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ADAPTIVE CONTROL OF UNCERTAIN SYSTEMS: A NEW UNITARY APPROACH

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The paper proposes a new and unitary approach of adaptive output feedback control for non-affine uncertain systems, about which the positive knowledge refers to the relative degree r. Given a reference model, the objective is to design a controller that forces the measured system output to track the reference model output with bounded error. The components of the so called pseudocontrol, thought on a superposition effects principle, are the following: 1) the output of reference model, 2) the output of a Kalman type stabilizing compensator of the pair of systems composed by a) an output dynamics of a set of integrators of order tantamount to the assumed known relative degree r of the controlled system and b) an internal model, of order r – 1, oriented to the tracking error decreasing in the presence of step input signals, and 3) the adaptive control designed to approximately cancel the error of approximate dynamic inversion by virtue of whom the real control is hereby determined from pseudocontrol. A single hidden layer neural network is used to counteract this dynamic inversion error. The classical approach of pseudocontrol design based on tracking error dynamics estimation is evaded. A proof of stable working of this intelligent type controller is sketched. The mathematical model for the longitudinal dynamics of an experimental helicopter is used as framework of synthesis and validation by numerical simulations.

Key words: Uncertain systems; Adaptive control; Dynamic inversion; Neural network; Kalman synthesis; Helicopter mathematical model; Numerical simulation.

1. INTRODUCTION

One of the most important problems in control theory is that of controlling a system in order to have its output tracking a given reference signal. In practice, whatever system appears as only approximately defined by differential equations, in other words, as uncertain. A way in treating the control of uncertain systems is the adaptive control. Research in this field is of particular importance, taking into account the emerging applications such as modern fighter and civilian aircrafts, unmanned aerial vehicles (UAV), flexible structures, robotics, flow physics, combustion processes and so on. Indeed, modelling for all these applications suffers of uncertainty, both in parameters and dynamics.

To highlight the framework of the paper, let the dynamics of an *observable* [1] nonlinear single-input-single-output (SISO) non affine system be given by the equations

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, u\right), y = g\left(\mathbf{x}\right), \tag{1}$$

where $\mathbf{x} \in D_{\mathbf{x}} \subset \mathbb{R}^n$ is the state vector, $u, y \in \mathbb{R}$ (for sake of simplicity) are the input signal (control), respectively, the output signal (measurement), and $\mathbf{f}$, g *are uncertain functions*, sufficiently smooth; moreover, n *need not be necessary prescribed!* For this real or virtual system, e. g. a helicopter or its mathematical model, various problems are stated in control theory. Let consider such a problem: design (more specific, synthesize) a control law $u(y)$, which uses the available measurement y, so that the measured and controlled output y to follow asymptotically a prescribed reference signal $y_{rm}(t) \in C^r$ (the class of continuous functions with continuous derivatives until de order r). In fact, this is the problem of trajectory tracking for an airplane or rocket, for example. In addition, the control law u is subjected to saturating restrictions, $|u| < u_M$.

A premise of solving the problem is the ability of the artificial intelligence techniques – of *neural networks (NNs)*, for example – in compensating the lack of system knowledge, in other works, in compensating the uncertainties in its modeling. For systems such as (1), let consider a relative degree $r < n$ and the fulfilment of feedback linearization conditions stated in [1]. This means that a certain state coordinates transformation involving Lie derivatives $L_f^{(i)}h$ will operate, and the system will be rewritten in the normal form

$$\dot{\varphi} = \mathbf{f}_0(\varphi, \xi), \dot{\xi}_i = \xi_{i+1}, ..., i = 1, ..., r-1, \dot{\xi}_r = g_r(\xi, \chi, u), y = \xi_1, g_r(\xi, \chi, u) := L_f^{(r)} g . \quad (2)$$

χ is the state vector associated with the zero dynamics $\dot{\chi} = \mathbf{f}_0(0, \chi)$. These considerations are not hazardous, because in any system the output depends finally on input. Feedback linearization is then performed by a transformation of variable

$$v = \hat{g}_r(y, u),\ u = \hat{g}_r^{-1}(y, v), \quad (3)$$

where v is the so-called *pseudo control* and $\hat{g}_r(y, u)$ represents any available approximation of $g_r(\xi, \chi, u)$ that is invertible with respect to its second argument. Thus, the uncertain system (1) will be represented by a linear dynamics of r integrators

$$y^{(r)} = v + \Delta,\ \Delta := g_r(\xi, \chi, u) - \hat{g}_r(y, u), \quad (4)$$

where Δ is the dynamic inversion error, which acts as a disturbance signal on system. In fact, making $(n-1)$ times Lie derivatives of the function $g(\mathbf{x})$ yields

$$y = g(\mathbf{x}), \dot{y} = L_f g(x), \ldots, y^{(n-1)} = L_f^{n-1} g(\mathbf{x}). \quad (5)$$

Observability hypothesis in (1) ensures that the right side of system (5) has a full rank and, taking into account (3) and the condition of relative degree r, the following implicit dependence can be stated

$$\mathbf{x} = \mathbf{F}\left(y, \dot{y}, \ldots, y^{(n-1)}, v, \dot{v}, \ldots, v^{(n-r-1)}\right). \quad (6)$$

A similar expression is obtained for the error

$$\Delta(\mathbf{x}, y, v) = G\left(y, \dot{y}, \ldots, y^{(n-1)}, v, \dot{v}, \ldots, v^{(n-r-1)}\right). \quad (7)$$

A theorem of Kolmogorov-Sprecher type [2], [3] ensures the existence of a NN so that Δ may be approximated with good accuracy when the network is operating only on the input-output data (d – a sample time)

$$y(t), y(t-d), \ldots, y(t-(N_1-1)d), v(t), v(t-d), \ldots.., v(t-(N_1-r-1)d),\quad N_1 \geq n,\quad d > 0. \quad (8)$$

2. A NEW AND UNITARY DESIGN OF ADAPTIVE CONTROLLER

During the last decade, adaptive methods based on NNs have been developed to control uncertain systems. Remarkable results are reported in the literature [4–9]. Pioneering works in the field are the references [10, 11]. A state-of-art of the adaptive output feedback control of uncertain systems, in which both the dynamics and the dimension of the regulated plant may be uncertain, and only the knowledge of the relative degree r is claimed, is summarized in [12]. The present paper proposes *a new and unitary strategy of adaptive controller design*, centered on the works [13, 14], wherein a stabilizing compensator was built for a pair plant-internal model of exogeneous signals. This strategy is presented step by step with reference to Figs. 1, 2 and to a service model – the pitch channel dynamics of R-50 experimental helicopter [6] (τ is actuator time constant)

$$\dot{\mathbf{x}}(t) = \boldsymbol{A}\mathbf{x}(t) + \mathbf{B}\delta_c(t),\ y = \theta, \tag{9}$$

$$\mathbf{x} := \begin{bmatrix} u \\ q \\ \theta \\ \beta \\ w \\ \delta \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1/\tau \end{bmatrix}, \boldsymbol{A} := \begin{bmatrix} X_u & X_q & X_\theta & X_\beta & X_w & X_\delta \\ M_u & M_q & 0 & M_\beta & M_w & M_\delta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ B_u & -1 & 0 & B_\beta & 0 & B_\delta \\ Z_u & Z_q & Z_\theta & Z_\beta & Z_w & Z_\delta \\ 0 & 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix}, \begin{array}{lll} X_u = -0.0553 & X_q = 1.413 & X_\theta = -32.1731 \\ X_\beta = -19.9033 & X_w = 0.0039 & X_\delta = 11.2579 \\ M_u = 0.2373 & M_q = -6.9424 & M_\beta = 68.2896 \\ M_w = 0.002 & M_\delta = -38.6265 & B_u = 0.0101 \\ B_\beta = -2.1633 & B_\delta = -4.2184 & Z_u = 0.0027 \\ Z_q = -0.0236 & Z_\theta - 0.2358 & Z_\beta = -0.1233 \\ Z_w = -0.5727 & Z_\delta = 0.0698 & \tau = 0.04\,\text{s}. \end{array} \tag{10}$$

The state variables are: u – forward velocity; w – vertical velocity; q – pitch rate; θ – pitch angle; β – control rotor longitudinal tilt angle; δ – actuator state; $\delta_c \equiv u$ – the control variable, longitudinal cyclic input. *Worthy noting, the system (9)-(10) is only a pretext in view of controller validating by numerical simulations. Control objective is the following:* the system output y is required to track a known bounded input y_c. Main sources of unmodeled dynamics are the control rotor dynamics and control delay time: $\delta_c(t)$ is really $\delta_c(t - T_d)$. As main assumption on system, *the relative degree was assumed*: one can see that in (9–10) *the controlled output* θ *has relative degree* $r = 3$.

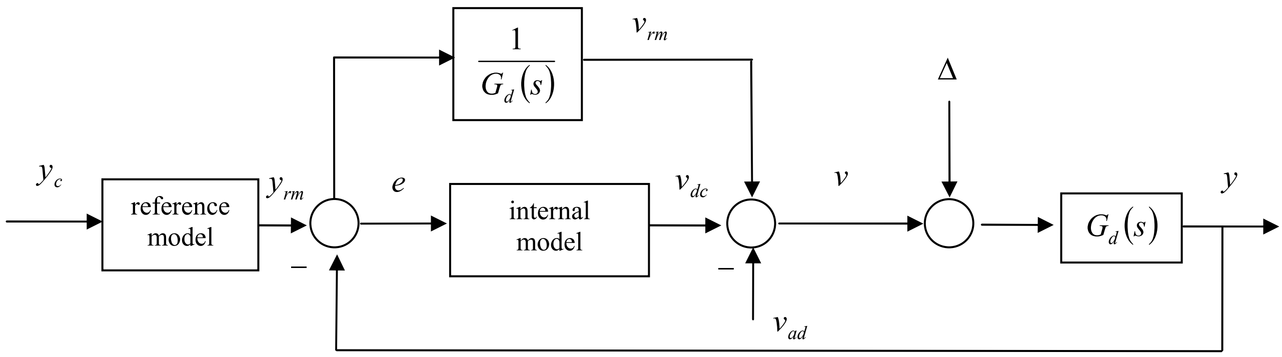

Fig. 1 – Control system architecture.

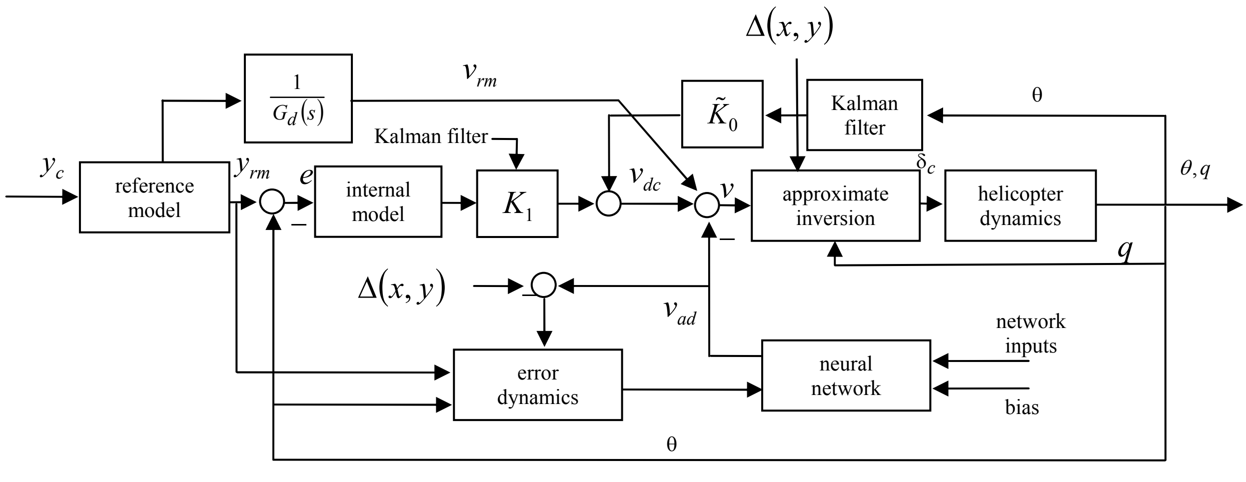

Fig. 2 - Implementation block diagram.

The pseudo control in (3) is chosen to have the form [4–9]

$$v = v_{rm} + v_{dc} - v_{ad}\,. \tag{11}$$

The three components are: v_{rm} – the output of a reference model, v_{dc} – the output of a stabilizing compensator for the linearized dynamics in (4) with $\Delta = 0$ and v_{ad} – the adaptive control signal designed to approximately cancel Δ. Thus, the control objective is the following: the system output y is required to track a known bounded input y_{rm}, rather than y_c

By virtue of assuming in synthesis only the minimal knowledge about relative degree, the dynamics of the output, with a key parameter b_0 in design, is (see Fig. 1) (with Laplace variable s)

$$y = G_d(s)(v+\Delta),\; G_d(s) := b_0 / s^3\,. \tag{12}$$

Aiming to correlate the blocks in view of structure simplifying, the block in upper loop is conceived as

$$v_{rm} = y_{rm} / G_d(s) \tag{13}$$

and substituting (12) in (11), one get, taking into account the substratum of v_{ad} synthesis, the error dynamics

$$e + b_0\left(v_{dc} - v_{ad} + \Delta\right)/s^3 = 0, e := y_{rm} - y \text{ or } e + b_0 v_{dc}/s^3 \cong 0\,. \tag{14}$$

At this point, an ordinary reflection concerns the necessity of completion the error dynamics by introducing a stabilizing compensation by means of a v_{dc} control component. Let note that the treatment of the question in the quoted references (e.g., [6]) suffers of some lack of coherence and clarity concerning the theory and exemplification of compensation selection. Another, unitary viewpoint of approach is now proposed. Consider, thence, the output dynamics as object and framework of v_{dc} component synthesis. The procedure used in [13–14] is invoked: for the order 3 integrator plant (12) (with the state vector $\mathbf{x}$), two compensators are designed, a) a servocompensator (internal model) (with the state vector η) and b) a stabilizing compensator (with the state vector $\hat{\mathbf{x}}_E$), see Fig. 2. As stabilizing compensator for the system ($\mathbf{x}, \eta$) is considered the well known Kalman filter [15, 16, 3]. Two pairs of matrices are thus introduced: the pair $\left(\boldsymbol{Q}_w, Q_\xi\right)$ of white noise matrices, parameters of the estimation Riccati equation, which provides filter gain $\mathbf{K}_f$, and the pair of weighting matrices $\left(Q_J, R_J\right)$, parameters of the control Riccati equation, which provides control gain $\boldsymbol{K}_R$. Q_J weights the herein performance output $y_p := \eta = \boldsymbol{C}_p \mathbf{x}_E$, as a measure of error e integral, and R_J weights the control $v_{dc} := u$. Then, the open loop triplet system will have the form

$$\begin{gathered}
\dot{\mathbf{x}} = \boldsymbol{A}\mathbf{x} + \mathbf{B}v_{dc},\, \boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}, \dot{\eta} = \boldsymbol{A}_c\eta + \mathbf{B_c}\left(y_{rm} - \boldsymbol{C}_o\mathbf{x}\right), \\
\dot{\hat{\mathbf{x}}}_\mathbf{E} = \boldsymbol{A_E}\hat{\mathbf{x}}_\mathbf{E} + \mathbf{B_E}v_{dc} + \mathbf{K}_f\left(y - \boldsymbol{C_E}\hat{\mathbf{x}}_\mathbf{E}\right) + \mathbf{D_E}y_{rm}, \boldsymbol{C}_E = \begin{bmatrix} \boldsymbol{C}_o & 0 & 0 \end{bmatrix}, \\
\mathbf{x_E} := \begin{bmatrix} \mathbf{x} \\ \eta \end{bmatrix}, \hat{\mathbf{x}}_\mathbf{E} := \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\eta} \end{bmatrix}, \boldsymbol{A_E} := \begin{bmatrix} \boldsymbol{A} & \mathbf{0} \\ -\mathbf{B_c}\boldsymbol{C}_o & \boldsymbol{A}_c \end{bmatrix}, \mathbf{B_E} := \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \mathbf{D_E} := \begin{bmatrix} \mathbf{0} \\ \mathbf{B_c} \end{bmatrix}, \boldsymbol{C}_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, y := \boldsymbol{C}_o\mathbf{x}.
\end{gathered} \tag{15}$$

The state vector η must have in principle the dimension $r-1=2$ [4], and the selection of matrices $\boldsymbol{A}_c, \mathbf{B_c}$ aims to obey the property of internal model for step input signals [14]: $\|\boldsymbol{A}_c\| \ll \|\boldsymbol{B}_c\|$. The Kalman filter output $\boldsymbol{K_R}\hat{\mathbf{x}}_\mathbf{E}$ will be used as control variable v_{dc} in the manner (available η is taken in calculation)

$$\boldsymbol{K_R} = \left[\boldsymbol{K}_0, \boldsymbol{K}_1\right],\; v_{dc} = -\boldsymbol{K}_0\hat{\mathbf{x}} - \boldsymbol{K}_1\eta = -\tilde{\boldsymbol{K}}_0\mathbf{x_E} - \boldsymbol{K}_1\eta,\; \tilde{\boldsymbol{K}}_0 = \begin{bmatrix} \boldsymbol{K}_0 & 0 & 0 \end{bmatrix}. \tag{15′}$$

Thus, the closed loop system for the stabilized output dynamics, with Hurwitz matrix $\boldsymbol{A}_{cl}$, is given by

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\boldsymbol{\eta}} \\ \dot{\hat{\mathbf{x}}}_{\mathbf{E}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & -\mathbf{B}\boldsymbol{K}_1 & -\mathbf{B}\tilde{\boldsymbol{K}}_0 \\ -\mathbf{B_c}\boldsymbol{C}_o & \boldsymbol{A}_c & \boldsymbol{0} \\ \boldsymbol{K}_f\boldsymbol{C}_o & -\mathbf{B_E}\boldsymbol{K}_1 & \boldsymbol{A}_E - \mathbf{B_E}\tilde{\boldsymbol{K}}_0 - \boldsymbol{K}_f\boldsymbol{C}_E \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\eta} \\ \hat{\mathbf{x}}_{\mathbf{E}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{B_c} \\ \mathbf{D_E} \end{bmatrix} y_{rm} := \boldsymbol{A}_{cl} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\eta} \\ \hat{\mathbf{x}}_{\mathbf{E}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{B_c} \\ \mathbf{D_E} \end{bmatrix} y_{rm}. \tag{16}$$

Bringing now in attention the complete equation of error (14), let proceed therein to the substitution of the control component value $v_{dc} = -\tilde{\boldsymbol{K}}_0\mathbf{x_E} - \boldsymbol{K}_1\boldsymbol{\eta}$. The *error dynamics* system will have the form ($e_1 := e$)

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{\boldsymbol{\eta}} \\ \dot{\hat{\mathbf{x}}}_{\mathbf{E}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \boldsymbol{0} & \boldsymbol{0} \\ 0 & 0 & 1 & \boldsymbol{0} & \boldsymbol{0} \\ 0 & 0 & 0 & b_0\boldsymbol{K}_1 & b_0\tilde{\boldsymbol{K}}_0 \\ \mathbf{B_c} & 0 & 0 & \boldsymbol{A}_c & \boldsymbol{0} \\ 0 & 0 & 0 & -\mathbf{B_E}\boldsymbol{K}_1 & \boldsymbol{A}_E - \mathbf{B_E}\tilde{\boldsymbol{K}}_0 - \mathbf{K}_f\boldsymbol{C}_E \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \boldsymbol{\eta} \\ \hat{\mathbf{x}}_{\mathbf{E}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{0} \\ \mathbf{K}_f \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ b_0 \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \left(v_{ad} - \Delta\right) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{0} \\ \mathbf{D_E} \end{bmatrix} y_{rm} \tag{17}$$

or, in matrix-vector describing

$$\dot{\mathbf{E}} = \overline{\boldsymbol{A}}\mathbf{E} + \overline{\mathbf{B}}\left(v_{\text{ad}} - \Delta\right) + \overline{\mathbf{D}}_1 y + \overline{\mathbf{D}}_2 y_{rm} := \overline{\boldsymbol{A}}\mathbf{E} + \overline{\mathbf{B}}\left(v_{\text{ad}} - \Delta\right) + \overline{\boldsymbol{D}}\overline{\mathbf{y}},\ \overline{\boldsymbol{D}} := \begin{bmatrix} \overline{\mathbf{D}}_1 & \overline{\mathbf{D}}_2 \end{bmatrix},\ \overline{\mathbf{y}} := \begin{bmatrix} y \\ y_{rm} \end{bmatrix}. \tag{17'}$$

Summarizing until this point, we have to run on computer the system (17) with the inputs $v_{ad} - \Delta$, $\overline{\mathbf{y}}$ and providing the output $\mathbf{E}$ as input for the equations of control component v_{ad} synthesis, see below. Thus, it is important to underline the evading in this work, as not being strictly necessary, the use of an error estimation $\hat{\mathbf{E}}$ [5]-[7], [9]. Worthy mentioning, also: the choice of key parameters $b_0, \boldsymbol{Q}_w, Q_\xi, Q_J, R_J, \boldsymbol{C}_p$ is performed until leads, by a *trial and error* process, to a *stable matrix* $\overline{\boldsymbol{A}}$.

The following step of design concerns the getting of adaptive control component v_{ad}. As mentioned in Introduction, the dynamic inversion error described in (2)-(4) will be counteracted using the property of universal approximator of a NN [2], [3].

Given $\mathbf{z} \in R^{n_1}$, a three layer-layer NN (with a single hidden layer) has an output given by

$$f_k = b_W\theta_{Wk} + \sum_{j=1}^{n_2} w_{j,k}\sigma_j,\ k = 1,\ldots,n_3,\ \sigma_j = \sigma\left(b_V\theta_{Vj} + \sum_{i=1}^{n_1} v_{i,j}z_i\right),\ j = 1,\ldots,n_2, \tag{18}$$

where n_1, n_2, and n_3 are the numbers of input nodes, hidden layer nodes, and outputs, respectively. $\sigma(\cdot)$ is so called activation function, $v_{i,j}$ are the first-to-second layer interconnection weights, $w_{j,k}$ are the second to third layer interconnection weights, b_V, b_W are bias terms, θ_{Vj} acts as thresholds for each neuron, θ_{Wk} allows the bias term b_W to be weighted in each output channel. In fact, such architecture

$$\mathbf{f} = \boldsymbol{W}^{\mathrm{T}}\sigma(\mathbf{z}) \tag{19}$$

is a universal approximator of piecewise continuous nonlinearities with "squashing" activation functions [17]. Accordingly, a general function $\mathbf{y}(\mathbf{z}) \in C^k$, $\mathbf{z} \in D \subset R^{n_1}$ can be written as

$$\mathbf{y}(\mathbf{z}) = \boldsymbol{W}^{\mathrm{T}}\sigma(\mathbf{z}) + \varepsilon(\mathbf{z}), \tag{20}$$

where $\varepsilon(\mathbf{z})$ is the functional reconstruction error. The essential results are expressed as **Theorem**, hereby:

Given $\varepsilon^* > 0$, *there exists a set of bounded ideal weights,* $\mathbf{W}$, *such that* Δ (7), *associated with the system* (1)-(4), *can be approximated over a compact set* $D \subset D_{\mathbf{x}} \times R$ *by a linearly parameterized NN*

$$\Delta = \mathbf{W}^{\mathrm{T}}\sigma\left(\mathbf{V}^{\mathrm{T}}\mu\right) + \varepsilon(\mu), \|\varepsilon\| < \varepsilon^* \tag{21}$$

*using the **past** input/output history vector derived from* (8) *and* $\sigma(\cdot)$ *as any squashing function,*

$$\mu(t) = \begin{bmatrix} 1 & \bar{\boldsymbol{v}}_d^{\mathrm{T}}(t) & \bar{\boldsymbol{y}}_d^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \mu \in R^{n_1+1} \tag{22}$$

(Note that for systems with full relative degree no *past* input/output history is required [5, 18]).

Thus, the *scalar* output of the adaptive element in Fig. 2 will be designed as

$$v_{ad} := \hat{\mathbf{W}}^{\mathrm{T}}\sigma\left(\hat{\boldsymbol{V}}^{\mathrm{T}}\mu\right), \tag{23}$$

(see [4–9]) with $\mathbf{W} \in R^{(n_2+1)\times(n_3)}, \boldsymbol{V} \in R^{(n_1+1)\times(n_2)}$, if threshold and bias terms are considered, $n_3 = 1$ herein, and the following *weights adaptation laws*

$$\dot{\hat{\boldsymbol{V}}} = -\boldsymbol{\Gamma}_V\left[\boldsymbol{\mu}\mathbf{E}^{\mathrm{T}}\boldsymbol{P}\bar{\mathbf{B}}\hat{\mathbf{W}}^{\mathrm{T}}\boldsymbol{\sigma}' + k\left(\hat{\boldsymbol{V}} - \boldsymbol{V}_0\right)\right], \dot{\hat{\mathbf{W}}} = -\boldsymbol{\Gamma}_{\mathrm{W}}\left[\left(\boldsymbol{\sigma} - \boldsymbol{\sigma}'\hat{\boldsymbol{V}}^{\mathrm{T}}\boldsymbol{\mu}\right)\mathbf{E}^{\mathrm{T}}\boldsymbol{P}\bar{\mathbf{B}} + k\left(\hat{\mathbf{W}} - \mathbf{W}_0\right)\right], \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\bar{\boldsymbol{A}} = -\boldsymbol{Q} \tag{24}$$

$(\boldsymbol{V}_0, \mathbf{W}_0)$ are initial guess of NN weights, $\boldsymbol{Q} > 0$ is a suitable matrix, $k > 0$ is a *sufficiently large* constant adaptation gain, and $\boldsymbol{\Gamma}_V$, $\boldsymbol{\Gamma}_W > 0$, of appropriate dimensions. $\hat{\boldsymbol{V}}$ and $\hat{\mathbf{W}}$ are inner (hidden) layer weight matrix and, respectively, outer layer weight vector, which must to be updated on line. The other notations stand for

$$\sigma_i = \sigma(z_i) = \frac{1}{1 + \mathrm{e}^{-a_i z_i}}, \sigma := \sigma\left(\boldsymbol{V}^{\mathrm{T}}\mu\right), \sigma'(z) := \mathrm{diag}\left(\frac{\partial\sigma_i}{\partial z_i}\right), \tag{25}$$

where σ is a sigmoidal function [17] and $\boldsymbol{a}$ is an activation potential. Worthy noting, in all above quoted references, instead of output vector $\mathbf{E}$, an estimate $\hat{\mathbf{E}}$ was considered, but evaded herein, as mentioned before.

Following now an used machinery [4–7], define

$$\tilde{\mathbf{W}} = \hat{\mathbf{W}} - \mathbf{W}, \tilde{\boldsymbol{V}} = \hat{\boldsymbol{V}} - \boldsymbol{V}, \tilde{\boldsymbol{Z}} = \begin{bmatrix} \tilde{\mathbf{W}} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{V}} \end{bmatrix}, \|\hat{\mathbf{W}}\| < \mathbf{W}^*, \|\hat{\boldsymbol{V}}\| < \boldsymbol{V}^*. \tag{26}$$

Thus

$$v_{ad} - \Delta = \hat{\mathbf{W}}^{\mathrm{T}}\sigma\left(\hat{\boldsymbol{V}}^{\mathrm{T}}\mu\right) - \mathbf{W}^{\mathrm{T}}\sigma\left(\boldsymbol{V}^{\mathrm{T}}\mu\right) - \varepsilon, \|v_{ad} - \Delta\| \le \alpha_1\|\tilde{\boldsymbol{Z}}\| + \alpha_2, \alpha_1 > 0, \alpha_2 > 0. \tag{27}$$

Then

$$\hat{\mathbf{W}}^{\mathrm{T}}\sigma\left(\hat{\boldsymbol{V}}^{\mathrm{T}}\mu\right) - \mathbf{W}^{\mathrm{T}}\sigma\left(\boldsymbol{V}^{\mathrm{T}}\mu\right) = \tilde{\mathbf{W}}^{\mathrm{T}}\left(\hat{\sigma} - \hat{\sigma}'\hat{\boldsymbol{V}}^{\mathrm{T}}\mu\right) + \hat{\mathbf{W}}^{\mathrm{T}}\hat{\sigma}'\tilde{\boldsymbol{V}}^{\mathrm{T}}\mu + w, \hat{\sigma} := \sigma\left(\hat{\boldsymbol{V}}\mu\right), w := \tilde{\mathbf{W}}^{\mathrm{T}}\hat{\sigma}'\boldsymbol{V}^{\mathrm{T}}\mu - \mathbf{W}^{\mathrm{T}}\mathcal{O}\left(\tilde{\boldsymbol{V}}^{\mathrm{T}}\mu\right)^2. \tag{28}$$

Other very important assumptions are made

$$\|\mu\| \le \mu^*, \mu^* > 0, \|\bar{\mathbf{y}}\| \le y_M. \tag{29}$$

A bound over the compact set D thus holds

$$\|w - \varepsilon\| \le \gamma_1\|\tilde{\boldsymbol{Z}}\| + \gamma_2, \gamma_1 > 0, \gamma_2 > 0. \tag{30}$$

Therefore, the error dynamics in (17′) can be written

$$\dot{\mathbf{E}} = \bar{\boldsymbol{A}}\mathbf{E} + \bar{\mathbf{B}}\left(\tilde{\mathbf{W}}^{\mathrm{T}}\left(\hat{\sigma} - \hat{\sigma}'\hat{\boldsymbol{V}}^{\mathrm{T}}\mu\right) + \hat{\mathbf{W}}^{\mathrm{T}}\hat{\sigma}'\tilde{\boldsymbol{V}}^{\mathrm{T}}\mu + w - \varepsilon\right) + \bar{\boldsymbol{D}}\bar{\mathbf{y}} \tag{31}$$

subject to (29), (30).

The stability of the closed loop dynamics (17′) is a consequence of a *mutatis mutandis* similar reasoning as that developed in [5], for example. Consider the following Lyapunov function candidate

$$L = \mathbf{E}^{\mathrm{T}} \boldsymbol{P} \mathbf{E} + \mathrm{tr}\left(\tilde{\mathbf{W}}^{\mathrm{T}} \boldsymbol{\Gamma}_W^{-1} \tilde{\mathbf{W}}\right)/2 + \mathrm{tr}\left(\boldsymbol{V}^{\mathrm{T}} \boldsymbol{\Gamma}_V^{-1} \boldsymbol{V}\right)/2 . \tag{32}$$

The derivative of L along (17′), after some not very difficult calculations, will be so bounded

$$\begin{gathered}\dot{L} = -\mathbf{E}^{\mathrm{T}} \boldsymbol{Q} \mathbf{E} + 2\mathbf{E}^{\mathrm{T}} \boldsymbol{P} \bar{\mathbf{B}}\left[w - \varepsilon\right] + 2\mathbf{E}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{D} \mathbf{y} - k\mathrm{tr}\left[\tilde{\mathbf{W}}\left(\hat{\mathbf{W}} - \mathbf{W}_0\right)\right] - k\mathrm{tr}\left[\tilde{\boldsymbol{V}}\left(\hat{\boldsymbol{V}} - \boldsymbol{V}_0\right)\right], \\ \dot{L} \le -\lambda_{\min}(Q)\|\mathbf{E}\|^2 + 2\|\boldsymbol{P}\bar{\mathbf{B}}\|\|\mathbf{E}\|\left[\gamma_1\|\tilde{\boldsymbol{Z}}\| + \gamma_2\right] - k\|\tilde{\boldsymbol{Z}}\|^2/2 + \bar{Z} + \|\bar{\mathbf{D}}\| y_M, \bar{Z} := k\left(\|\mathbf{W} - \mathbf{W}_0\|^2 + \|\boldsymbol{V} - \boldsymbol{V}_0\|^2\right)/2\end{gathered} \tag{33}$$

and further allows for the following upper bound (note that $\|\boldsymbol{A}\|^2 = \mathrm{tr}\left(\boldsymbol{A}^{\mathrm{T}} \boldsymbol{A}\right)$) with appropriate upper bounds

$$\begin{gathered}\dot{L} \le -\|\mathbf{E}\|\left[\left(\lambda_{\min}(\boldsymbol{Q}) - 1\right)\|\mathbf{E}\| - 2\gamma_2\|\boldsymbol{P}\bar{\mathbf{B}}\|\right] - \left[\left(k/2 - \gamma_1^2\|\boldsymbol{P}\bar{\mathbf{B}}\|^2\right)\|\tilde{Z}\|^2 - \bar{Z} - \|\bar{\mathbf{D}}\| y_M\right], \\ \|\mathbf{E}\| > 2\gamma_2\|\boldsymbol{P}\bar{\mathbf{B}}\|/\left(\lambda_{\min}(\boldsymbol{Q}) - 1\right), \|\tilde{\boldsymbol{Z}}\| > \sqrt{\left(\bar{Z} + \|\bar{\boldsymbol{D}}\| y_M\right)/\left(k/2 - \gamma_1^2\|\boldsymbol{P}\bar{\mathbf{B}}\|^2\right)},\end{gathered} \tag{34}$$

which will render $\dot{L} < 0$ outside a compact set. γ_1 includes the unknown constant μ^*, γ_2 includes ε^*. A standard reasoning [5] ends proof: the feedback control law given by (30), (11), (13), (15′), (27) guarantees that all signals $\mathbf{E}, \tilde{\mathbf{W}}, \tilde{V}$ are ultimately bounded, provided compact set D is sufficiently large and other three specific assumptions are fulfilled [9]: A1) stable zero dynamics; A2) $\partial g_r(\mathbf{x},u)/\partial u$ is continuous and nonzero for every $(\mathbf{x},u) \in D_{\mathbf{x}} \times R$; A3) $\partial \hat{g}_r(y,u)/\partial u$ is continuous and nonzero for every $(y,u) \in D_y \times R$ and

$$\mathrm{sign}\, \partial \hat{g}_r(y,u)/\partial u = \mathrm{sign}\, \partial g_r(\mathbf{x},u)/\partial u , \tag{35}$$

for every $(\mathbf{x}, y, u) \in D_{\mathbf{x}} \times D_y \times R$.

As concerning the reference model, the representation is chosen as a third order filter

$$y_{rm} = \frac{\omega_1 \omega_2^2}{(s + \omega_1)\left(s^2 + 2\zeta\omega_2 s + \omega_2^2\right)} y_c , \tag{36}$$

where $\omega_1, \zeta, \omega_2$ stockpile some information – if this is available – about the basic, low modes, of the plant (in our case, represented by the system (10)).

The following point of design must provide an approximate inversion law as well as (3) and, consequently, the real control $\delta_c \equiv u$. To be consequent in our approach, let assume in the sequel an enhanced level of uncertainty and evade the direct use of equations (10). A simple, heuristic approach on flight mechanics enables us a series of inferences on the dynamics of output $y = \theta$

$$y = \theta, \dot{y} := q, \ddot{y} \approx M_q q + M_\delta \delta, \dddot{y} := g_r = M_q\left(M_q q + M_\delta \delta\right) - M_\delta \delta/\tau + M_\delta \delta_c/\tau, \hat{g}_r = M_q^2 q + M_\delta \delta_c/\tau , \tag{37}$$

Assume, however, that below g_r is not the exact expression derived from applying feedback linearization mapping on (10). Now taking into account (12),

$$b_0 v \cong \dddot{y} = M_\delta \delta_c/\tau + M_q^2 q , \tag{38}$$

therefore the inversion is performed

$$u := \delta_c = \tau\left[b_0 v - \hat{M}_q^2 q\right]/\hat{M}_\delta , \tag{39}$$

where $\hat{M}_q, \hat{M}_\delta$ were introduced to account for parametric uncertainties in $\hat{M}_q, \hat{M}_\delta$, respectively. Note that with measured output $y = \theta$, the derivative $\dot{y} = q$ is available and considered in the above relations, but the variable δ is ignored. Given that our approach is rather physical than theoretical, assumptions A2, A3 are implicit.

3. NUMERICAL SIMULATION AND CONCLUSION

Numerical simulations have been performed to investigate the performance of the proposed adaptive controller. For the sake of rigor, canonical coordinates transformation [1] on system (10) was done and the zero dynamics were proved to be asymptotically stable, but with correction $M_\beta = 100$. Further on, the system parameters were as follows: $\tau = 0.05\,\mathrm{s}$, $\omega_1 = 10$ rad/s, $\zeta = 0.25$, $\omega_2 = 5.52$ rad/s, $n_1 = 15$, $n_2 = 5$, $\hat{M}_\delta = 0.6 M_\delta$, , $\hat{M}_q = 2M_q$ $\boldsymbol{C}_p = [0 \quad 0 \quad 0 \quad 1000 \quad 1000]$, $R_J = 10^{-9}$, $Q_J = 100$, $Q_\xi = 100$, $\boldsymbol{Q}_w = [1 \quad 10 \quad 100 \quad 0.1 \quad 0.01]$, $\boldsymbol{\Gamma}_\mathrm{v} = \mathbf{I}_{16}$, $\boldsymbol{\Gamma}_\mathrm{w} = \mathrm{I}_6$, $\mathbf{W}_0 = 0.001 \times [1 \quad \cdots \quad 1]^\mathrm{T} \in R^{6\times 1}$, $\boldsymbol{V}_0 = 0.001 \times \boldsymbol{U} \in \mathbb{R}^{16\times 5}$, $\boldsymbol{A}_c = \begin{bmatrix} -0.001 & 0 \\ 0 & -0.001 \end{bmatrix}$, $\mathbf{B_c} = [100 \quad 10000]^\mathrm{T}$. $\boldsymbol{U}$ is a matrix with all entries 1; $\boldsymbol{Q} = \boldsymbol{I}_{10}$. A stable matrix $\overline{\boldsymbol{A}}$ occurs. The stabilizing compensator then emerged:

$$\tilde{\boldsymbol{K}}_0 = 10^9 \times [3.51 \quad 0.001 \quad 0.000000621 \quad 0 \quad 0]^\mathrm{T},\ \boldsymbol{K}_1 = 10^9 \times [-0.316 \quad -0.316]^\mathrm{T},$$

$$\mathbf{K}_f = [2.01 \ 2.03 \ 0.99 \ -99.99 \ -9999.99]^\mathrm{T}$$

An excellent working of the proposed control law, in conditions of increased parameter and structural uncertainty concerning the system in comparison with usual approaches [5]-[8], is illustrated in Fig. 3. Worthy noting, an improving of tracking properties y versus y_{rm} by *trail and error* procedure can be still reached.

The main conclusion of the work refers to the proposal of a new, unitary conceived, control law builded in an uncertainty framework and based on use of Kalman synthesis dynamic compensation and NN to counteract the feedback linearization errors. A sketch of stable work of the controlled system is presented.

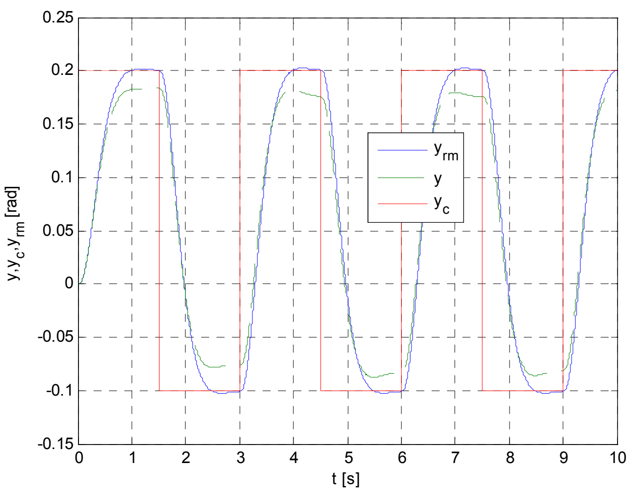

Fig. 3 – Time history of controlled output y in the presence of step cascade input signal y_c; y_{rm} – reference model.

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