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BAYESIAN AND NON-BAYESIAN ESTIMATORS USING RECORD STATISTICS OF THE MODIFIED-INVERSE WEIBULL DISTRIBUTION

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This paper develops Bayesian and non-Bayesian analysis in the context of record statistics values from the modified-inverse Weibull distribution. We obtained non-Bayes estimators using MLE and Bayes estimators using the squared error loss function (quadratic loss) and LINEX loss function. This was done with respect to the conjugate prior for the shape parameter. The results may be of interest in a situation where only record values are stored.

Key words: Bayesian estimation; Maximum likelihood estimates; Modified Inverse Weibull distribution; Record values.

1. INTRODUCTION

Drapella in [9] calls the inverse Weibull distribution the complementary Weibull distribution, Jiag et al. in [11] have discussed some useful measures for the inverse Weibull distribution.

The inverse Weibull distribution plays an important role in many applications, including the dynamic components of Diesel engines and several dataset such as the times to breakdown of an insulating fluid subject to the action of a constant tension; see [14]. Calabria and Pulcini in [8] provide an interpretation of the inverse Weibull distribution in the context of the load-strength relationship for a component and Maswadah in [12] has the fitted inverse Weibull distribution to the flood data. For more details on the inverse Weibull distributions see for example [13].

Record values and the associated statistics are of interest and importance in many areas of real life applications involving data relating to industry, economics, lifetesting, meteorology, hydrology, seismology, athletic events, and mining. Many authors have studied records and associated statistics. Among others are Ahsanullah in [1, 2], Arnold et al. in [3], [4], Gulati and Padgett in [10], Raqab and Ahsanullah in [24], Raqab in [23], Sultan in [28] and Preda et al. in [22]. On the line of papers [17, 18, 19] and [20] we can formulate some problems of such type.

2. PRELIMINARIES

Let $X_1, X_2, X_3 \ldots$ a sequence of independent and identically distributed (iid) random variables with cdf $F(x)$ and pdf $f(x)$. Setting $Y_n = \min(X_1, X_2, X_3, \ldots, X_n)$, $n \geq 1$, we say that X_j is a lower record and denoted by $X_{L(j)}$ if $Y_j < Y_{j-1}$, $j > 1$. Assume that $X_{L(1)}, X_{L(2)}, X_{L(3)}, \ldots, X_{L(n)}$ are the first *n* lower record values arising from a sequence $\{X_i\}$ of iid modified-inverse Weibull variables with pdf

$$f(x) = \alpha(\beta + \lambda x)x^{-\beta-1}e^{-\lambda x - \alpha x^{-\beta}e^{-\lambda x}}, \quad x \geq 0, \quad \alpha, \beta, \lambda > 0 \tag{1}$$

and cdf

$$F(x) = \mathrm{e}^{-\alpha x^{-\beta} e^{-\lambda x}}, \quad x \geq 0, \quad \alpha, \beta, \lambda > 0 \tag{2}$$

where α is the scale parameter and β, λ are the shape parameters.

The reliability function $R(t)$, and the hazard (instantaneous failure rate) function $H(t)$ at mission time t for the modified exponential distribution are respectively given by

$$R(t) = 1 - \mathrm{e}^{-\alpha t^{-\beta} e^{-\lambda t}}, \quad x \geq 0, \quad \alpha, \beta, \lambda > 0 \tag{3}$$

and

$$H(t) = \frac{\alpha t^{-\beta-1}(\beta + \lambda t) \cdot \mathrm{e}^{-\lambda t - \alpha t^{-\beta} e^{-\lambda t}}}{1 - \mathrm{e}^{-\alpha t^{-\beta} e^{-\lambda t}}}, \quad x \geq 0, \quad \alpha, \beta, \lambda > 0. \tag{4}$$

In Bayesian estimation, we consider two types of loss functions. The first is the squared error loss function (quadratic loss) which is classified as a symmetric function and associates equal importance to the losses for overestimation and underestimation of equal magnitude. The second is the LINEX (linear-exponential) loss function which is asymmetric, that was introduced by Varian in [29]. These loss functions were widely used by several authors; among of them Basu and Ebrahimi in [5], Pandey in [16], Soliman in [27] and Nassar and Eissa in [15]. This function rises approximately exponentially on one side of zero and approximately linearly on the other side.

Under the assumption that the minimal loss occurs at $\phi = \phi^*$ the LINEX loss function for can be expressed as

$$L(\Delta) \propto \mathrm{e}^{c\Delta} - c\Delta - 1, \quad c \neq 1, \tag{5}$$

where $\Delta = \left(\phi - \phi^*\right)$, ϕ^* is an estimate of ϕ. The sign and magnitude of the shape parameter c represents the direction and degree of symmetry, respectively. (If $c > 0$, the overestimation is more serious than underestimation, and vice-versa.) For c close to zero, the LINEX loss is approximately s.e.l. and therefore almost symmetric.

The posterior expectation of the LINEX loss function (6) is

$$E_\phi\left[L(\phi - \phi^*)\right] \propto \mathrm{e}^{c\phi^*} E_\phi\left[\mathrm{e}^{-c\phi}\right] - c\left(\phi^* - E_\phi\left[\phi\right]\right) - 1, \tag{6}$$

where $E_\phi(\cdot)$ denotes the posterior expectation with respect to the posterior density of ϕ. By a result of Zellner in [30], the (unique) Bayes estimator of ϕ, denoted by ϕ^*_{BL} under the LINEX loss function is the value ϕ^* which minimizes (6). It is

$$\phi^*_{BL} = -\frac{1}{c}\ln\left\{E_\phi\left[\mathrm{e}^{-c\phi}\right]\right\} \tag{7}$$

provided that the expectation $\left\{E_\phi\left[\mathrm{e}^{-c\phi}\right]\right\}$ exists and is finite. The problem of choosing the value of the parameter c is discussed in Calabria and Pulcini in [6].

3. MAXIMUM LIKELIHOOD ESTIMATION (MLE)

The joint density function of the first n upper record values $x \equiv (x_{L(1)}, x_{L(2)}, \ldots, x_{L(n)})$ is given by

$$f_{1,2,\ldots,n}(x_{U(1)}, x_{U(2)}, \ldots, x_{U(n)}) = f(x_{L(n)}) \prod_{i=1}^{n-1} \frac{f(x_{L(i)})}{F(x_{U(i)})}, \quad 0 \leq x_{L(n)} < \ldots < x_{L(2)} < x_{L(1)}, \tag{8}$$

where $f(x)$, and $F(x)$ are given, respectively, by (1) and (2) after replacing x by $x_{L(i)}$. The likelihood function based on the n lower record values x is given by

$$\ell(\alpha,\beta,\lambda \mid x) = \alpha^n \cdot \mathrm{e}^{-\alpha x_{L(n)}^{-\beta} e^{-\lambda x_{L(n)}}} \prod_{i=1}^{n} \{(\beta + \lambda x_{L(i)}) x_{L(i)}^{-\beta-1} \mathrm{e}^{-\lambda x_{L(i)}}\}. \tag{9}$$

Assuming that the shape parameters β and λ are known, the maximum likelihood estimator (MLE), $\hat{\alpha}_{ML}$ of the scale parameter α can be shown by using (9) to be $\hat{\alpha}_{ML} = n x_{L(n)}^{\beta} \mathrm{e}^{\lambda x_{L(n)}}$.

If only the shape parameter λ is known, the MLEs of the scale parameter α and the shape parameter β, $\hat{\alpha}_{ML}$ and $\hat{\beta}_{ML}$, can be obtained as solutions of the equations

$$\begin{cases} \alpha - n x_{L(n)}^{\beta} \mathrm{e}^{\lambda x_{L(n)}} = 0 \\ \alpha x_{L(n)}^{-\beta} \mathrm{e}^{-\lambda x_{L(n)}} \ln x_{L(n)} - \sum_{i=1}^{n} \ln x_{L(i)} + \sum_{i=1}^{n} \dfrac{1}{\beta + \lambda x_{L(i)}} = 0 \end{cases} \tag{10}$$

which be solved using for example, Newton-Raphson iteration scheme.

If $0 < \lambda < \dfrac{\sum_{i=1}^{n} \dfrac{1}{x_{L(i)}}}{\sum_{i=1}^{n} \ln x_{L(i)} - n \ln x_{L(n)}}$ then the maximum likelihood estimate of β, $\hat{\beta}_{ML}$ is the (unique) solution of the equation in β obtained by eliminating α in (11). Then the maximum likelihood estimate of α is $\hat{\alpha}_{ML} = n x_{L(n)}^{\hat{\beta}_{ML}} \mathrm{e}^{\lambda x_{L(n)}}$.

If only the shape parameter β is known, the MLEs of the scale parameter α and the shape parameter λ, $\hat{\alpha}_{ML}$ and $\hat{\lambda}_{ML}$, can be obtained as solutions of the equations

$$\begin{cases} \alpha - n x_{L(n)}^{\beta} \mathrm{e}^{\lambda x_{L(n)}} = 0 \\ \alpha x_{L(n)}^{-\beta+1} \mathrm{e}^{-\lambda x_{L(n)}} - \sum_{i=1}^{n} x_{L(i)} + \sum_{i=1}^{n} \dfrac{x_{L(i)}}{\beta + \lambda x_{L(i)}} = 0 \end{cases}, \tag{11}$$

which be solved using a iteration scheme. Then, if $0 < \beta < \dfrac{\sum_{i=1}^{n} x_{L(i)}}{\sum_{i=1}^{n} x_{L(i)} - n x_{L(n)}}$, we obtain the MLE of λ, $\hat{\lambda}_{ML}$, the (unique) solution of the equation in λ obtained by eliminating α in (11). Then the maximum likelihood estimate of α is $\hat{\alpha}_{ML} = n x_{L(n)}^{\beta} \mathrm{e}^{\hat{\lambda}_{ML} x_{L(n)}}$. If the three parameters α, β and λ are unknown, using the first likelihood equation (of α), we obtain α, and, after replacing α in (9), we get

$$\widetilde{L}(\beta,\lambda) = n \ln n - n\beta \ln x_{L(n)} - n\lambda x_{L(n)} - n - \lambda \cdot \sum_{i=1}^{n} x_{L(i)} - (\beta+1) \sum_{i=1}^{n} \ln x_{L(i)} + \sum_{i=1}^{n} \ln(\beta + \lambda x_{L(i)}).$$

The Hessian of $\widetilde{L}$ is negative defined matrix and then $\widetilde{L}$ is a concave application on the admissible region ($\beta > 0$, $\lambda > 0$). Then, the Newton-Raphson algorithm converges to the global optimum, assuming that it does not go outside the admissible region. The Newton-Raphson algorithm requires initial parameter estimates. Different types of initialization are discussed in [25] and [31].

However, in this case we can use the likelihood equations for this $\widetilde{L}$. After some transformations, we get

$$\sum_{i=1}^{n}\left(n+\lambda\left(x_{L(i)}\left(\sum_{j=1}^{n}\ln x_{L(j)}-n\ln x_{L(n)}\right)-\left(\sum_{j=1}^{n}x_{L(j)}-nx_{L(n)}\right)\right)\right)^{-1}=1$$

which, again, maybe solve using an iteration scheme. We note that this equation has a solution (unique) only if

$$n\left(\sum_{i=1}^{n}x_{L(i)}-nx_{L(n)}\right)<\left(\sum_{i=1}^{n}\ln x_{L(i)}-n\ln x_{L(n)}\right)\sum_{i=1}^{n}x_{L(i)}$$

$$\sum_{i=1}^{n}\frac{1}{x_{L(i)}^{2}}>\left(\sum_{i=1}^{n}\ln x_{L(i)}-n\ln x_{L(n)}\right)\left(\sum_{i=1}^{n}x_{L(i)}-nx_{L(n)}\right)\sum_{i=1}^{n}x_{L(i)}\ \text{ and}$$

$$\frac{\sum_{i=1}^{n}\frac{1}{x_{L(i)}}}{n}\left(\sum_{i=1}^{n}x_{L(i)}-nx_{L(n)}\right)>\sum_{i=1}^{n}\ln x_{L(i)}-n\ln x_{L(n)}.$$

So first, we get $\hat{\lambda}_{ML}$, the MLE of λ, and then $\hat{\beta}_{ML}$ and $\hat{\alpha}_{ML}$, the MLEs of α and β

$$\hat{\beta}_{ML}=\frac{n-\hat{\lambda}_{ML}\left(\sum_{i=1}^{n}x_{L(i)}-nx_{L(n)}\right)}{\sum_{i=1}^{n}\ln x_{L(i)}-n\ln x_{L(n)}}\ \text{ and }\ \hat{\alpha}_{ML}=nx_{L(n)}^{\hat{\beta}_{ML}}\mathrm{e}^{\hat{\lambda}_{ML}x_{L(n)}}.$$

Finally, the corresponding MLE's $\hat{R}_{ML}(t)$, and $\hat{H}_{ML}(t)$ of $R(t)$ and $H(t)$ are given by (3) and (4) after replacing α, β and λ by $\hat{\alpha}_{ML}$, $\hat{\beta}_{ML}$ and $\hat{\lambda}_{ML}$, respectively.

4. BAYES ESTIMATION

In this section, considering the symmetric (squared error) loss function and the asymmetric (LINEX) loss function, we estimate α, β and λ, and *R(t)* and *H(t)*.

4.1. Known shape parameters λ and β

Under the assumption that the shape parameter λ is known, we assume a gamma $\gamma(a,b)$ conjugate prior for α as

$$\pi(\alpha)=\frac{b^{a}\alpha^{a-1}e^{-b\alpha}}{\Gamma(a)},\alpha>0,a,b>0$$

Applying Bayes theorem, we obtain from the likelihood function and the prior density, the posterior density of α in the form

$$\pi^{*}(\alpha\mid x)=\frac{v^{(n+a)}\alpha^{(n+a-1)}\mathrm{e}^{-\alpha v}}{\Gamma(n+a)}\ \text{ with }\ v=b+x_{L(n)}^{-\beta}\cdot\mathrm{e}^{-\lambda x_{L(n)}}$$

where Γ(□) is gamma function.

Theorem 1. *If shape parameters λ and β are known, under the squared error loss function (BS) and the LINEX loss function (BL), the Bayes estimators for* α, *R(t) and H(t), are given by*

$$\tilde{\alpha}_{BS} = \frac{n+a}{\nu},\ \tilde{\alpha}_{BL} = \frac{n+a}{c}\log\left(1+\frac{c}{v}\right)$$

$$\tilde{R}_{BS}(t) = 1-\left(1+\frac{t^{-\beta}\mathrm{e}^{-\lambda t}}{\nu}\right)^{-(n+a)},\ \tilde{R}_{BL}(t) = -\frac{1}{c}\log\left[\mathrm{e}^{-c}\sum_{m=0}^{\infty}\frac{c^m}{m!}\left(1+\frac{mt^{-\beta}\mathrm{e}^{-\lambda t}}{v}\right)^{-(n+a)}\right]$$

and respectively

$$\tilde{H}_{BS}(t) = (\beta+\lambda t)(n+a)\nu^{n+a}t^{(n+a)\beta-1}\mathrm{e}^{\lambda t(n+a)}\xi\left(n+a+1,1+t^{\beta}\mathrm{e}^{\lambda t}v\right)$$

$$\tilde{H}_{BL}(t) = -\frac{1}{c}\log\left[\frac{v^{n+a}}{\Gamma(n+a)}\sum_{i=0}^{\infty}\sum_{j=0}^{i}\frac{(-1)^i}{i!}\binom{i+j-1}{j}\frac{\left[c(\beta+\lambda t)t^{-\beta-1}e^{-\lambda t}\right]^i\Gamma(n+a+i)}{\left[v+(i+j)t^{-\beta}e^{-\lambda t}\right]^{n+a+i}}\right]$$

4.2. Known shape parameter λ

It is well known that, for Bayes estimators, the performance depends on the form of the prior distribution and the loss function assumed. Under the assumption that both parameters α and β are unknown, no analogous reduction via sufficiency is possible for the likelihood corresponding to a sample of records from the modified Weibull density (1). Also, specifying a general joint prior for α and β leads to computational complexities. In trying to solve this problem and simplify the Bayesian analysis, we use Soland's method. Soland [26] considered a family of joint prior distributions that places continuous distributions on the scale parameter and discrete distributions on the shape parameter.

We assume that the shape parameter β is restricted to a finite number of values $\beta_1,\beta_2,...,\beta_k$ with respective prior probabilities $\eta_1,\eta_2,...,\eta_k$ such that $0 \le \eta_j \le 1$ and $\sum_{j=1}^{k}\eta_j = 1$ [i.e. $P(\beta=\beta_j) = \eta_j$]. Further, suppose that conditional upon $\beta = \beta_j$ there is a natural conjugate prior with distribution a gamma (a_j, b_j) with density

$$\pi(\alpha) = \frac{b_j^{a_j}\alpha^{a_j-1}\mathrm{e}^{-b_j\alpha}}{\Gamma(a_j)}, \alpha>0, a_j, b_j>0,$$

where a_j and b_j are chosen so as to reflect prior beliefs on α given that $\beta=\beta_j$. Then given the set of the first *n* upper record values *x*, the conditional posterior pdf of α is given by

$$\pi^*(\alpha\,|\,\beta=\beta_j,\mathrm{x}) = \frac{B_j^{A_j}\alpha^{A_j-1}\mathrm{e}^{-B_j\alpha}}{\Gamma(A_j)},\quad \alpha>0,\quad A_j,B_j>0,$$

which is a gamma (A_j, B_j), where $A_j = a_j + n$ and $B_j = b_j + x_{L(n)}^{-\beta_j}\mathrm{e}^{-\lambda x_{L(n)}}$.

The marginal posterior probability distribution of β_j obtained by applying the discrete version of Bayes' theorem, is given by

$$P_{j(\beta)} = A_{(\beta)}\int_0^{\infty}\frac{\eta_j b_j^{a_j}\alpha^{n+a_j-1}u_{j(\beta)}}{\Gamma(a_j)}\mathrm{e}^{-\alpha\left(b_j+x_{L(n)}^{-\beta_j}e^{-\lambda x_{L(n)}}\right)}\mathrm{d}\alpha = A_{(\beta)}\frac{\eta_j b_j^{a_j}u_{j(\beta)}\Gamma(A_j)}{\Gamma(a_j)B_j^{A_j}},$$

where $A_{(\beta)}$ is a normalized constant given by

$$\left(A_{(\beta)}\right)^{-1} = \sum_{j=1}^{k} \frac{\eta_j b_j^{a_j} u_{j(\beta)} \Gamma(A_j)}{\Gamma(a_j) B_j^{A_j}} \text{ and } u_{j(\beta)} = \prod_{i=1}^{n} x_{L(i)}^{-\beta_j - 1} \left(\beta_j + \lambda x_{L(i)}\right) \mathrm{e}^{-\lambda x_{L(i)}} .$$

Under the general the symmetric (squared error) loss function (5) and the asymmetric (LINEX) loss function (6), the Bayes estimator of ϕ_{BL}^* a function $\phi(a,b)$ is given by (7).

Theorem 2. *If the shape parameter λ is known, under the squared error loss function (BS) and the LINEX loss function (BL), the Bayes estimators for* α, β, *R(t) and H(t) are given by*

$$\tilde{\alpha}_{BS} = \sum_{j=1}^{k} P_{j(\beta)} \frac{A_j}{B_j}, \quad \tilde{\alpha}_{BL} = -\frac{1}{c} \log\left[\sum_{j=1}^{k} P_{j(\beta)} \left(1 + \frac{c}{B_j}\right)^{-A_j}\right]$$

$$\tilde{\beta}_{BS} = \sum_{j=1}^{k} P_{j(\beta)} \beta_j, \quad \tilde{\beta}_{BL} = -\frac{1}{c} \log\left[\sum_{j=1}^{k} P_{j(\beta)} \mathrm{e}^{-c\beta_j}\right]$$

$$\tilde{R}_{BS}(t) = 1 - \sum_{j=1}^{k} P_{j(\beta)} \left(1 + \frac{t^{-\beta_j} \mathrm{e}^{-\lambda t}}{B_j}\right)^{-A_j}, \quad \tilde{R}_{BL}(t) = -\frac{1}{c} \log\left[\mathrm{e}^{-c} \sum_{j=0}^{k} \sum_{m=0}^{\infty} \frac{P_{j(\beta)} c^m}{m!} \left(1 + \frac{m t^{-\beta_j} \mathrm{e}^{-\lambda t}}{B_j}\right)^{-A_j}\right],$$

and respectively

$$\tilde{H}_{BS}(t) = \sum_{j=1}^{k} \frac{P_{j(\beta)} (\beta_j + \lambda t) B_j^{A_j} t^{-1} (A_j + 1)}{\left(t^{-\beta_j} e^{-\lambda t}\right)^{A_j}} \xi\left(A_j + 1, 1 + B_j t^{\beta_j} \mathrm{e}^{\lambda t}\right)$$

$$\tilde{H}_{BL}(t) = -\frac{1}{c} \log\left[\sum_{j=1}^{k} \sum_{m=0}^{\infty} \sum_{z=0}^{m} P_{j(\beta)} \frac{B_j^{A_j}}{\Gamma(A_j)} \frac{(-1)^m}{m!} \binom{m+z-1}{z} \frac{\left[c(\beta_j + \lambda t) t^{-\beta_j - 1} \mathrm{e}^{-\lambda t}\right]^m \Gamma(A_j + m)}{\left[B_j + (z+m) t^{-\beta_j} \mathrm{e}^{-\lambda t}\right]^{A_j + m}}\right].$$

The case of known shape parameter β is similar with the case of known shape parameter λ.

4.3. Unknown scale parameter α and shape parameters β and λ

We assume that the shape parameters β and λ are restricted to a finite number of values $\beta_1, \beta_2, ..., \beta_k$ and respective $\lambda_1, \lambda_2, ..., \lambda_p$ with prior probabilities $\eta_1, \eta_2, ..., \eta_k$ and $\xi_1, \xi_2, ..., \xi_p$ such that $0 \le \eta_j, \xi_i \le 1$, $\sum_{j=1}^{k} \eta_j = 1$ and $\sum_{i=1}^{p} \xi_i = 1$ (i.e. $P(\beta = \beta_j) = \eta_j$ and $P(\lambda = \lambda_i) = \xi_i$). Further, suppose that conditional upon $\beta = \beta_j$ and $\lambda = \lambda_i$ there is a natural conjugate prior with distribution a gamma (g_{ij}, h_{ij}) with density

$$\pi(\alpha) = \frac{h_{ij}^{g_{ij}} \alpha^{g_{ij} - 1} \mathrm{e}^{-h_{ij}\alpha}}{\Gamma(g_{ij})}, \quad \alpha > 0, \quad g_{ij}, h_{ij} > 0,$$

where g_{ij} and h_{ij} are chosen so as to reflect prior beliefs on α given that $\beta = \beta_j$ and $\lambda = \lambda_i$, $i = \overline{1, p}$, $j = \overline{1, k}$. Then given the set of the first *n* upper record values *x*, the conditional posterior pdf of α is given by

$$\pi^*(\alpha \mid \beta = \beta_j, \lambda = \lambda_i, x) = \frac{H_{ij}^{G_{ij}} \alpha^{G_{ij} - 1} \mathrm{e}^{-H_{ij}\alpha}}{\Gamma(G_{ij})}, \quad \alpha > 0, \quad G_{ij}, \quad H_{ij} > 0,$$

which is a gamma (G_{ij}, H_{ij}), where $G_{ij} = g_{ij} + n$ and $H_{ij} = h_{ij} + x_{L(n)}^{-\beta_j} \mathrm{e}^{-\lambda_i x_{L(n)}}$.

The marginal posterior probability distribution of β_j and λ_i obtained by applying the discrete version of Bayes' theorem, is given by

$$P_{ij(\beta\lambda)} = A_{(\beta\lambda)} \frac{\eta_j \xi_i h_{ij}^{g_{ij}} u_{ij(\beta\lambda)} \Gamma(G_{ij})}{\Gamma(g_{ij}) H_{ij}^{G_{ij}}},$$

where $A_{(\beta\lambda)}$ is a normalized constant given by

$$\left(A_{(\beta\lambda)}\right)^{-1} = \sum_{j=1}^{k}\sum_{i=1}^{p} \frac{\eta_j \xi_i h_{ij}^{g_{ij}} u_{ij(\beta\lambda)} \Gamma(G_{ij})}{\Gamma(g_{ij}) H_{ij}^{G_{ij}}} \text{ and } u_{ij(\beta\lambda)} = \prod_{z=1}^{n} x_{L(z)}^{-\beta_j - 1}\left(\beta_j + \lambda_i x_{L(z)}\right) \mathrm{e}^{-\lambda_i x_{L(z)}}.$$

Theorem 3. *If all parameters are unknown, under the squared error loss function (BS) and the LINEX loss function (BL), the Bayes estimators for* α, β, λ, *R(t) and H(t) are given by*

$$\tilde{\alpha}_{BS} = \sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \frac{G_{ij}}{H_{ij}},\ \tilde{\alpha}_{BL} = -\frac{1}{c}\log\left[\sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \left(1 + \frac{c}{H_{ji}}\right)^{-G_j}\right],$$

$$\tilde{\beta}_{BS} = \sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \beta_j,\ \tilde{\beta}_{BL} = -\frac{1}{c}\log\left[\sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \mathrm{e}^{-c\beta_j}\right],$$

$$\tilde{\lambda}_{BS} = \sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \lambda_i,\ \tilde{\lambda}_{BL} = -\frac{1}{c}\log\left[\sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \mathrm{e}^{-c\lambda_i}\right],$$

$$\tilde{R}_{BS}(t) = 1 - \sum_{i=1}^{p}\sum_{j=1}^{k} P_{ij(\beta\lambda)} \frac{H_{ij}^{G_{ij}} \Gamma(G_{ij})}{\left(H_{ij} + t^{-\beta_j} e^{-\lambda_i t}\right)^{G_{ij}}},$$

$$\tilde{R}_{BL}(t) = -\frac{1}{c}\log\left[\mathrm{e}^{-c} \sum_{i=1}^{p}\sum_{j=0}^{k}\sum_{m=0}^{\infty} \frac{P_{ij(\beta\lambda)} c^m}{m!} \left(1 + \frac{m t^{-\beta_j} \mathrm{e}^{-\lambda_i t}}{H_{ij}}\right)^{-G_{ij}}\right],$$

and respectively

$$\tilde{H}_{BS}(t) = \sum_{i=1}^{p}\sum_{j=1}^{k} \frac{P_{ij(\beta\lambda)} (\beta_j + \lambda_i t) H_{ij}^{G_{ij}} t^{-1} (G_{ij} + 1)}{\left(t^{-\beta_j} e^{-\lambda_i t}\right)^{G_{ij}}} \xi\left(G_{ij} + 1, 1 + H_{ij} t^{\beta_j} \mathrm{e}^{\lambda_i t}\right),$$

$$\tilde{H}_{BL}(t) = -\frac{1}{c}\log\left[\sum_{i=1}^{p}\sum_{j=1}^{k}\sum_{m=0}^{\infty}\sum_{z=0}^{m} P_{ij(\beta\lambda)} \frac{H_{ij}^{G_{ij}}}{\Gamma(G_{ij})} \frac{(-1)^m}{m!} \binom{m+z-1}{z} \frac{\left[c(\beta_j + \lambda_i t) t^{-\beta_j - 1} \mathrm{e}^{-\lambda_i t}\right]^m \Gamma(G_{ij} + m)}{\left[H_{ij} + (z+m) t^{-\beta_j} \mathrm{e}^{-\lambda_i t}\right]^{G_{ij} + m}}\right].$$

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