THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 11, Number 2/2010, pp. 188–194

TOPOLOGICAL GEOMETRY AND DIRECT KINEMATICS OF PARALLEL MANIPULATORS – 4 ACTUATORS, RSS TYPE

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The parallel manipulator is a mobile mechanic structure, having one or more levels, with closed kinematical chains, which are driven by actuators. The parallel manipulator is a complex topological structure, having the joints (as kinematical pairs) of various types. The most frequently utilized joints are: the mono-mobile kinematical pairs (revolute (R) or prismatic), the bi-mobile cylindrical pairs (C) and tri-mobile spherical pairs (S). The paper presents a new way for the structural - topological analysis (with referring to the calculation of mobility) and a new method for the direct kinematics of parallel manipulators with four actuators, RSS (Revolute – Spherical – Spherical pairs) type.

Key words: Geometry; Kinematics; Mobility; Parallel Manipulator.

1. INTRODUCTION

The paper presents a new way of the topological geometry for the Parallel Manipulators (PMp) regarding the calculation of mobility (degree of freedom (d.o.f.) of complex mechanisms. The parallel manipulators are multi-mobile planar or spatial mechanisms with two to six kinematical chains, which connect two rigid bodies, generically named platforms. Mobility or d.o.f. is the main structural – topological parameter of a mechanism and manipulator-robot; this influences the kinematical and dynamic modeling of mobile mechanical systems [1, 2, 5].

The kinematical chains mounted in parallel between two platforms are composed of two or more kinematical elements, these being realized as open chains or as complex chains with closed and open contours [6]. Gough's machine (1947) and the Stewart's platform (1965) are structured as spatial mechanisms with six identical kinematical chains (KC) [12].

These mobile platforms constitute the first structural-topological model, from which subsequent research was started. They are named parallel manipulator (PMp). The PMp-s can be connected in series, obtaining overlapping PMp-s, similar to the structural-topological of serial elephant trunk-type robots.

Unlike the serial manipulators, to PMp-s, practically all types of kinematical pairs are used [1, 10], that allows a larger diversity of kinematical structures.

In the last period, since 1990, the attention of more researches [8, 10, 11, 14, 16] targeted the potential possibilities of PMp-s. The results of numerous studies [6] show that the concept of the parallel manipulator has been generalized and can be used as a model of geometrical and kinematical analysis and synthesis in various domains, from the simulation platforms (in the aeronautic and automotive industries) and drilling petroleum platforms, to industrial robots and walking robots. Defining PMp, J. P. Merlet [12], compares them with a terminal organ with n degrees of freedom and a fixed base, linked together by at least two independent kinematical chains, which are actuated by n simple actuators.

One of the most important activities in the design and development of parallel robots is the creation of new manipulator type [14]. The forward geometrics and kinematics of parallel robots consists in determining the position and the orientation of the platform when the joints variables are known [15]. G. Gogu [9] makes a critical review on the mobility calculation for mechanisms/manipulators, and presents a critical analysis of various methods existing in the literature in the last 150 years.

2. SPATIAL PARALLEL MANIPULATOR (SPMP) WITH 4 ACTUATORS, RSS TYPE

2.1. Mobility calculation

For this type of parallel manipulator (Fig.1a), the mobile platform is linked to the fixed platform through four actuators. Each actuator has a motor kinematical chain $A_iB_iC_i$ (with elements *1* and *2*), with RSS *(Revolute – Spherical – Spherical pairs)* structure. The platforms are linked by a central kinematical chain *OP* with a single element 1*, having one cylindrical (C) joint and one spherical (*S*) joint (Fig.1b). The fixed axes can be situated on any four sides from the fixed pentagon (Fig.1a) or can be situated on the sides of a square (Fig.1c).

The mobility (Fig.1a) is calculated with general formula [1]:

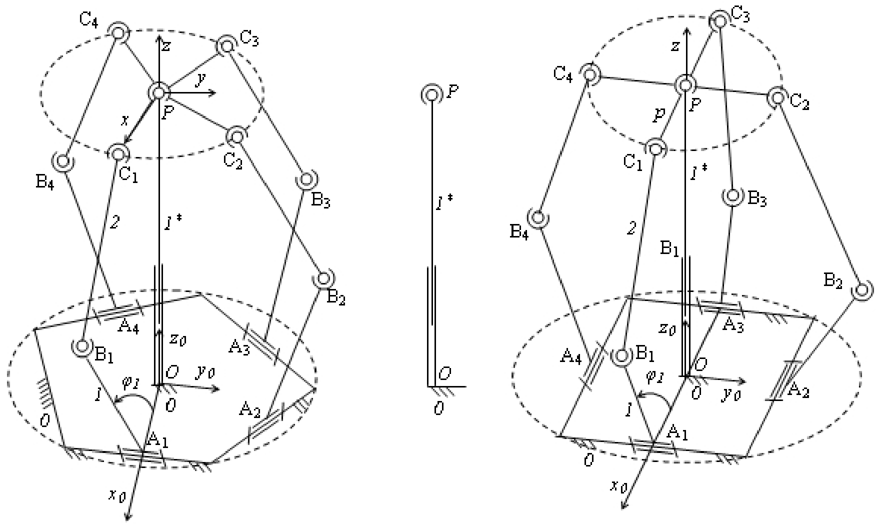
$$M = \sum_{m=1}^{5} mC_m - \sum_{r=2}^{6} rN_r \ . \tag{1}$$

In formula (1), it was noted: N_r - the number of closed contours, with the r rank; C_m - the number of the joints of functional class $m \in [1,(r-1)]$).

For the parallel manipulator presented in Fig.1a, c: m=1 is the mobility of the A_i joints; m=2 is the mobility of O joint; m=3 is the mobility of B_i, C_i and P joints; C_m is the number of kinematical joints with m mobility: C_1=4, C_2=1, C_3=9; r=6 is the rank of associated space, for a independent closed contour; N_r=4 is the number of independent closed contours.

By replacing these values in (1), results: $M = (1\times 4 + 2\times 1 + 3\times 9) - 6\times 4 = 9$.

From the nine mobility degree, four are *active mobility's* (by the four actuators) and five mobility's are *passive* (independent rotations of the four bars 2 and of the vertical rod 1*).



a) b) c)

Fig. 1 – Kinematic schema of SPMp with four legs, RSS type.

2.2. Direct kinematics

The direct kinematics requires the knowledge of bars 1 positions (Fig.1a), given by φ_i (i=1, 2, 3, 4). The A_i (i=1, 2, 3, 4) coordinates can be obtained in function of the circumscribed circle radius R of the regular fixed pentagon (Fig.1a):

$$x_{A_1} = R\cos 36^0;\ x_{A_2} = R\cos 36^0 \cos 72^0;\ x_{A_3} = R\cos 36^0 \cos 144^0;\ x_{A_4} = R\cos 36^0 \cos 144^0;\ y_{A_1} = 0;$$
$$y_{A_2} = R\cos 36^0 \sin 72^0;\ y_{A_3} = R\cos 36^0 \sin 144^0;\ y_{A_4} = R\cos 36^0 \sin 144^0.$$

The coordinates of mobile points B_i (i = 1, 2, 3, 4) are expressed in function of A_i (i = 1, 2, 3, 4) coordinates, A_iB_i=l_1 and φ_i (i = 1, 2, 3, 4).

Thus, in the first case (Fig.1a):

$$x_{B_1} = R\cos 36^0 - l_1 \cos\varphi_1;\ x_{B_2} = (R\cos 36^0 - l_1\cos\varphi_2)\cos 72^0;$$
$$x_{B_3} = (R\cos 36^0 - l_1\cos\varphi_3)\cos 144^0;\ x_{B_4} = (R\cos 36^0 - l_1\cos\varphi_4)\cos 144^0; \tag{2}$$

$$y_{B_1} = 0;\ \ y_{B_2} = (R\cos 36^0 - l_1\cos\varphi_2)\sin 72^0;\ y_{B_3} = (R\cos 36^0 - l_1\cos\varphi_3)\sin 144^0;$$
$$y_{B_4} = -(R\cos 36^0 - l_1\cos\varphi_4)\sin 144^0; \tag{3}$$

$$z_{B_1} = l_1\sin\varphi_1;\ z_{B_2} = l_1\sin\varphi_2;\ z_{B_3} = l_1\sin\varphi_3;\ \ z_{B_4} = l_1\sin\varphi_4; \tag{4}$$

In the second case, presented in Fig.1c, the A_i (i = 1, 2, 3, 4) coordinates are expressed in function of the radius of the square circumscribed circle:

$$x_{A_1} = \tfrac{1}{2}R\sqrt{2};\ x_{A_2} = 0;\ x_{A_3} = -\tfrac{1}{2}R\sqrt{2};\ x_{A_4} = 0;$$
$$y_{A_1} = 0;\ \ y_{A_2} = \tfrac{1}{2}R\sqrt{2};\ y_{A_3} = 0;\ y_{A_4} = -\tfrac{1}{2}R\sqrt{2}.$$

In this case, the Cartesian coordinates B_i (i = 1, 2, 3, 4) are:

$$x_{B_1} = \tfrac{1}{2}R\sqrt{2} - l_1\cos\varphi_1;\ x_{B_2} = 0;\ x_{B_3} = -\tfrac{1}{2}R\sqrt{2} - l_1\cos\varphi_3;\ x_{B_4} = 0; \tag{2'}$$

$$y_{B_1} = 0;\ \ y_{B_2} = \tfrac{1}{2}R\sqrt{2} + l_1\cos\varphi_2;\ y_{B_3} = 0;\ y_{B_4} = -\tfrac{1}{2}R\sqrt{2} + l_1\cos\varphi_4; \tag{3'}$$

$$z_{B_1} = l_1\sin\varphi_1;\ z_{B_2} = l_1\sin\varphi_2;\ z_{B_3} = l_1\sin\varphi_3;\ z_{B_4} = l_1\sin\varphi_4; \tag{4'}$$

The sphere equations with B_i (i=1, 2, 3, 4) centers and B_iC_i=l_2 radius is:

$$(x_{C_i} - x_{B_i})^2 + (y_{C_i} - y_{B_i})^2 + (z_{C_i} - z_{B_i})^2 = l_2^2,\ i = 1,\cdots,4 \tag{5,6,7,8}$$

where the B_i (i=1, 2, 3, 4) coordinates are given by (2/2'), (3/3') and (4/4').

The distances between C_1, C_2, C_3 and C_4 (the vertices of the pentagon platform p) are known constant lengths:

$$\overline{C_1C_2} = l_{12};\ \overline{C_2C_3} = l_{23};\ \overline{C_3C_4} = l_{34};\ \ \overline{C_4C_1} = l_{41}.$$

These constrains are given by four scalar equations:

$$(x_{C_2} - x_{C_1})^2 + (y_{C_2} - y_{C_1})^2 + (z_{C_2} - z_{C_1})^2 = l_{12}^2; \tag{9}$$

$$(x_{C_3} - x_{C_2})^2 + (y_{C_3} - y_{C_2})^2 + (z_{C_3} - z_{C_2})^2 = l_{23}^2; \tag{10}$$

$$(x_{C_4} - x_{C_3})^2 + (y_{C_4} - y_{C_3})^2 + (z_{C_4} - z_{C_3})^2 = l_{34}^2; \tag{11}$$

$$(x_{C_4} - x_{C_1})^2 + (y_{C_4} - y_{C_1})^2 + (z_{C_4} - z_{C_1})^2 = l_{14}^2. \tag{12}$$

Because the pentagon platform p is rigid, the diagonals of quadrilateral $C_1C_2C_3C_4$ are constant: $\overline{C_1C_3} = l_{13}$. This means:

$$(x_{C_3} - x_{C_1})^2 + (y_{C_3} - y_{C_1})^2 + (z_{C_3} - z_{C_1})^2 = l_{13}^2. \tag{13}$$

Considering the triangle formed by the last three vertices $C_2C_3C_4$ of the mobile platform p, another equation results from condition that C_1 to be included in $C_2C_3C_4$ plane:

$$\begin{vmatrix} x_{C_1}-x_{C_2} & y_{C_1}-y_{C_2} & z_{C_1}-z_{C_2} \\ x_{C_3}-x_{C_2} & y_{C_3}-y_{C_2} & z_{C_3}-z_{C_2} \\ x_{C_4}-x_{C_2} & y_{C_4}-y_{C_2} & z_{C_4}-z_{C_2} \end{vmatrix} = 0\,. \tag{14}$$

The fifth kinematical chain *OP*, with *CS* structure, which links the two platforms (Fig.1b), introduces the restriction that *P* has to be situated on the vertical axis Oz_0:

$$x_P = 0\,, \quad y_P = 0\,. \qquad (15,\ 16)$$

P is the quadrilateral $C_1C_2C_3C_4$ center, meaning:

$$x_{C_1} + x_{C_2} + x_{C_3} + x_{C_4} = 0; \qquad (15')$$

$$y_{C_1} + y_{C_2} + y_{C_3} + y_{C_4} = 0\,. \qquad (16')$$

Thus, the equations (5, 6, ..., 14, 15', 16') form a system of 12 *scalar equations* with *12 parameters* (the C_1, C_2, C_3, C_4 Cartesian coordinates).

The 12 equations were solved for the case presented in Fig.1c, for the following values: $\varphi_1 = 120^0$, $\varphi_2 = 110^0$, $\varphi_3 = 100^0$, $\varphi_4 = 115^0$, $R = 20$, $l_1 = 20$. We need the C_i points to be the square vertices, which has the circumscribed circle radius of 40 units. Therefore, the square lengths of the mobile platform must be: $l_{12}=l_{23}=l_{34}=l_{41}=20\sqrt{2}$ and $l_{13}=40$ units.

First, we determined the B_i values, using the following syntax in Mathematica:

$$\begin{aligned}
&\{X_{B1} \to R * N[\tfrac{\sqrt{2}}{2}] - l_1 \,Cos[\varphi_1],\\
&\quad X_{B2} \to 0,\\
&\quad X_{B3} \to -R * N[\tfrac{\sqrt{2}}{2}] + l_1 \,Cos[\varphi_3],\\
&\quad X_{B4} \to 0,\\
&\quad Y_{B1} \to 0,\\
&\quad Y_{B2} \to R * N[\tfrac{\sqrt{2}}{2}] - l_1 \,Cos[\varphi_2],\\
&\quad Y_{B3} \to 0,\\
&\quad Y_{B4} \to -R * N[\tfrac{\sqrt{2}}{2}] + l_1 \,Cos[\varphi_4],\\
&\quad Z_{B1} \to\\
&\qquad N[l_1 \,Sin[\varphi_1]\ /.\ \{R \to 20,\ l_1 \to 20,\ \varphi_1 \to 120\ Degree,\ \varphi_2 \to 110\ Degree,\\
&\qquad\quad \varphi_3 \to 100\ Degree,\ \varphi_4 \to 115\ Degree\}],\\
&\quad Z_{B2} \to\\
&\qquad N[l_1 \,Sin[\varphi_2]\ /.\ \{R \to 20,\ l_1 \to 20,\ \varphi_1 \to 120\ Degree,\ \varphi_2 \to 110\ Degree,\\
&\qquad\quad \varphi_3 \to 100\ Degree,\ \varphi_4 \to 115\ Degree\}],\\
&\quad Z_{B3} \to\\
&\qquad N[l_1 \,Sin[\varphi_3]\ /.\ \{R \to 20,\ l_1 \to 20,\ \varphi_1 \to 120\ Degree,\ \varphi_2 \to 110\ Degree,\\
&\qquad\quad \varphi_3 \to 100\ Degree,\ \varphi_4 \to 115\ Degree\}],\\
&\quad Z_{B4} \to\\
&\qquad N[l_1 \,Sin[\varphi_4]\ /.\ \{R \to 20,\ l_1 \to 20,\ \varphi_1 \to 120\ Degree,\ \varphi_2 \to 110\ Degree,\\
&\qquad\quad \varphi_3 \to 100\ Degree,\ \varphi_4 \to 115\ Degree\}]\}\ /.\\
&\{R \to 20,\ l_1 \to 20,\ \varphi_1 \to 120\ Degree,\ \varphi_2 \to 110\ Degree,\ \varphi_3 \to 100\ Degree,\\
&\ \varphi_4 \to 115\ Degree\}
\end{aligned}$$

We obtained the following values:

$$\{X_{B1}\to 24.1421,\ X_{B2}\to 0,\ X_{B3}\to -17.6151,\ X_{B4}\to 0,\ Y_{B1}\to 0,\ Y_{B2}\to 20.9825,\ Y_{B3}\to 0,\ Y_{B4}\to -22.5945,\ Z_{B1}\to 17.3205,\ Z_{B2}\to 18.7939,\ Z_{B3}\to 19.6962,\ Z_{B4}\to 18.1262\}$$

Second, we built the first equation, using the syntax:

$$\begin{aligned}&(X_{C1}-X_{B1})^2+(Y_{C1}-Y_{B1})^2+(Z_{C1}-Z_{B1})^2==l_2{}^2\ /.\\ &\{R\to 20,\ l_1\to 20,\ l_2\to 30,\ l_{12}\to 28.284271247461902`,\ l_{23}\to 28.284271247461902`,\\ &l_{34}\to 28.284271247461902`,\ l_{14}\to 28.284271247461902`,\ l_{13}\to 40,\\ &\varphi_1\to 120\,\mathrm{Degree},\ \varphi_2\to 110\,\mathrm{Degree},\ \varphi_3\to 100\,\mathrm{Degree},\ \varphi_4\to 115\,\mathrm{Degree}\}\ /.\\ &\{X_{B1}\to 24.14213562373095`,\ X_{B2}\to 0,\ X_{B3}\to -17.615099177069556`,\ X_{B4}\to 0,\\ &Y_{B1}\to 0,\ Y_{B2}\to 20.982538490244323`,\ Y_{B3}\to 0,\ Y_{B4}\to -22.594500858544937`,\\ &Z_{B1}\to 17.32050807568877`,\ Z_{B2}\to 18.79385241571817`,\ Z_{B3}\to 19.69615506024416`,\\ &Z_{B4}\to 18.126155740733`\}\end{aligned}$$

We obtained:

$$(-24.1421+X_{C1})^2+Y_{C1}^2+(-17.3205+Z_{C1})^2==900$$

Similarly, we built all the 12 equations.
The syntax for solving the equations system is:

$$\begin{aligned}&\mathbf{FindRoot}\Big[\Big\{(-24.14213562373095`+X_{C1})^2+Y_{C1}^2+(-17.32050807568877`+Z_{C1})^2==900,\\ &X_{C2}^2+(-20.982538490244323`+Y_{C2})^2+(-18.79385241571817`+Z_{C2})^2==900,\\ &(17.615099177069556`+X_{C3})^2+Y_{C3}^2+(-19.69615506024416`+Z_{C3})^2==900,\\ &X_{C4}^2+(22.594500858544937`+Y_{C4})^2+(-18.126155740733`+Z_{C4})^2==900,\\ &(-X_{C1}+X_{C2})^2+(-Y_{C1}+Y_{C2})^2+(-Z_{C1}+Z_{C2})^2==800,\\ &(-X_{C2}+X_{C3})^2+(-Y_{C2}+Y_{C3})^2+(-Z_{C2}+Z_{C3})^2==800,\\ &(-X_{C3}+X_{C4})^2+(-Y_{C3}+Y_{C4})^2+(-Z_{C3}+Z_{C4})^2==800,\\ &(-X_{C1}+X_{C4})^2+(-Y_{C1}+Y_{C4})^2+(-Z_{C1}+Z_{C4})^2==800,\\ &(-X_{C1}+X_{C3})^2+(-Y_{C1}+Y_{C3})^2+(-Z_{C1}+Z_{C3})^2==1600,\\ &-(-X_{C2}+X_{C4})(-Y_{C2}+Y_{C3})(Z_{C1}-Z_{C2})+(-X_{C2}+X_{C3})(-Y_{C2}+Y_{C4})(Z_{C1}-Z_{C2})+\\ &\quad(-X_{C2}+X_{C4})(Y_{C1}-Y_{C2})(-Z_{C2}+Z_{C3})-(X_{C1}-X_{C2})(-Y_{C2}+Y_{C4})(-Z_{C2}+Z_{C3})-\\ &\quad(-X_{C2}+X_{C3})(Y_{C1}-Y_{C2})(-Z_{C2}+Z_{C4})+(X_{C1}-X_{C2})(-Y_{C2}+Y_{C3})(-Z_{C2}+Z_{C4})==0,\\ &X_{C1}+X_{C2}+X_{C3}+X_{C4}==0,\ Y_{C1}+Y_{C2}+Y_{C3}+Y_{C4}==0\Big\},\ \{X_{C1},12\},\ \{Y_{C1},0\},\\ &\{Z_{C1},45\},\ \{X_{C2},0\},\ \{Y_{C2},12\},\ \{Z_{C2},45\},\ \{X_{C3},-13\},\ \{Y_{C3},10\},\ \{Z_{C3},45\},\\ &\{X_{C4},0\},\ \{Y_{C4},-12\},\ \{Z_{C4},45\}\Big]\end{aligned}$$

We obtained the solution:

$$\{X_{C1}\to 16.7779,\ Y_{C1}\to 10.7563,\ Z_{C1}\to 44.3403,\ X_{C2}\to -10.7572,\ Y_{C2}\to 16.8539,\ Z_{C2}\to 46.4929,\ X_{C3}\to -16.7779,\ Y_{C3}\to -10.7563,\ Z_{C3}\to 47.689,\ X_{C4}\to 10.7572,\ Y_{C4}\to -16.8539,\ Z_{C4}\to 45.5365\}$$

For any start values in iteration, we obtained the same solution.
This solution is verified with the following syntax:

$$\begin{aligned}&\Big\{(-24.14213562373095`+X_{C1})^2+Y_{C1}^2+(-17.32050807568877`+Z_{C1})^2,\\ &X_{C2}^2+(-20.982538490244323`+Y_{C2})^2+(-18.79385241571817`+Z_{C2})^2,\\ &(17.615099177069556`+X_{C3})^2+Y_{C3}^2+(-19.69615506024416`+Z_{C3})^2,\\ &X_{C4}^2+(22.594500858544937`+Y_{C4})^2+(-18.126155740733`+Z_{C4})^2,\end{aligned}$$

$$
\begin{aligned}
&(-X_{C1}+X_{C2})^2+(-Y_{C1}+Y_{C2})^2+(-Z_{C1}+Z_{C2})^2,\\
&(-X_{C2}+X_{C3})^2+(-Y_{C2}+Y_{C3})^2+(-Z_{C2}+Z_{C3})^2,\\
&(-X_{C3}+X_{C4})^2+(-Y_{C3}+Y_{C4})^2+(-Z_{C3}+Z_{C4})^2,\\
&(-X_{C1}+X_{C4})\,\hat{}\,2+(-Y_{C1}+Y_{C4})^2+(-Z_{C1}+Z_{C4})^2,\\
&(-X_{C1}+X_{C3})^2+(-Y_{C1}+Y_{C3})^2+(-Z_{C1}+Z_{C3})^2,\\
&-(-X_{C2}+X_{C4})(-Y_{C2}+Y_{C3})(Z_{C1}-Z_{C2})+(-X_{C2}+X_{C3})(-Y_{C2}+Y_{C4})(Z_{C1}-Z_{C2})+\\
&\quad(-X_{C2}+X_{C4})(Y_{C1}-Y_{C2})(-Z_{C2}+Z_{C3})-(X_{C1}-X_{C2})(-Y_{C2}+Y_{C4})(-Z_{C2}+Z_{C3})-\\
&\quad(-X_{C2}+X_{C3})(Y_{C1}-Y_{C2})(-Z_{C2}+Z_{C4})+(X_{C1}-X_{C2})(-Y_{C2}+Y_{C3})(-Z_{C2}+Z_{C4}),\\
&X_{C1}+X_{C2}+X_{C3}+X_{C4},\ Y_{C1}+Y_{C2}+Y_{C3}+Y_{C4}\}\ /.\\
&\{X_{C1}\to 16.77794006821789`,\ Y_{C1}\to 10.756266623814613`,\\
&Z_{C1}\to 44.340340646184046`,\ X_{C2}\to -10.75723143100907`,\\
&Y_{C2}\to 16.85388111019362`,\ Z_{C2}\to 46.49288056072506`,\\
&X_{C3}\to -16.7779400682179`,\ Y_{C3}\to -10.756266623814602`,\\
&Z_{C3}\to 47.68904510296569`,\ X_{C4}\to 10.75723143100908`,\\
&Y_{C4}\to -16.853881110193626`,\ Z_{C4}\to 45.536505188424684`\}
\end{aligned}
$$

We obtained for the equations left side, the following numerical results:

$$\{900.,\ 900.,\ 900.,\ 900.,\ 800.,\ 800.,\ 800.,\ 800.,\ 1600.,\ -6.48015\times 10^{-12},\ 0.,\ 0.\}$$

It is observed that the solution verifies the equations system.
The graphical representation of the obtained solution is shown below (Fig. 2):

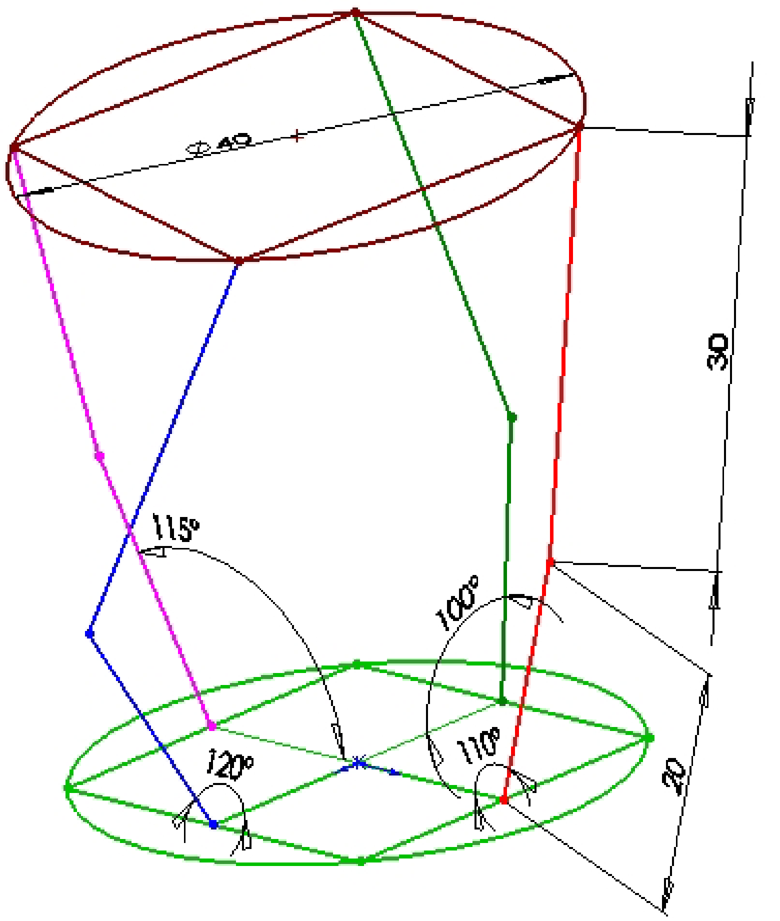


Fig. 2 – Graphical representation of the obtained solution for SPMp, with four legs, RSS type.

3. CONCLUSIONS

The specialized software allows a quick solving of the PMp direct kinematics, without making guesses or matches. It offers precise solutions, easily to be verified on the virtual model made by means of SolidWorks. This 3D PMp model can be kinematically simulated in order to find the right adjustments that are needed in the design process. The main purpose of this paper was the use of specialized software to obtain the right design solution.

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Received November 4, 2009