometric lines along the path 8–5–2–1.

– in comparison with the results obtained by MVE, the ones obtained by optimization models are more economical, a substantial reduction of specific energy consumption for water distribution is achieved (MOM – 21.3 %, MOL-N – 41.3 %, MOL-D – 45.3%) as well as a reduction of pumping energy (MOM – 6,4%, MOL-N – 10.3%, MOL-D – 10.6%), at the same time the objective function has also smaller values (MOM – 2.3 %, MOL-N – 4.5 %, MOL-D – 4.8 %);

– the optimal results obtained using MOL are superior energetically to those offered by MOM, lea-ding to pumping energy savings of 5 %;

– also, the application of MOL for uniform outflow along the lenght of the pipes, has led to the minimum deviation from the optimal form of the pie-zometric line, especially to a more uniform distribution of the pumping energy, by elimination of a high level of available pressure at some nodes even at maximum consumption. The smallest value of the specific energetic consumption, namely that of 0.0041 kWh/$m^3$, also supports this assertion;

– reduction of the pressure in the distribution network achieved in this way, is of major practical import, contributing to the diminishing of water losses from the system.

## 6. CONCLUSIONS

The mathematical programming, as a fundamental procedure for optimizing the structures in general, together with graph theory and the increasing implication of computers in solving mathematical formulations have created conditions for solving efficiently some optimization problems of design of water distribution networks. The different types of programming which exist (linear, nonlinear, whole, geometric etc.) provide multiple possibilities for solving specific problems.

The proposed optimization model, a very general and practical one, offers the possibility of optimal design of water supply networks using multiple optimization criteria and considers the transitory or quadratic turbulence flow. It has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, and other criteria can be expressed by simple options in the objective function (7).

The model of linear optimization could be applied to any looped or tree–shaped network, either when piezometric heads at pressure devices (pump stations or tanks) must be determined or when these heads are given. It permits the determination of an optimal distribution of comercial diameters along the length of each pipe of the network and the length of pipe sectors corresponding to these diameters. Also, this facilitates the consideration of uniform outflow along the length of the pipes network. A more uniform distribution of pumping energy is achieved so that head losses and parameters of pump stations can be determined more precisely.

For different analized networks, the saving of electrical energy due to diminishing pressure losses and operation costs when applying the model of linear optimization represents about 10...35 %, which is of great importance, considering the general energy issues.

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