

ON THE VARIATIONAL FORMULATION OF A CHEMICALLY ACTIVE SHALE CONSTITUTIVE MODEL

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This paper deals with the domain of the physico-chemical exchanges coupling and of the poro-mechanical behavior of clay. A short presentation of the constitutive model, a semianalytical solution for the borehole problem and the numerical formulation of a chemo-poroelastic model are presented. The variational formulation imposes certain restrictions on some constitutive parameters of the model that is important for the model calibration. The solution of a borehole drilled in a chemically active shale layer using the model is presented as a validation for further FEM implementation of the problem. Borehole mud pressure at rupture via Mohr-Coulomb criterion is also calculated in the framework of the model.

Key words: Rock mechanics; Variational formulation; Chemo-poroelasticity; Finite element; Chemo-osmotic effect; Wellbore stability.

1. INTRODUCTION

It has been devoted a lot of works to model the behavior of geomaterials. The constitutive models used for the domain of different problems from geomechanics are for instance, linear elastic or nonlinear [1–3], elastoplastic [5, 10], viscoelastic, viscoplastic [4, 8, 15–19], poroelastic [11, 12], thermoelastic [22], thermo-poroelastic [6], thermo-poro-viscoplastic [6], etc.

In this paper we present the variational approach of a chemo-poroelastic theory for describing chemically active shale behavior. The theory considers chemical and poroelastic processes and couples ion transfer in the mud/shale system to stresses and pore pressure. This theory can be degenerated into two limiting cases, one corresponding to the absence of any chemical interaction (but with ion motion) and the other one which can be characterized as a perfect ion exclusion membrane theory.

First, we summarize the Biot type of chemoporoelasticity. Then, the variational formulation of the problem is presented and some restrictions on material parameters, as well. The field equations are then specialized to solve initial boundary problems, characterized by an irrotational displacement field. The solution of the borehole drilled in an active chemically shale is presented by developing the solution given in Detournay&al [9]. The Laplace transformed field equations are solved explicitly for the borehole problem to yield the solute mass fraction, pore pressure and the stress distributions around the borehole. As time increases, the transfer of ions leads to osmotic pressure dissipation and re-establishment of a pore pressure regime characteristic of hydraulic flow. Furthermore, it is observed that while the well bore may be supported by a mud pressure of significant magnitude, the rock can experience a tensile effective radial stress due to the physicochemical interaction between the shale and the drilling mud. Hence, the contribution of the physicochemical processes can significantly affect the stress and pore pressure distributions around a borehole and should be considered when optimizing drilling mud properties. For this class of problems, all the material parameters can in principle be measured in oedometric tests. The calculation of borehole mud pressure at rupture p_{mR} via Mohr-Coulomb criterion is also presented.

Previous work on this topic was performed by Mody&Hale [14], Sherwood [20, 21], Tan&al [23], Ghassemi&Diek [13] on the same approach of macroscopic scale, while microscopic approach was accomplished by Coussy [6] and Coussy&al [7].

2. MODEL OF CHEMOPOROELASTICITY

2.1. The constitutive equations

The constitutive equations are expressed in terms of the conjugated "stress" $\{d\boldsymbol{\sigma}, dp, d\chi\}$ and "strain" $\{d\boldsymbol{\varepsilon}, d\zeta, d\theta\}$ (see [9]):

$$d\zeta = v_w dm_w + v_s dm_s, \quad d\theta = v/\chi dm_w + v/(1-\chi) dm_s \quad (1)$$

where χ is the solute molar fraction, $d\zeta$ the increment of the fluid content, $d\theta$ the relative increment of salt content, dm_w and dm_s are the water and salt mass respectively, expressed as moles per unit reference of the porous solid. The form of the constitutive relations can be written as (see [9]):

$$d\boldsymbol{\varepsilon} = C d\boldsymbol{\sigma} + b C dp - \alpha \beta C N d\chi, \quad d\zeta = b C d\boldsymbol{\sigma} + b C B^{-1} dp - \beta b B^{-1} C N d\chi, \quad d\theta = -\alpha b C d\boldsymbol{\sigma} - \beta b C B^{-1} dp + \gamma d\chi, \quad (2)$$

where the symmetry of the constitutive parameters derives directly from the symmetry in incremental form of the coupled constitutive equations and the relations that express the chemical potentials $d\mu_w$ and $d\mu_s$ in terms of the pore pressure increment dp and $d\chi$. Also, v_w, v_s and v denote the molar volume of the solvent, of the solute and total volume respectively. The parameters introduced above can be organized in three groups:

The first group corresponds to the classic poroelastic parameters, i.e. the drained volumetric compliance C , the Biot stress coefficient b and the Skempton pore pressure coefficient B , with $0 \leq b \leq 1$, $0 \leq B \leq 1$, but $b \cong B \cong 1$ for a shale. The second group consists of two numbers α and β , which quantifies the chemo-mechanical and the chemo-hydraulic coupling. It can be shown that α and β are constrained to be in the range $[0,1]$ considering the two limiting cases corresponding either to the absence of the chemical interaction or to the immobility of the salt ions referred to as "perfect ion exclusion membrane model". The third group comprises the number γ and a parameter N with dimension of a stress: $N = RT/v$, where R is the gas constant and T the absolute temperature. One can derive that $\gamma = \chi(1-\chi)/\varphi = X/\varphi$, where φ is the porosity of the shale.

2.2. Transport equations

We introduce (see [9]) the specific discharge \mathbf{q} and the relative solute flux \mathbf{r} as:

$$\mathbf{q} = v_w \mathbf{I}_w + v_s \mathbf{I}_s, \quad \mathbf{r} = v/\chi \mathbf{I}_w + v/(1-\chi) \mathbf{I}_s, \quad (3)$$

where \mathbf{I}_w and \mathbf{I}_s are the mass flux of water and salt respectively.

Hence the transport equations can be written as:

$$\mathbf{q} = -\kappa \nabla p + \mathfrak{R} \kappa \nabla \chi, \quad \mathbf{r} = \mathfrak{R} \kappa \nabla p - \gamma D_s \nabla \chi, \quad (4)$$

where κ is the mobility coefficient defined as the ratio of the intrinsic permeability k over the dynamic viscosity μ and \mathfrak{R} is a reflection coefficient, $0 \leq \mathfrak{R} \leq 1$.

2.3. Balance laws

The equilibrium equation in the absence of body forces is: $\nabla \cdot \boldsymbol{\sigma} = 0$. In terms of the specific discharge \mathbf{q} and the relative solute flux \mathbf{r} the balance laws are: $\nabla \cdot \mathbf{q} = -\zeta$, $\nabla \cdot \mathbf{r} = -\theta$.

3. DISCRETISATION OF THE PROBLEM

Let us note Ω the volume occupied by the chemically active shale. The fields of stress tensor $\boldsymbol{\sigma}$, the specific discharge \mathbf{q} , the relative solute flux \mathbf{r} , the fluid content ζ and the salt content θ are supposed known on the domain Ω at the instant t_{n-1} . So, the unknown are the increments $\Delta \boldsymbol{\sigma}, \Delta \mathbf{q}, \Delta \mathbf{r}, \Delta \zeta, \Delta \theta$ between t_{n-1} and t_n . The discretised conservation laws are, adopting an implicit scheme:

$$\operatorname{div} \Delta \boldsymbol{\sigma} + \rho \Delta F = 0, \quad \operatorname{div}(\mathbf{q}_{n-1} + \Delta \mathbf{q}) - \Delta \zeta / \Delta t = 0, \quad \operatorname{div}(\mathbf{r}_{n-1} + \Delta \mathbf{r}) - \Delta \theta / \Delta t = 0 \quad \text{in } \Omega, \quad (5)$$

where ρ is the shale density and ΔF is the increment of volume force.

The boundary conditions will be given in increments as well. Let us note S_a^d the part of surface of the domain Ω on which the increment Δa is imposed.

$$\begin{aligned} \Delta \boldsymbol{\sigma}_{ij} n_i &= \Delta T_i^d \text{ on } S_{T_i}^d, & \Delta u_i &= \Delta u_i^d \text{ on } S_{U_i}^d, & \Delta p &= \Delta p^d \text{ on } S_p^d, \\ \Delta \mathbf{q} \cdot \mathbf{n} &= \Delta q^d \text{ on } S_q^d, & \Delta \mathbf{r} \cdot \mathbf{n} &= \Delta r^d \text{ on } S_r^d, & \Delta s &= \Delta s^d \text{ on } S_s^d. \end{aligned} \quad (6)$$

The fields of displacement \mathbf{u} , the pore pressure p and the chemical stress s are supposed known at the instant t_{n-1} , so the discretised problem consists of determining the increments $\Delta \mathbf{u}, \Delta p$ and Δs so that the corresponding increments $\Delta \boldsymbol{\sigma}, \Delta \mathbf{q}, \Delta \mathbf{r}, \Delta \zeta, \Delta \theta$ through the constitutive equations and the transport equations satisfy the field equations and the boundary conditions (6).

3.1. Admissible fields. Dualisations

Let us note $S_\sigma(S_{T_i}^d, \Delta \mathbf{T}_i^d, \Delta \mathbf{F})$ the space of stress tensors fields statically admissible with the data $\Delta \mathbf{T}_i^d$ and $\Delta \mathbf{F}$ (the charge increment); $S_p(S_p^d, \Delta p^d)$ the space of pore pressure fields statically admissible with the data Δp^d ; $S_s(S_s^d, \Delta s^d)$ the space of chemical stress fields statically admissible with the data Δs^d .

In the same manner we note $K_u(S_{u_i}^d, \Delta u_i^d)$ the space of displacement fields cinematically admissible with the data Δu_i^d ; $S_q(S_q^d, \Delta \mathbf{q}^d)$ the space of the specific discharge fields cinematically admissible with the data $\Delta \mathbf{q}^d$; $S_r(S_r^d, \Delta \mathbf{r}^d)$ the space of relative solute flux fields cinematically admissible with the data $\Delta \mathbf{r}^d$.

The space S_σ can be defined classically by dualisation. Multiplying the equilibrium equation with a displacement field $\Delta \mathbf{u}'$, kinematically admissible with the null data and then integrating on Ω , we get:

$$\Delta \boldsymbol{\sigma}' \in S_\sigma(S_{T_i}^d, \Delta \mathbf{T}_i^d, \Delta \mathbf{F}) \Leftrightarrow \int_{\Omega} \Delta \boldsymbol{\sigma}' \cdot \Delta \boldsymbol{\varepsilon}' \, d\Omega = \int_{\Omega} \rho \Delta \mathbf{F} \cdot \Delta \mathbf{u}' \, d\Omega + \int_{S_{T_i}^d} \Delta \mathbf{T}_i^d \cdot \Delta \mathbf{u}' \, da, \quad \forall \Delta \mathbf{u}' \in K_u(S_{u_i}^d, 0), \quad (7)$$

where $\Delta \boldsymbol{\varepsilon}' = 1/2 (\operatorname{grad} \Delta \mathbf{u}' + {}^t \operatorname{grad} \Delta \mathbf{u}')$.

Proceeding in the same way, we get:

$$\begin{aligned} \Delta \mathbf{q}' \in S_q(S_q^d, \Delta \mathbf{q}^d), \Delta \zeta' &= -\Delta t \operatorname{div} \mathbf{q}' \text{ in } \Omega \Leftrightarrow \\ -\int_{\Omega} \Delta p \cdot \Delta \zeta' \, d\Omega + \Delta t \int_{\Omega} \operatorname{grad} \Delta p' \cdot \mathbf{q}' \, d\Omega &= \Delta t \int_{S_p^d} \Delta p' \cdot \mathbf{q}^d \, da, \quad \forall \Delta p' \in S_p(S_p^d, 0) \end{aligned} \quad (8)$$

and also

$$\begin{aligned} \Delta \mathbf{r}' \in S_r(S_r^d, \Delta \mathbf{r}^d), \Delta \theta' &= -\Delta t \operatorname{div} \mathbf{r}' \text{ in } \Omega \Leftrightarrow \\ -\int_{\Omega} \Delta s \cdot \Delta \theta' \, d\Omega + \Delta t \int_{\Omega} \operatorname{grad} \Delta s' \cdot \mathbf{r}' \, d\Omega &= \Delta t \int_{S_r^d} \Delta s' \cdot \mathbf{r}^d \, da, \quad \forall \Delta s' \in S_s(S_s^d, 0). \end{aligned} \quad (9)$$

The relations (7), (8), (9) can be written again as:

$$\begin{aligned} \Delta \boldsymbol{\sigma}' \in S_\sigma(S_{T_i}^d, \Delta \mathbf{T}_i^d, \Delta \mathbf{F}), \quad \mathbf{q}' &= \mathbf{q}_{n+1} + \Delta \mathbf{q}' \in S_q(S_q^d, \Delta \mathbf{q}^d), \quad \mathbf{r}' = \mathbf{r}_{n+1} + \Delta \mathbf{r}' \in S_r(S_r^d, \Delta \mathbf{r}^d), \\ \Delta \zeta' &= -\Delta t \operatorname{div} \mathbf{q}' \text{ in } \Omega, \quad \Delta \theta' = -\Delta t \operatorname{div} \mathbf{r}' \text{ in } \Omega \quad \Leftrightarrow \end{aligned} \quad (10)$$

$$\begin{aligned} \int_{\Omega} \left\{ \begin{array}{l} \Delta \boldsymbol{\sigma}' \\ \Delta \zeta' \\ \Delta \theta' \end{array} \right\} \left\{ \begin{array}{l} \Delta \boldsymbol{\varepsilon}' \\ \Delta p' \\ \Delta s' \end{array} \right\} \, d\Omega + \Delta t \int_{S_{T_i}^d} \left\{ \begin{array}{l} \Delta \mathbf{q}' \\ \Delta \mathbf{r}' \end{array} \right\} \left\{ \begin{array}{l} \operatorname{grad} \Delta p' \\ \operatorname{grad} \Delta s' \end{array} \right\} \, d\Omega &= \int_{\Omega} \rho \Delta \mathbf{F} \cdot \Delta \mathbf{u}' \, d\Omega + \int_{S_{T_i}^d} \Delta \mathbf{T}_i^d \cdot \Delta \mathbf{u}' \, da - \\ -\Delta t \int_{\Omega} \operatorname{grad} \Delta p' \cdot \mathbf{q}' \, d\Omega + \Delta t \int_{S_q^d} \Delta p' \cdot \mathbf{q}^d \, da - \Delta t \int_{\Omega} \operatorname{grad} \Delta s' \cdot \mathbf{r}' \, d\Omega + \Delta t \int_{S_r^d} \Delta s' \cdot \mathbf{r}^d \, da, \end{aligned} \quad (11)$$

$$\forall \Delta \mathbf{u}' \in K_u(S_{u_i}^d, 0), \quad \forall \Delta p' \in S_p(S_p^d, 0), \quad \forall \Delta s' \in S_s(S_s^d, 0).$$

3.2 Variational formulation in chemoporoelasticity

The discretised constitutive chemoporoelastic equation and the transport law are, respectively:

$$\begin{Bmatrix} \Delta \boldsymbol{\sigma} \\ -\Delta \zeta \\ -\Delta \theta \end{Bmatrix} = \begin{bmatrix} C & -b & \alpha b \\ -b & -1/M - b\eta/G & bC(\beta B^{-1} - \alpha b) \\ \alpha b & bC(\beta B^{-1} - \alpha b) & \alpha^2 b^2 C - \gamma/N \end{bmatrix} \begin{Bmatrix} \Delta \boldsymbol{\varepsilon} \\ \Delta p \\ \Delta s \end{Bmatrix}, \quad (12)$$

$$\begin{Bmatrix} \Delta \mathbf{q} \\ \Delta \mathbf{r} \end{Bmatrix} = \begin{bmatrix} \kappa & -\kappa \mathfrak{R} \\ -\kappa \mathfrak{R} & \gamma D_s / N \end{bmatrix} \begin{Bmatrix} -\Delta p \\ -\Delta s \end{Bmatrix}. \quad (13)$$

The variational formulation in chemoporoelasticity consists of finding the increments of displacement, pressure and chemical stress $ds = Nd\chi$, the solution of a chemoporoelastic problem characterized by the stationarity property of a scalar functional in the spaces $K_u(S_{u_i}^d, \Delta u_i^d)$, $S_p(S_p^d, \Delta p^d)$ and $S_s(S_s^d, \Delta s^d)$. We will prove this stationarity property.

Let us note $\Delta \mathbf{u}, \Delta p, \Delta s$ a solution of the problem. There are elements of the spaces $K_u(S_{u_i}^d, \Delta u_i^d)$, $S_p(S_p^d, \Delta p^d)$ and $S_s(S_s^d, \Delta s^d)$ so that the stress tensor increment $\Delta \boldsymbol{\sigma}$, the fluid content increment $\Delta \zeta$, the relative increment of salt content $\Delta \theta$, the specific discharge vector $\mathbf{q} = \mathbf{q}_{n-1} + \Delta \mathbf{q}$ and the relative solute flux $\mathbf{r} = \mathbf{r}_{n-1} + \Delta \mathbf{r}$ corresponding to $\Delta \mathbf{u}, \Delta p, \Delta s$ through the constitutive equation (12) and the transport law (13) satisfying the field equation and the boundary conditions in the local form (6) or their dual forms (11).

Let us be $\Delta \mathbf{u}', \Delta p', \Delta s'$ some admissible fields, elements of $K_u(S_{u_i}^d, \Delta u_i^d)$, $S_p(S_p^d, \Delta p^d)$ and $S_s(S_s^d, \Delta s^d)$. The fields $\Delta \mathbf{u} - \Delta \mathbf{u}', \Delta p - \Delta p', \Delta s - \Delta s'$ are then the admissible fields with the null data, i.e. the elements of spaces $K_u(S_{u_i}^d, 0)$, $S_p(S_p^d, 0)$ and $S_s(S_s^d, 0)$.

We consider now that $\Delta \boldsymbol{\sigma}, \Delta \mathbf{q}, \Delta \mathbf{r}, \Delta \zeta, \Delta \theta$, corresponding to $\Delta \mathbf{u}, \Delta p, \Delta s$ through the constitutive equation (12) and the transport law (13) satisfies equation (11), taking as test fields $\Delta \mathbf{u} - \Delta \mathbf{u}', \Delta p - \Delta p', \Delta s - \Delta s'$:

$$\begin{aligned} & \int_{\Omega} \begin{Bmatrix} \Delta \boldsymbol{\varepsilon} \\ \Delta p \\ \Delta s \end{Bmatrix} \begin{bmatrix} 1/C & -b & \alpha b \\ -b & -C(b/B - b^2) & bC(\beta B^{-1} - \alpha b) \\ \alpha b & bC(\beta B^{-1} - \alpha b) & \alpha^2 b^2 C - \gamma/N \end{bmatrix} \begin{Bmatrix} \Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}' \\ \Delta p - \Delta p' \\ \Delta s - \Delta s' \end{Bmatrix} d\Omega + \\ & + \Delta t \int_{\Omega} \begin{Bmatrix} \text{grad} \Delta p \\ \text{grad} \Delta s \end{Bmatrix} \begin{bmatrix} \kappa & -\kappa \mathfrak{R} \\ -\kappa \mathfrak{R} & \gamma D_s / N \end{bmatrix} \begin{Bmatrix} \text{grad}(\Delta p - \Delta p') \\ \text{grad}(\Delta s - \Delta s') \end{Bmatrix} d\Omega = F(\Delta \mathbf{u} - \Delta \mathbf{u}', \Delta p - \Delta p', \Delta s - \Delta s'), \\ & \forall \Delta \mathbf{u}' \in K_u(S_{u_i}^d, \Delta u_i^d), \quad \forall \Delta p' \in S_p(S_p^d, \Delta p^d), \quad \forall \Delta s' \in S_s(S_s^d, \Delta s^d), \end{aligned} \quad (14)$$

where the linear form $F(\Delta \mathbf{u}', \Delta p', \Delta s')$ is defined as:

$$\begin{aligned} F(\Delta \mathbf{u}', \Delta p', \Delta s') &= \Delta t \int_{\Omega} \begin{Bmatrix} \text{grad} p_{n-1} \\ \text{grad} s_{n-1} \end{Bmatrix} \begin{bmatrix} \kappa & -\kappa \mathfrak{R} \\ -\kappa \mathfrak{R} & \gamma D_s / N \end{bmatrix} \begin{Bmatrix} \text{grad} \Delta p' \\ \text{grad} \Delta s' \end{Bmatrix} d\Omega + \int_{\Omega} \rho \Delta \mathbf{F} \cdot \Delta \mathbf{u}' d\Omega + \\ & + \int_{S_i^d} \Delta \mathbf{T}_i^d \cdot \Delta \mathbf{u}' da + \Delta t \int_{S_q^d} \Delta p' \mathbf{q}^d da + \Delta t \int_{S_r^d} \Delta s' \mathbf{r}^d da. \end{aligned} \quad (15)$$

The first member of equation (14) can be written $G(\Delta \mathbf{u}, \Delta p, \Delta s) - G(\Delta \mathbf{u}', \Delta p', \Delta s') + G(\Delta \mathbf{u} - \Delta \mathbf{u}', \Delta p - \Delta p', \Delta s - \Delta s')$, where G is a quadratic form defined as:

$$\begin{aligned}
G(\Delta\mathbf{u}', \Delta p', \Delta s') = & \frac{1}{2} \int_{\Omega} \left\{ \begin{array}{l} \Delta\boldsymbol{\varepsilon}' \\ \Delta p' \\ \Delta s' \end{array} \right\} \left[\begin{array}{ccc} 1/C & -b & \alpha b \\ -b & -C(b/B - b^2) & bC(\beta B^{-1} - \alpha b) \\ \alpha b & bC(\beta B^{-1} - \alpha b) & \alpha^2 b^2 C - \gamma/N \end{array} \right] \left\{ \begin{array}{l} \Delta\boldsymbol{\varepsilon}' \\ \Delta p' \\ \Delta s' \end{array} \right\} d\Omega + \\
& + \frac{1}{2} \Delta t \int_{\Omega} \left\{ \begin{array}{l} \text{grad}\Delta p \\ \text{grad}\Delta s \end{array} \right\} \left[\begin{array}{cc} \kappa & -\kappa\mathfrak{R} \\ -\kappa\mathfrak{R} & \gamma D_s / N \end{array} \right] \left\{ \begin{array}{l} \text{grad}\Delta p \\ \text{grad}\Delta s \end{array} \right\} d\Omega. \quad (16)
\end{aligned}$$

If we note $g(\Delta\mathbf{u}', \Delta p', \Delta s') = G(\Delta\mathbf{u}', \Delta p', \Delta s') - F(\Delta\mathbf{u}', \Delta p', \Delta s')$ then the equation (14) can be written as:

$$\begin{aligned}
g(\Delta\mathbf{u}, \Delta p, \Delta s) - g(\Delta\mathbf{u}', \Delta p', \Delta s') = & -G(\Delta\mathbf{u} - \Delta\mathbf{u}', \Delta p - \Delta p', \Delta s - \Delta s'), \\
\forall \Delta\mathbf{u}' \in K_u(S_{u_i}^d, \Delta u_i^d), \quad \forall \Delta p' \in S_p(S_p^d, \Delta p^d), \quad \forall \Delta s' \in S_s(S_s^d, \Delta s^d). \quad (17)
\end{aligned}$$

As the quadratic form corresponding to the elasticity module tensor C is symmetric and positive defined we get that $G(\Delta\mathbf{u} - \Delta\mathbf{u}', 0, 0) > 0$ if $\Delta\mathbf{u} \neq \Delta\mathbf{u}'$ and the relation (17) leads to the following inequality:

$$g(\Delta\mathbf{u}, \Delta p, \Delta s) \leq g(\Delta\mathbf{u}', \Delta p', \Delta s'), \forall \Delta\mathbf{u}' \in K_u(S_{u_i}^d, \Delta u_i^d). \quad (18)$$

In the same manner, the properties of the coefficients $M, b, \eta, G, \kappa, \gamma, D_s, N$ (more precisely the negativity of $-1/M - b\eta/G, -\kappa, \alpha^2 b^2 C - \gamma/N, -\gamma D_s/N$) imply that $G(\Delta\mathbf{u} - \Delta\mathbf{u}', 0, 0) > 0$ if $\Delta\mathbf{u} \neq \Delta\mathbf{u}'$ and the relation (17) leads to the following inequality:

$$g(\Delta\mathbf{u}, \Delta p, \Delta s) \geq g(\Delta\mathbf{u}, \Delta p', \Delta s'), \forall \Delta p' \in S_p(S_p^d, \Delta p^d), \quad \forall \Delta s' \in S_s(S_s^d, \Delta s^d). \quad (19)$$

Moreover, the properties of the coefficients imply the strict convexity of $g(\Delta\mathbf{u}', \Delta p', \Delta s')$ according with $\Delta\mathbf{u}'$ and its strict concavity according with $\Delta p'$ and $\Delta s'$. Thus, the solution in increment of displacement, pore pressure and molar fraction is unique. Among all the fields of the spaces $K_u(S_{u_i}^d, \Delta u_i^d)$, $S_p(S_p^d, \Delta p^d)$ and $S_s(S_s^d, \Delta s^d)$ this solution minimizes the functional $g(\Delta\mathbf{u}', \Delta p', \Delta s')$ according with $\Delta\mathbf{u}'$ and maximizes it according with $\Delta p'$ and $\Delta s'$. We note: $\delta(\Delta\mathbf{u}) = \Delta\mathbf{u} - \Delta\mathbf{u}'$, $\delta(\Delta p) = \Delta p - \Delta p'$, $\delta(\Delta s) = \Delta s - \Delta s'$ the variations of $\Delta\mathbf{u}', \Delta p', \Delta s'$ corresponding to the admissible fields with the null data on the surfaces $S_{u_i}^d$, S_p^d and S_s^d . The equation (19) means in fact the following result of a variational calculus:

$$\delta g(\Delta\mathbf{u}, \Delta p, \Delta s) = 0 \quad (20)$$

and it represents the **stationarity of g** in $K_u(S_{u_i}^d, \Delta u_i^d)$, $S_p(S_p^d, \Delta p^d)$ and $S_s(S_s^d, \Delta s^d)$ for the solution of the chemoporoelastic problem.

4. THE SEMIANALYTICAL BOREHOLE SOLUTION

It is assumed that the porous rock mass is a semi-infinite body in which a vertical circular opening is made. Therefore only plane strain condition is considered. The depth of the well is much longer than its diameter, so the influence of the ground surface is neglected. It is further assumed that the borehole is drilled instantaneous, provided that the time needed to drill over a distance to about 5 times the radius a of the borehole is much smaller than a characteristic time, $\tau = tD/a^2$ (see [9]). The borehole drilling is simulated by removing at $t = 0$ the stresses that are acting on the borehole wall and setting the pore pressure to zero.

The solution will be found in the vicinity of a certain fixed depth h where the horizontal and vertical components of the initial stress σ_h and σ_v are known and, generally, distinct. It is assumed that in the vicinity of borehole these components are constant and equal with their corresponding value for well depth.

Cylindrical coordinate system is chosen for convenience with axis Oz being the symmetry axis of the well. It is also assumed that the stress components $\sigma_r, \sigma_\theta, \sigma_z$ are principal components. We note P_0 the far-field mean stress, S_0 the stress deviator, p_0 the virgin pore pressure. The non-hydrostatic *in situ* stress field is:

$$\sigma_{xx} = -(P_0 - S_0), \quad \sigma_{yy} = -(P_0 + S_0), \quad \sigma_{xy} = 0, \quad p = p_0. \quad (21)$$

It is denoted by Δ the drilling induced field quantities (stress, pore pressure and salt molar fraction) that are responsible for the deformation of the shale and the flux of the water and salt:

$$\sigma_r = \sigma_0 + \Delta\sigma_r, \quad \sigma_\theta = \sigma_0 + \Delta\sigma_\theta, \quad p = p_0 + \Delta p, \quad x = x_0 + \Delta x. \quad (22)$$

The boundary conditions are then given by:

$$\Delta\sigma_r = -(p_m + \sigma_0)H(t), \quad \Delta p = (p_m - p_0)H(t), \quad \Delta x = (x_m - x_0)H(t), \quad \text{for } r = a \quad (\text{borehole wall}), \quad (23)$$

where p_m represents the mud pressure, x_m the mud salt molar fraction and $H(t)$ the Heaviside function and

$$\Delta\sigma_r = \Delta\sigma_\theta = \Delta p = \Delta x = 0 \quad \text{for } r = \infty. \quad (24)$$

The loading is decomposed into two fundamental modes in order to facilitate the physical interpretation of the problem. It is denoted by the superscript (i) the stress induced by loading mode i . The boundary conditions at $r = a$ for each of the loading modes can be written as follows:

- Mode 1 is associated with a variation of the normal stress at the boundary:

$$\Delta\sigma_r^{(1)} = -(p_m + \sigma_0)H(t), \quad \Delta p^{(1)} = 0, \quad \Delta x^{(1)} = 0 \quad \text{at } r = a. \quad (25)$$

- Mode 2 is associated with a perturbation of the pore pressure and salt molar fraction at the boundary:

$$\Delta\sigma_r^{(2)} = 0, \quad \Delta p^{(2)} = (p_m - p_0)H(t), \quad \Delta x^{(2)} = (x_m - x_0)H(t) \quad \text{at } r = a. \quad (26)$$

The solution derived below is expressed in terms of a dimensionless radial coordinate and a dimensionless characteristic time, respectively: $\vartheta = r/a$ and $\tau = tD/a^2$.

4.1. Field equations and borehole solution

Combining all the governing equations we form a set of field equations consisting of a Navier-type equation and two coupled diffusions equations that are solved for a class of problems characterized by an irrotational displacement field involving infinite or semi-infinite domains. Thus, the volumetric strain ε , pore pressure p and molar fraction x are governed by the following equations (we are referring here to small perturbations from a known equilibrated state):

$$G\varepsilon - \eta p + \alpha\eta s = 0, \quad A_{hh}D\nabla^2 p + A_{hc}D\nabla^2 s = \dot{p}, \quad A_{hc}D\nabla^2 p + A_{cc}D\nabla^2 s = \dot{s}, \quad (27)$$

where $\eta = b(1 - 2\nu)/2(1 - \nu)$, so $\eta \in [0, 1/2]$ and coefficients $A_{hh}, A_{hc}, A_{hc}, A_{cc}$ are defined as in [9].

Mode 1 of loading. For loading mode 1, the solution corresponds to the classical Lamé solution in elasticity. Since the displacement field is characterized by zero volumetric strain, there is no mechanism for pore pressure and change of salt molar fraction and their subsequent diffusion.

Mode 2 of loading. It is convenient to introduce the following scaled quantities: $\Pi(\rho, \theta)$, $\Sigma_r(\rho, \theta)$, $\Sigma_\theta(\rho, \theta)$, $X(\rho, \theta)$, $E(\rho, \theta)$, $U(\rho, \theta)$ where:

$$\begin{aligned} \Pi &= \Delta p^{(2)} / (p_m - p_0), \quad X = \Delta x^{(2)} / (x_m - x_0), \quad E_r = \Gamma \varepsilon_r, \quad E_\theta = \Gamma \varepsilon_\theta, \quad E = \Gamma \varepsilon, \quad U = \Gamma u_r / a, \\ \Sigma_r &= \Delta\sigma_r^{(2)} / \eta(p_m - p_0), \quad \Sigma_\theta = \Delta\sigma_\theta^{(2)} / \eta(p_m - p_0) \end{aligned} \quad (28)$$

and two numbers Γ and Ω characterizing the boundary conditions for mode 2: $\Gamma = G/\eta(p_m - p_0)$ and $\Omega = S(x_m - x_0)/\eta(p_m - p_0)$ where $S = \rho RT$.

The complete solution can be obtained explicitly in terms of the Laplace transform of the various quantities [9]. The solution in time is then calculated using a numerical inversion technique, using Stehfest

method. The approximate solution in time is given by the formula $f(t) \approx \ln 2/t \sum_{s=1}^N C_n \tilde{f}(n \ln 2/t)$ with the

coefficients C_n given by: $C_n = (-1)^{n+N/2} \sum_{s=(n+1)/2}^{\min(n, N/2)} k^{N/2} (2k)! / (N/2 - k)! k! (k-1)! (n-k)! (2k-n)!$. The number

of terms N in the series is even and is typically in the range of 5-20. In our solution is used $N = 5$.

Stress concentration at the borehole can be calculated for $r = a$. For instance, the hoop stress at the borehole wall is given after combining the mode 1 and mode 2 solution as:

$$\sigma_{\theta} = -2\sigma_0 + p_m - 2\eta\varphi_{mh}(p_m - p_0) + 2\eta(1 - \alpha)S(x_m - x_0). \quad (29)$$

4.2. Numerical results

The solution presented above was obtained by a C++ program with following input data: $a = 0.1\text{m}$, $\sigma_0 = 31\text{MPa}$, $p_0 = 15\text{MPa}$, $x_0 = 0.05$, $p_m = 18\text{MPa}$, $x_m = 0.04$, $T = 341\text{K}$; $G = 1000\text{MPa}$, $\eta = 0.25$, $\alpha = 0.6$, $\rho = 10^{-4}\text{Mole/mm}^3$, $\varphi = 0.8$. Three cases were taken into account for the input data: case 1: *chemical and hydraulic loading*, case 2: *chemical loading only*, case 3: *hydraulic loading only*.

Figure 1 represents the isochrones of the pore pressure with radius for various values of the characteristic time in the case 1. The pore pressure varies monotonically under that loading, unlike the loading in case 2 when we observe a peak in the pore pressure profile (fig. 2). The radial stress for the chemical effect only is exhibits in Fig. 3. Figures 4, 5 and 6 represents the isochrones of the hoop stress, salt molar fraction and radial displacement respectively, for various values of characteristic time for chemical and hydraulic loading.

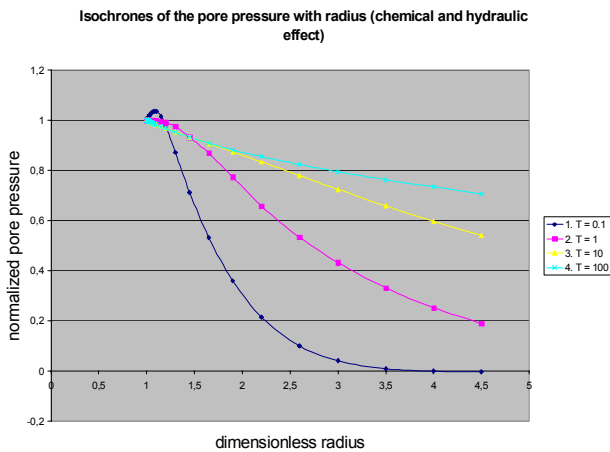


Fig. 1 – Pore pressure vs. radius (case 1).

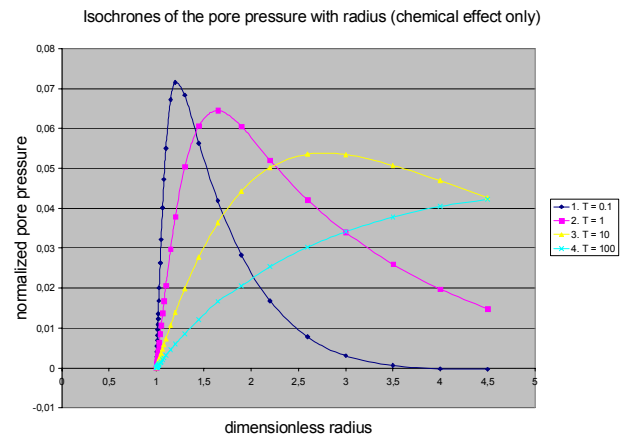


Fig. 2 – Pore pressure vs. radius (case 2).

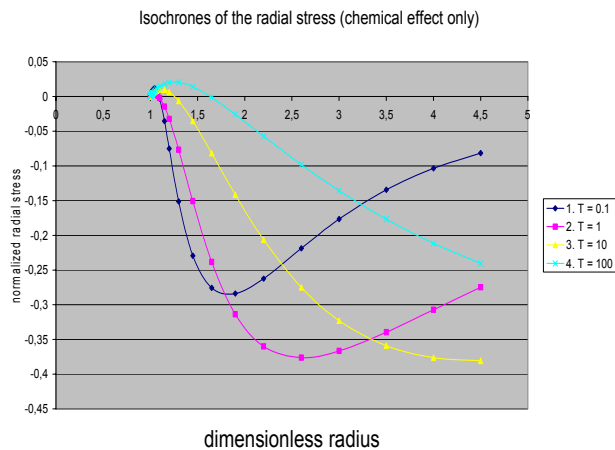


Fig. 3 – Radial stress vs. radius (case 2).

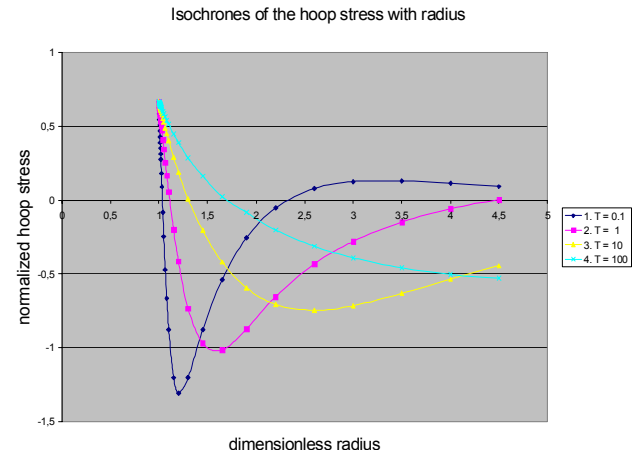


Fig. 4 – Hoop stress vs. radius (case 1).

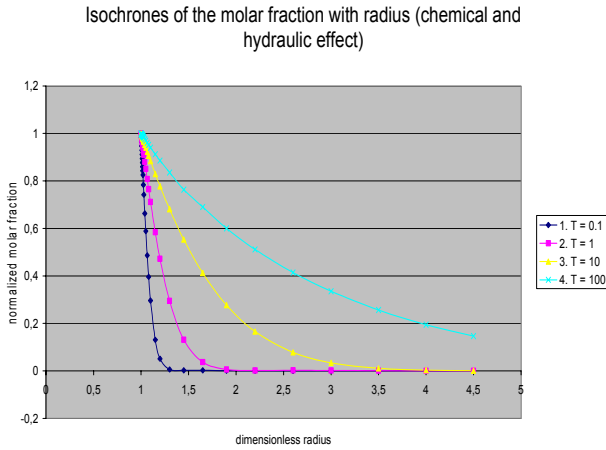


Fig. 5 – Molar fraction vs. radius (case 1).

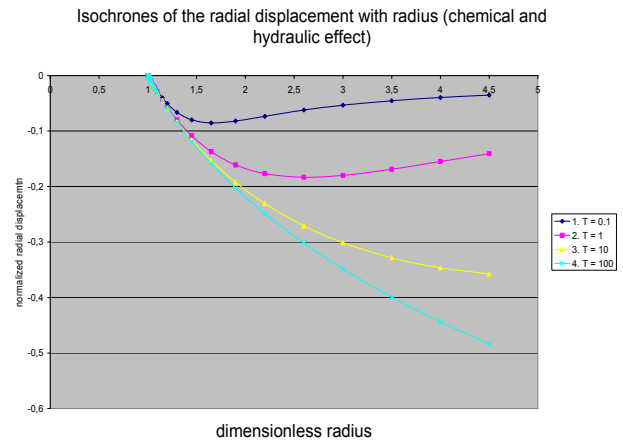


Fig. 6 – Radial displacement vs. radius (case 1).

4.3. Borehole mud pressure at rupture p_{mR} via Mohr-Coulomb criterion

Combining the relation (29) with the Mohr - Coulomb criterion we got:

$$p_{mR} = [2c \cos \phi (1 - \sin \phi) + 2\sigma_0 - 2\eta \varphi_{mh} p_0 - 2\eta(1 - \alpha)N(x_m - x_0)](1 - 2\eta \varphi_{mh}). \quad (30)$$

We note the mud pressure at rupture for the case with no chemical effect (classical poroelasticity):

$$p_{mR}^0 = [2c \cos \phi (1 - \sin \phi) + 2\sigma_0 - 2\eta \varphi_{mh} p_0](1 - 2\eta \varphi_{mh}) \quad (31)$$

and p_{mR}^1, p_{mR}^2 the mud pressure at rupture for the case $x_m < x_0$ and the mud pressure at rupture for the case $x_m > x_0$, respectively, we obtain via (30) and (31): for $x_m < x_0$, $p_{mR}^0 < p_{mR}^1$, and for $x_m > x_0$, $p_{mR}^0 > p_{mR}^1$.

So, the mud pressure at rupture for the case $x_m < x_0$ which is the case less stable, is greater than the mud pressure at rupture for the case with no chemical effect (classical poroelasticity). Similarly, the mud pressure at rupture for the case $x_m > x_0$ which is the case more stable. is smaller than the mud pressure at rupture for the case with no chemical effect.

5. CONCLUSIONS

In this paper, we report a coupled chemo-physical formulation within the framework of Biot's poroelasticity to describe the phenomenon of hydration swelling and chemo-osmotic effects. The effective stress equation is modified by adding an additional swelling term due to the chemical gradient, while the Darcian flux is altered to allow for the membrane effect. A semianalytical solution of the well bore problem was developed to calculate the state of stress, strain pore pressure, salt molar fraction and the pressure at rupture. A detailed variational formulation is provided for the chemo-poroelasticity theory. Further developments concern the FEM solution for more complex geometry of the domain and faster algorithms for the numerical techniques.

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