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MULTIPLE RESONANCES OF THE SITE OSCILLATING SYSTEMS

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The object of this paper is to estimate the resonance behavior of the oscillating systems with dynamic degradation materials like soils. Using a nonlinear Kelvin-Voigt model (NKV model) with material functions in terms of the strain or stress level, by numerical simulation, one puts into evidence the resonant frequency dispersion and the consequences of this nonlinear behavior upon resonant avoidance strategy.

Key words: Resonance of nonlinear systems; Dynamic degradation; Soils dynamic response.

1. INTRODUCTION

Resonance is a well-known concept in linear system analysis. At a resonance, the frequency of an exciting force reaches the natural frequency of the system so that the amplitude of vibration can become significant even for small dynamic imput [11, 18].

The resonance may cause violent swaying motions and even “resonance disaster”, a catastrophic failure in improperly constructed structures. Therefore, the operating resonant condition must to be avoided. But, for the structural-site oscillating systems the resonance avoidance need a correct evaluation both natural frequencies and the oscillating frequencies of expected ground motion. However, in civil engineering all soil - structure oscillating systems have nonlinear components because of nonlinear behavior of the site materials, which cannot simply be described by a linear model [5, 6, 8, 9].

The resonances of the oscillating systems with nonlinear materials have some specific behavior which can modify the linear “natural frequency” concept [1, 15]. For this reason, the nonlinear properties of the site materials must be included into analysis model.

Assuming that the geological site materials are nonlinear viscoelastic materials in the previous author's papers [5, 6, 7] this nonlinear behavior was modeled with the aid of the nonlinear Kelvin-Voigt model (NKV model). This model describes the nonlinearity by the dependence on the material mechanical parameters: shear modulus G and damping ratio ζ in terms of shear strain invariant γ: $G = G(\gamma)$, $\zeta = \zeta(\gamma)$ and accordingly, all of the dynamic characteristics of the oscillating system acquire strain dependence.

It is experimentally observed that when the external loads are increasing, the soils rigidity diminishes, because of the dynamic degradation effect [3, 4] and the material damping increase [5, 6, 7]. Thereby, the modulus function $G = G(\gamma)$ is a decreasing function and damping function $\zeta = \zeta(\gamma)$ is an increasing function. These contradictory material evolutions have contradictory effects on dynamic structural response, including the resonance behavior evaluation.

The NKV model is appropriate to evaluate the resonance behavior of the soil-structure system under these simultaneous and opposite material effects, because this model includes both nonlinear material tendencies and can offer a relevant description of the nonlinear response as result of both nonlinear phenomenon – decreasing rigidity and increasing damping. As one can see in the next paragraphs, the nonlinear magnification functions of the NKV model are proper tools for the qualitative and quantitative description of the nonlinear resonance.

2. RESONANCE OF THE LINEAR SYSTEMS

For a qualitative evaluation of the linear resonance, one can consider a linear single degree of freedom system (sdof system) subjected to harmonic abutment acceleration:

$$\ddot{x}_g(t) = \ddot{x}_g^0 \sin \omega t, \tag{2.1}$$

where $\ddot{x}_g^0$ is the acceleration amplitude (usually connected with peak ground acceleration – *PGA* [6]) and ω is the pulsation of the excitation.

In linear dynamics, a usual description of such sdof system behavior is given by the Kelvin-Voigt model consisting of a mass *m* supported by a spring (with a stiffness *k*) and a dashpot (with a viscosity *c*) connected in parallel. The governing equation of this system is [18]:

$$m\ddot{x} + c \cdot \dot{x} + k \cdot x = -m\ddot{x}_g^0 \tag{2.2}$$

or:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = -\omega_0^2 \ddot{x}_g^0 , \tag{2.3}$$

where ω_0 is undamped natural pulsation and ζ is the damping ratio:

$$\omega_0 = \sqrt{\frac{k}{m}} \quad ; \quad \zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_0}. \tag{2.4}$$

By using the change of variable $\tau = \omega_0 t$ and by introducing a new "time" function $\varphi(\tau) = x(t) = x(\tau/\omega_0)$ [7] one obtains from eq. (2.3) a dimensionless form of the equation of motion:

$$\varphi'' + C\varphi' + K\varphi = \mu \sin \upsilon\tau, \tag{2.5}$$

where the superscript accent denotes the time derivative with respect to τ and:

$$C = \frac{c}{m\omega_0} = 2\zeta \quad ; \quad K = \frac{k}{m\omega_0^2} = 1 \quad ; \quad \mu = \frac{\ddot{x}_g^0}{\omega_0^2} \quad ; \quad \upsilon = \frac{\omega}{\omega_0}. \tag{2.6}$$

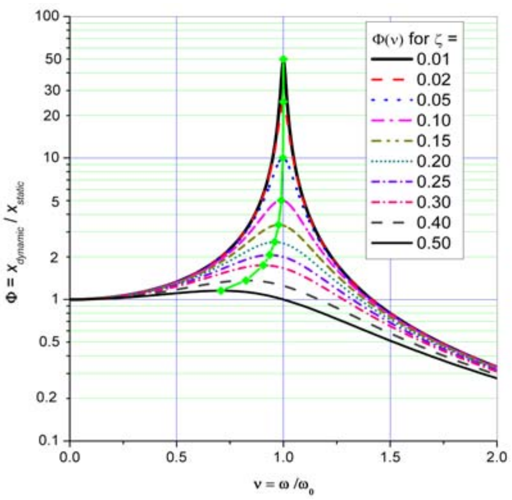


Fig. 2.1 – Linear magnification functions.

The steady-state solution of the equation (2.5) reads as:

$$\varphi(\tau,\upsilon,\zeta) = \mu\Phi(\upsilon,\zeta)\sin(\upsilon\tau - \psi), \tag{2.7}$$

where $\Phi(\upsilon,\zeta)$ is *the magnification function* (Fig. 2.1):

$$\Phi(\upsilon;\zeta) = \frac{\max\limits_{\tau}\left[\varphi(\tau,\upsilon,\zeta)\right]}{\mu} = \frac{x_{dynamic}}{x_{static}}. \tag{2.8}$$

As one can see from Fig. 2.1, the dynamic magnification functions have a maximum value at $\omega \simeq \omega_0$ (for usual small damping), that is at resonance when the input frequency reaches (or is in close proximity) the system natural frequency.

Also, from the same Fig. 2.1 one can see the decreasing effect on dynamic magnification due to the increasing system damping.

3. DYNAMIC DEGRADATION EFFECT

Considering soils as nonlinear viscoelastic materials, the material response under torsional loading is characterized by nonlinear torsional modulus-function $G = G(\gamma)$ [5, 6]. A value of this function is, by definition, given by the ratio of the amplitudes of the invariants τ and γ, where:

$$\tau = \sqrt{[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]/3} \quad ; \quad \gamma = \sqrt{[(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2]/3}\,. \tag{3.1}$$

The values of the function G differ from one cycle to another and this variation can be an experimental evidence in the resonant column apparatus, the dynamic or classical triaxial devices, shear or torsional devices, a.s.o. [3, 4]. Thus, in a strain-controlled test under constant amplitude γ it is found that the amplitudes τ are reduced and in the experiments with controlled-stress under constant amplitudes τ the amplitudes γ were found to become larger. Hence, the cyclic loading induces in both cases a reduction of the sample stiffness, i.e. a ***degradation*** of its mechanical-strength properties.

A measure of the degradation after n cycles is the ratio between the value of the modulus-function G at the cycle n and this initial value. As this ratio depends on both the number and amplitudes of cycles, it was called ***degradation function***: $d = d(\tau, n)$ in the stress-controlled tests or $d = d(\gamma, n)$ in strain-controlled test [1].

One can use the normalized values of the invariants τ and γ with respect to their statically failure values: $r = \tau/\tau_f$ or $r = \gamma/\gamma_f$, such that the degradation function may be written as:

$$d(r,n) = G(r,n)/G_0\,. \tag{3.2}$$

The degradation function (3.2) can be determined using either strain or stress-controlled tests, depending on the available device. As an example, in Fig. 3.1 are given such determination using the triaxial strain-controlled tests performed on clay samples [4]. After the statistical processing of resulting data the following form of the degradation function was obtained (Fig. 3.2):

$$d(r,n) = \frac{a(r)+0.5n}{1+b(r)\cdot n}, \quad \text{with} \quad \begin{aligned} a(r) &= 0.9+0.1\cdot\exp(5.97r) \\ b(r) &= 0.45+0.05\cdot\exp(5.73r). \end{aligned} \tag{3.3}$$

Depending on the amplitude and number of loading cycles, the degradation may increase until failure. Hence, the failure in dynamic conditions can be defined as the minimum of the degradation function [4]:

$$d_f = \min\left[d(r,n)\right] = G_f/G_0\,. \tag{3.4}$$

The minimum values of the degradation function describe in the space (d, r, n) a spatial curve $r = r(n)$ obtained by intersecting the surface $d = d(r,n)$, with the plane $d = d_f$ (Fig.3.2).

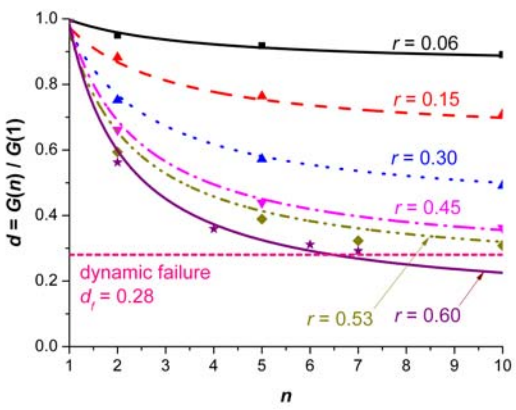


Fig. 3.1 – Some degradation functions for clay.

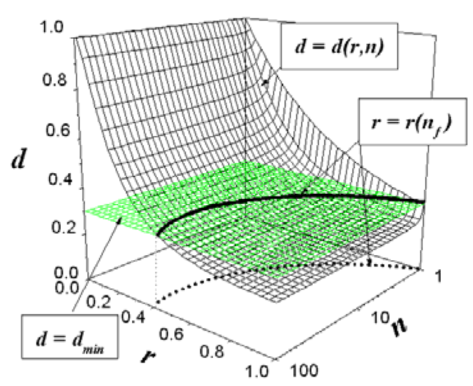


Fig. 3.2 – Spatial diagram of the degradation function.

Dynamic degradation of the site materials lead to a decreasing rigidity in terms of number and amplitude of external cyclic loading. The rigidity behavior can be quantified by dynamic degradation function (3.2) or by dynamic modulus function in normalized form $G_n(\gamma)=G(\gamma)/G_0$, both decreasing functions in terms of strain invariant γ. [1], [2]. We can remark that for a certain cycle *n* the degradation function is reduced to normalized modulus function, first in terms of normalized strain *r* and then in terms of strain γ:

$$d(r,n)\Big|_{n=ct.}=\frac{G(r)}{G_0}=G_n(r)\ \xrightarrow{r=\gamma/\gamma_f}\ G_n(\gamma). \tag{3.5}$$

It was experimentally observed that the decreasing rigidity is accompanied by an important increasing of the internal dissipated energy [5, 6, 8, 12]. This phenomenon can be modeled by the damping function, an increasing function in terms of strain. Thus, for the same material (clay) used for degradation evaluation (3.3) the degradation function was obtained from resonant column test in the form:

$$\zeta(\gamma)=0.153-0.134\exp(-7.965\gamma). \tag{3.6}$$

4. RESONANCE OF NONLINEAR SYSTEMS

Using nonlinear dynamic material functions $G_n=G_n(x)$ and $\zeta=\zeta(x)$, from eqs. (3.3) and (3.6), the differential equation of the nonlinear sdof system in harmonic abutment conditions (Fig. 4.1) can be written as an extension of eq. (2.2) [6, 7]:

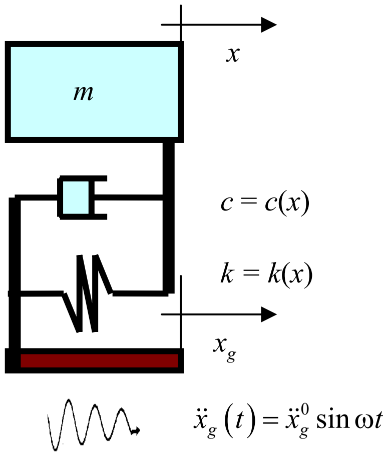


Fig. 4.1 – NKV model with abutment excitation.

$$\ddot{x}+2\omega_0\zeta(x)\cdot\dot{x}+\omega_0^2k_n(x)\cdot x=-\ddot{x}_g^0\sin\omega t \tag{4.1}$$

or in the normalized form:

$$\varphi''+C(\varphi)\cdot\varphi'+K(\varphi)\cdot\varphi=\mu\sin\upsilon\tau, \tag{4.2}$$

where

$$\begin{aligned}C(\varphi)&\equiv C(x)=\frac{c(x)}{m\omega_0}=2\zeta(x),\\ K(\varphi)&\equiv K(x)=\frac{k(x)}{m\omega_0^2}=\frac{k(x)}{k(0)}=k_n(x),\\ \mu&=-\frac{\ddot{x}_g^0}{\omega_0^2}\quad;\quad \upsilon=\frac{\omega}{\omega_0}.\end{aligned} \tag{4.3}$$

The solution of the equation (4.2) can be given a similar form as the solution of eq. (2.7):

$$\varphi(\tau,\upsilon,\mu;\zeta)=\mu\Phi(\upsilon,\mu;\zeta)\sin(\upsilon\tau-\psi), \tag{4.4}$$

where the magnification function Φ has in this nonlinear case the same form:

$$\Phi(\upsilon,\mu;\zeta)=\frac{\max_\tau\left[\varphi(\tau,\upsilon,\mu;\zeta)\right]}{\mu}=\frac{x_{dynamic}}{x_{static}}. \tag{4.5}$$

but becomes a load dependent function by normalized load μ. Therefore, in this nonlinear case there is for each μ a magnification function Φ.

The resonant frequency of the linear or nonlinear system is defined as the frequency at which an extreme (maximum value) of the magnification function occurs [1, 11, 15, 18]. Therefore, a nonlinear oscillating system has multiple resonant frequencies. The location of these extremes is a difficult problem. It requires the solution of the nonlinear ordinary differential eq. (4.1). Unfortunately, there are no analytical approaches that are able to obtain this solution, in general. But, for a given amplitude μ and a given cycle *n* the nonlinear equation (4.2) can be numerically solved using a computer program based on Newmark algorithm [10, 13, 16].

5. NUMERICAL SIMULATION

As an example, in this paper the resonance behavior of the oscillating systems with degradation materials was investigated with the aid of the numerical simulation using the NKV model. For this simulation was used the following material functions – the degradation function (3.3) and the damping function (3.6).

The excitation used in this simulation process is of harmonic abutment type $\ddot{x}_g = \mu \sin \upsilon\tau$ with the normalized amplitude values μ corresponding to several strain values *r* (0.06, 0.15, 0.30, 0.45) using the relationship $\mu = 0.045r$ obtained from resonant column test result. Also, the damping values ζ (0.05, 0.09, 0.15, 0.20) at these strain levels *r* were obtained from eq. (3.6) by introducing $\gamma = r \cdot \gamma_f$, where $\gamma_f = 0.01$.

The simulation results are given in Figs. 5.1, 5.2, 5.3 and 5.4. As one can see from these figures, after each cycle the external loading meets another material with other dynamic properties and with another dynamic response. Also, one can observe some features of the nonlinear behavior, namely that the peak amplitude of the nonlinear magnification functions depends on the strain level *r* and damping ratio ζ and the resonance peaks occur at different normalized pulsation υ situated before the linear resonant pulsation (usually soils have a softening nonlinearity type). The dependence of the maximum peak amplitude $\Phi_{\max}$ in terms of normalized pulsation υ: $\Phi_{\max} = \Phi_{\max}(\upsilon)$ is illustrated in these figures under "resonant peaks locus" denomination.

These simulation results make apparent both different effects – degradation and damping – on resonant behavior of the nonlinear oscillating systems. The material degradation leads to a weak increasing of resonant peak values together with an important frequency dispersion of these maximum values. However, the increasing of resonant peak values is counteracted by the strong decreasing of these values due to the material damping.

To make more obvious the dynamic magnification as a result of the degradation in Figs. 5.5, 5.6, 5.7 and 5.8 is given a reorganization of the simulation results by illustration of the magnification function variation in terms of number of cycles under constant strain and damping. One can remark from these figures that the degradation leads to dynamic amplification but fortunately this increase is covered by damping effect.

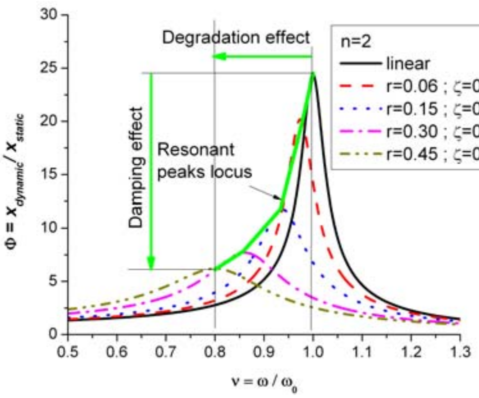


Fig. 5.1 – Magnification functions for second cycle.

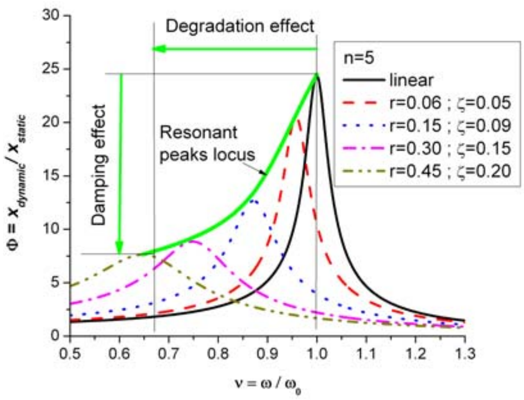


Fig. 5.2 – Magnification functions for the 5th cycle.

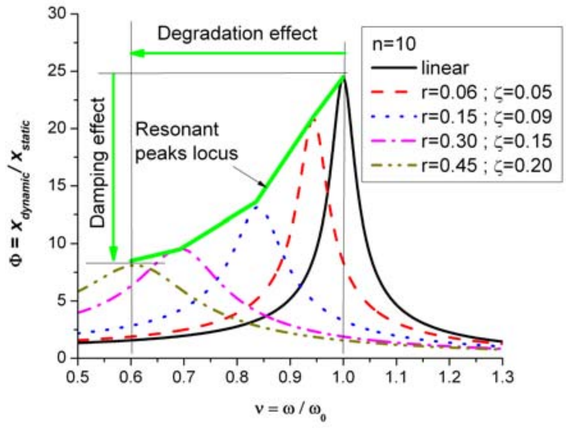


Fig. 5.3 – Magnification functions for the 10th cycle.

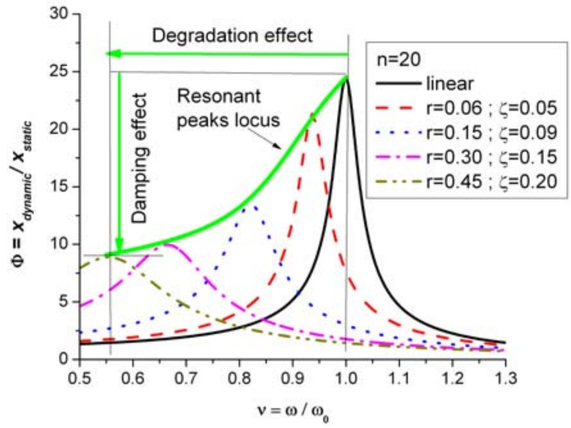


Fig. 5.4 – Magnification functions for the 20th cycle.

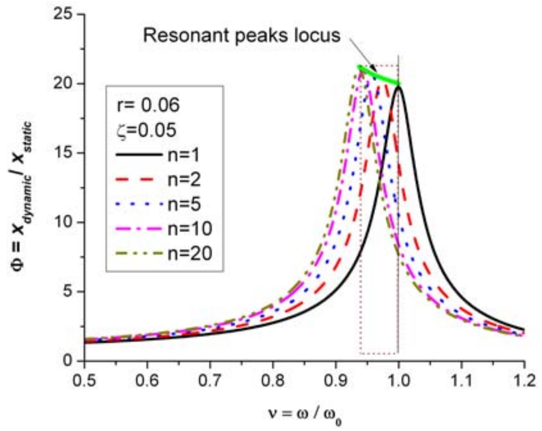


Fig. 5.5 – Magnification functions for different number of cycles under $r = 0.06$ and $\zeta = 0.05$.

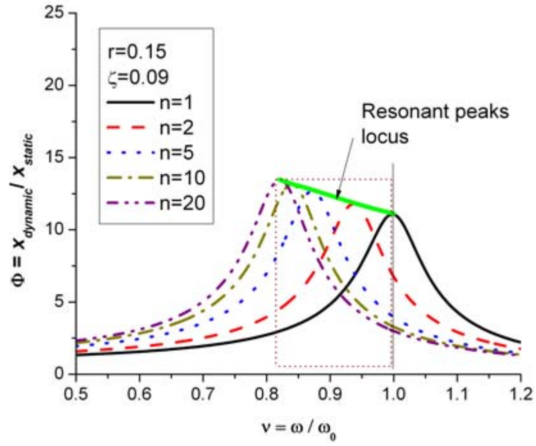


Fig. 5.6 – Magnification functions for different number of cycles under $r = 0.15$ and $\zeta = 0.09$.

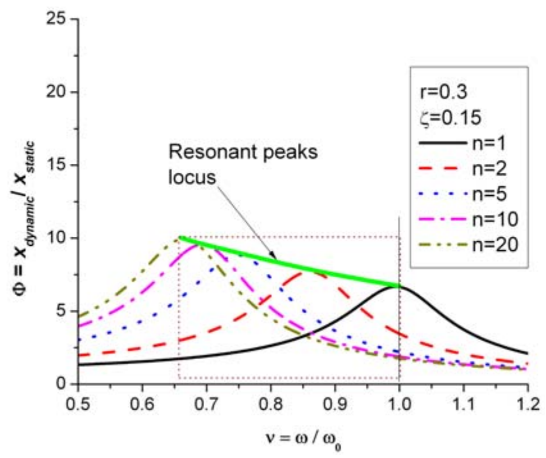


Fig. 5.7 – Magnification functions for different number of cycles under $r = 0.3$ and $\zeta = 0.15$.

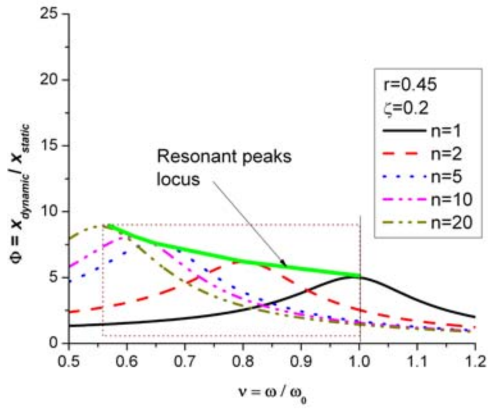
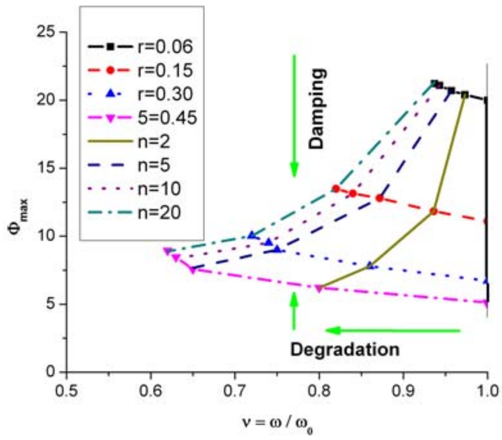
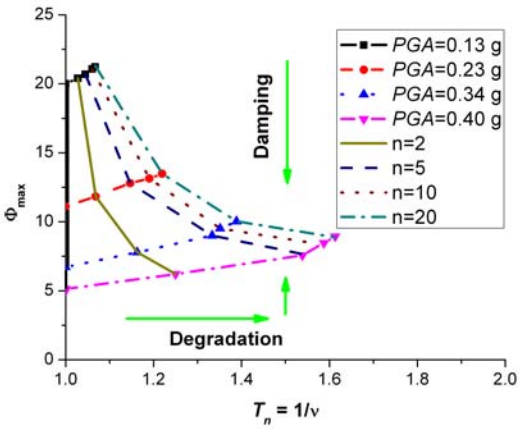


Fig. 5.8 – Magnification functions for different number of cycles under $r = 0.45$ and $\zeta = 0.2$.

As a synthesis, in Fig. 5.9 a and b is presented the evolution of the magnification function peak values Φ_{max} under both degradation and damping effects, in Fig.5.9a in terms of normalized pulsation υ and in Fig. 5.9b in terms of normalized natural period $T_n = T_0^{nonlinear} / T_0^{linear} = 1/\upsilon$ [9]. Also, in Fig. 5.9b the strain levels r_i was replaced by peak ground acceleration (*PGA*) which can generate these strain levels using the relationship between r and *PGA* obtained from resonant column test: $PGA = 0.458 - 0.432\exp(-4.32r)$.



a $\Phi_{\max} = \Phi_{\max}(\upsilon)$



b $\Phi_{\max} = \Phi_{\max}(T_n)$

Fig. 5.9 – Resonant peak values.

One can observe that as result of increasing loadings the degradation + damping combined effects lead to the reduction of the dynamic magnification peaks together with the frequency dispersion of these maximum values.

Certainly, the reduction of the dynamic amplification due to material damping is a favorable effect. But the frequency dispersion of the maximum resonance values can affect the frequency resonance evaluation for a nonlinear system. The frequency dispersion, visible in all presented figures, proves that the nonlinear oscillating system has not a unique resonant frequency and thereby has not a unique natural period [9].

Accordingly, due to nonlinear behavior of the site materials all structural-site systems have multiple resonant frequencies in terms of loading level. This real nonlinear behavior must modify the linear strategy of the resonant avoidance because the period dispersion can take important values. For example, as can see from Fig. 5.9b a strong earthquake with $PGA = 0.3 \div 0.4$ g can enlarge the linear estimation of the natural period with $40 \div 60\%$.

6. CONCLUDING REMARKS

The above results allow us to make the following remarks:

- Majority of the linear oscillating systems can pass over resonant conditions without material damages. Not the same affirmation can be making regarding to the nonlinear oscillating systems that contain the materials with dynamic characteristics depending on loading levels. In resonant conditions, these nonlinear materials undergo important modifications of their dynamic properties.
- When the external loads are increasing, the rigidity of the nonlinear materials is reduced, due to the dynamic degradation effect, and the material damping increases. These contradictory material evolutions have contradictory effects on resonance behavior.
- The NKV model is appropriate to evaluate the resonance behavior of the nonlinear system under simultaneous and opposite nonlinear material effects – decreasing rigidity and increasing damping, because this model is built up upon two nonlinear material functions – one for material strength modeling and the other including material damping.
- The nonlinear magnification functions of the NKV model are proper tools for the qualitative and quantitative description of the nonlinear resonance.
- Whereas the linear oscillating systems have a unique resonance value, the nonlinear oscillating systems have multiple resonant values in terms of excitation amplitudes. The nonlinear resonance peaks occurs at different normalized pulsation υ situated before the excitation pulsation (frequency dispersion) and under linear resonant value.

• These multiple resonant peaks may generate some difficulties for the resonant frequency or natural period evaluation of the structural-site system. This real nonlinear behavior must modify the linear strategy of the resonant avoidance because the period dispersion can take important values.

• Because whatever structural-site system is a nonlinear oscillating system there are no unique "natural periods" of a certain building, irrespective of his emplacement. Thus, the natural period of the structural-site system is a function of excitation level.

• Neglecting this nonlinear aspect by using only a linear natural period value, the resonance avoidance strategy may be compromised.

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REFERENCES

1. ALEXANDER Nicholas-Andrew, *Estimating the nonlinear resonant frequency of a single pile in nonlinear soil,* Journal of Sound and Vibration, **329**, pp. 1137–1153, 2010.
2. BĂLAN Stefan, CRISTESCU Valeriu, CORNEA Ion (Editors), *Romanian earthquakes from March 4, 1977*(in Romanian), Publishing House of the Romanian Academy, 1982.
3. BRATOSIN D., *On Dynamic Failure of Soils*, Rev. Roum. Sci. Techn.–Méc. Appl., **1**, 1989.
4. BRATOSIN Dinu, *Soils fatigue modeling by dynamic degradation effect,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **2**, *3*, pp. 101–104, 2001.
5. BRATOSIN Dinu, *Soil dynamics elements* (in Romanian), Publishing House of the Romanian Academy, 2002.
6. BRATOSIN Dinu, A *dynamic constitutive law for soils,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **3**, *1-2*, pp. 37–44, 2002.
7. BRATOSIN Dinu, SIRETEANU Tudor, *Hysteretic damping modeling by nonlinear Kelvin-Voigt model,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **3**, pp. 99–104, 2002.
8. BRATOSIN Dinu, *Non-linear attenuation in soils and rocks,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **7**, *3*, pp. 209–214, 2006.
9. BRATOSIN Dinu, BĂLAN Florin-Ştefan, CIOFLAN Carmen-Ortanza, S*oils nonlinearity effects on dominant site period evaluation,* Proceedings of the Romanian Academy – Series A: Mathematics, Physics, Technical Sciences, Information Science, **10**, *3*, pp. 261–268, 2009.
10. CARNAHAN, B., LUTHER, H.A., WILKES, J.O., *Applied numerical methods*, J.Willey, New York, 1969.
11. CHEN Wai-Fah (Editor), *Structural Engineering Handbook,* CRC Press LLC, 1999.
12. ISHIHARA K., *Soil Behavior in Earthquake Geotechnics,* Clarendon Press, Oxford, 1996.
13. LEVY S., WILKINSON J.P.D., *The Component Element Method in Dynamics*, McGraw-Hill Book Company, 1976.
14. MĂRMUREANU Gh., MĂRMUREANU Al., CIOFLAN C.O., BĂLAN S.F., *Assessment of Vrancea earthquake risk in a real/nonlinear seismology,* Proc.of the 3rd Conf. on Structural Control, Vienna, 12–15 July, 2004, pp. 29–32.
15. PENG Z.K., LANG Z.Q., BILLINGS S.A., *Resonances and resonant frequencies for a class of nonlinear systems,* Journal of Sound and Vibration, **300**, pp. 993–1014, 2007.
16. PRESS W.H., FLANNERY B.P., TEUKOLSKY S.A., VETTERLING W.T., *Numerical Recipes, The art of Scientific Computing,* Cambridge University Press, 1990.
17. MÂNDRESCU Nicolae, RADULIAN Mircea, MĂRMUREANU Gheorghe, *Geological, geophysical and seismological criteria for local response evaluation in Bucharest area,* Soil Dynamics and Earthquake Engineering, **27**, pp. 367–393, 2007.
18. DeSILVA, C., *Vibrations: Fundamentals and Practice,* CRC Press, 2000.

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