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AN APPROACH TO WALKING ROBOTS PLANNING AND CONTROL

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Vehicles which operate more or less autonomously are considered belonging to the robot's family. The legged vehicles have the potential advantage to be able to operate over irregular terrain. The planning and control of these robots is considered hierarchically organized on two levels. At the high level, the robot is piloted in the task world. At the local level – operative level – the locomotion function is performed. In this paper, walking orthogonal hexapods, for moving along uneven terrain are the object of study. By kinematic analysis the direct and inverse geometrical models and a differential model are developed. The accommodation to terrain using force sensing is approached too. This approach is useful to write the control algorithms of the analyzed robots or of other structures by analogy.

Key words: Robot control; Walking robots; Mathematic modelling.

1. INTRODUCTION

Vehicles which operate more or less autonomously are considered as belonging to the robot's family. Betwixt these, the legged vehicles have the potential advantage to be able to operate over irregular terrain. Walking robots (WR), in comparison with industrial robots which are dextrous arms (6 or more D.O.F.) able to ensure the precise positioning of the end-effector, comprise a body and legs, each kinematic chains with no more than 3 D.O.F. which support the body and ensure the gait, the locomotion mechanism. Terrain – adaptive walkers have the ability to individually place their feet at desired position on the terrain [1, 2]. The legs can be in supporting state when the legs are in contact with the ground, and the body is moving or not, with respect to the footholds, or in stepping or transfer state when the leg is detached and has a relative movement to the robot's body, somehow looking for a place for the new foothold [3, 9, 10].

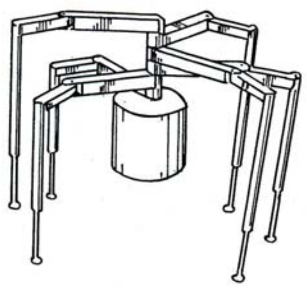

Fig. 1 – Walking vehicle with weaving gait.

With orthogonal walking robots, the body movement is carried out in a plane with a group of actuators, and the vertical movement is performed by another group of actuators [6]. The three-degree-of-freedom orthogonal legs decouple horizontal and vertical motion, leading to simplified planning and control. The legs are kinematic chains with driving (propulsive) joints revolute (R) or prismatic (T), RTT or RRT, with the supporting element pedestal with prismatic joint; the supporting element is always vertical to the foothold [4, 8]. In this paper two orthogonal-legged walker configurations are tackled. The first is termed a weaving walker (Fig. 1) because recovering legs thread through supporting legs.

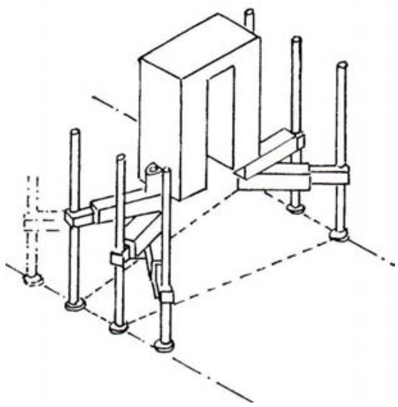

Fig. 2 – Walking robot with circular gait.

The second is termed a circulating walker, because its legs continuously rotate with respect to the body during the course (Fig. 2) of walking. Both have the feature that legs are recovered from rear to place ahead of supporting legs. The legs are arranged in a single stack or in two stacks respectively. The Ambles rover is an example of such a six-legged robot with circulating gait.

2. AN APPROACH OF PLANNING AND CONTROL OF WALKING ROBOTS.

The planning and control of these robots may be hierarchically organized on two levels. A highest level on which the robot is piloted in the task world, along a prescribed trajectory defined in an inertial frame of this world and avoiding unforeseen obstacles based on sensory information and a local one on which the locomotion function is performed [5, 8]. At the highest level the position and orientation of the robot can be piecewise generated at each step, following the imposed trajectory and/or avoiding the unpredictable obstacles perceived by sensors [5, 7].

The piecewise movement of the body may be related to the gait cycle. During the body displacement its characteristic point M, placed in the center of gravity and the origin of a mobile frame suitably attached is considered. For the trajectory generation let's consider an inertial frame O attached to the robot's task world and the mobile frame in M (Fig. 3). A homogenous transformation H defines the position and orientation (PO) of the robot in the frame O. The piecewise displacement provides the displacement of the point M from a point denoted M_0 along the approximated trajectory.

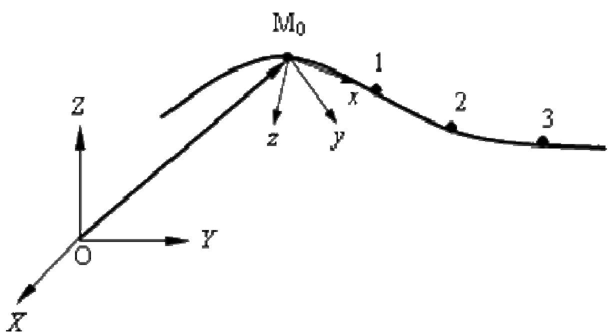

Fig. 3 – The fixed frame and the piecewise displacement.

We start from the premise that the position and orientation of the robot, in the fixed frame are known. For a prescribed trajectory, the intermediate position corresponding to the elementary displacement are generated, with a sampling τ period. Period τ is conditioned by the prescribed velocity $v_t = d_s/d_t$, on the curvilinear abscissa, so that the displacement of the body characteristic point M to be according to the step length. Two kinds of approximation of trajectory are considered by straight line or by circular arc. If the piece of trajectory is approximated by a straight line segment, the body will move from a point M_0 to a point M_1. If the piece of trajectory is approximated by a circular arc, the destination is a point M_2, passing through a point M_1. Along such a segment, in the frame located in M_0, the position references for the intermediate instant points M will be generated with a sampling period $T<<\tau$. The homogenous matrix H describe the position and orientation in a point $M(X, Y, Z)$:

$$H = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & X \\ -\sin\varphi & -\cos\varphi & 0 & Y \\ 0 & 0 & -1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where φ is the orientation angle. Let's consider an elementary displacement on the straight line segment M_0M_1. If the homogenous transformation matrix T_0 describes the PO in M_0 and T_1 the PO in M_1, as in Fig. 4, the transformation P determines the PO in M_1 viewed from M_0, $P = T_0^{-1}T_1$.

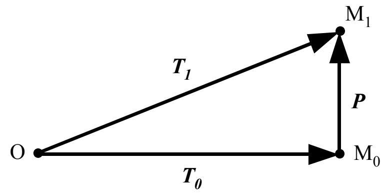

Fig. 4 – The graph of homogeneous transformation for an elementary displacement on a straight line.

If the matrix P is of the form:

$$P = \begin{bmatrix} \xi_{1x} & \xi_{2x} & \xi_{3x} & p_x \\ \xi_{1y} & \xi_{2y} & \xi_{3y} & p_y \\ \xi_{1z} & \xi_{2z} & \xi_{3z} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

p_x, p_y are coordinates of the point M_1 in the M_{oxy} plane and the orientation variation (if any) $\beta=\Delta\varphi=\varphi_1-\varphi_0$ is given by:

$$\beta = \mathrm{tg}^{-1}\frac{\xi_{1y}}{\xi_{1x}}. \tag{3}$$

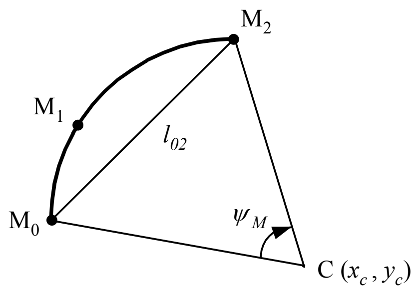

Fig. 5 – Elementary displacement on a circular arc.

The intermediate references can be generated by period T. For an elementary displacement on a circular arc, the circle center $C(x_c, y_c)$ in the frame M_0 (Fig. 5) and two points are considered, M_1 an intermediate one and M_2 at the end of the arc. The relation between these points direct to obtain the coordinates of the circle center x_c, y_c and its radius R. For the circular arc generation with a parameter ψ, in the range $[0, \psi_M]$, ψ_M must be formed. By solving the triangle CM_0M_2 (Fig. 5), one obtains the chord l_{o2} and the angle ψ_M.

$$l_{02} = \sqrt{x_2^2 + y_2^2}, \quad \Psi_M = 2\mathrm{tg}^{-1} \frac{l_{02}}{\sqrt{4R^2 - l_{02}^2}} \tag{4}$$

Given the tangential velocity $v_t = d_s/d_t$, using finite differences, one gets:

$$\Delta\psi = v_t \frac{T}{R}, \tag{5}$$

which is the step of the parameter for the circular arc generation. The generated position references are:

$$x = \pm R(1 - \cos\psi), \quad y = R\sin\Psi, \tag{6}$$

where the sign is dependent on the variation of angle ψ (clockwise or counter clockwise). When the end of a segment is reached (M_1 or M_2), this becomes M_0 for the next elementary segment.

3. THE LOCOMOTION FUNCTION

Being established a method for generating the position of the robot body in the local Cartesian frame, some control equations must be written in terms of joint control variables to obtain the derived movement.

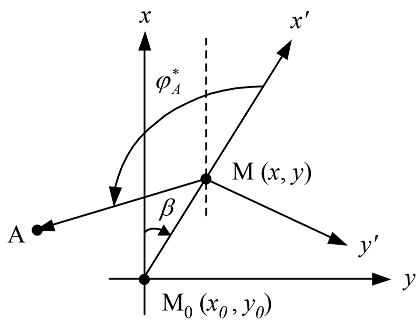

Fig. 6 – Reference systems.

For an elementary displacement M_0M, the scheme shown in Fig. 6, is considered where:

- M_0 is the origin of the reference frame for a segment M_0M;
- M is the origin of the mobile frame in the instant point;
- ρ_s the position vector of the foothold A, in the frame M;
- β the variation of orientation angle;
- φ^*_A the new orientation angle in M.

The locomotion function for a weaving walker. For this purpose it is convenient to take two legs as basic, determining a penthalater planar loop, the propulsive mechanism. The kinematic propulsive chain is different if the legs are RRTR or RTTR chains. The propulsive mechanism for a chain RRRRR is shown in Fig. 7. The position of the point M is univocally determined by two active joints convenient chosen.

The body being fixed on the axis in M, for the orientation control, this must be active connected to an element (MA′ or MB′). The direct geometric model for RRRRR chain (Fig. 7) is:

$$\begin{aligned} \rho_{Ax} &= l_1 \cos\varphi_A - l_2 \cos(\varphi_A + \theta_A) \;;\quad \rho_{Ay} = -l_1 \sin\varphi_A + l_2 \sin(\varphi_A + \theta_A), \\ \rho_{Bx} &= l_1 \cos\varphi_B - l_2 \cos(\varphi_B + \theta_B) \;;\quad \rho_{By} = l_1 \sin\varphi_B - l_2 \sin(\varphi_B + \theta_B), \end{aligned} \tag{7}$$

where:

$$\rho_{Ax} = x_A - x\,; \qquad \rho_{Bx} = x_B - x\,;\; \rho_{Ay} = y_A - y\,; \qquad \rho_{By} = y_B - y \tag{8}$$

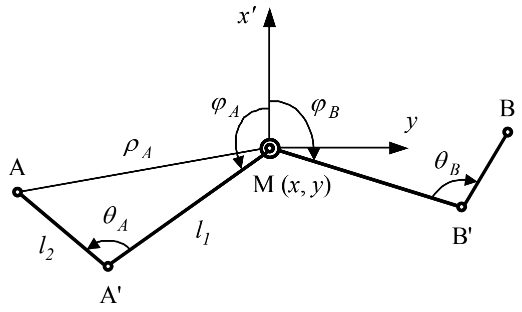

Fig. 7 – The propulsive mechanism chain, type RRRRR.

Angles θ_A, θ_B and φ being known the position of footholds with respect to the frame attached to the body are expressed in equations (7) and (8). On the beginning of an elementary displacement the point M is in M_0. For an incremental displacement in the place M(x,y) angles θ_A, θ_B and φ can be obtained from the inverse geometric model:

$$\cos\theta_A = \frac{(l_1^2 + l_2^2 - \rho_A{}^2)}{2l_1l_2} \;\; ; \;\; \sin\theta_A = +\sqrt{1-\cos^2\theta_A} \;\; ; \;\; \theta_A = \mathrm{tg}^{-1}\frac{\sin\theta_A}{\cos\theta_A}, \tag{9}$$

where $\theta \in (0, \pi)$ and $\rho^2=(x_A-x)^2+(y_A-y)^2$. The same is applicable for θ_B. To get the desired orientation the angle between the axis Mx' and one of the contiguous element (MA′ or MB′) must be controlled. For the taken configuration in Fig. 7, the angle φ_A:

$$\varphi_A = \mathrm{tg}^{-1}\frac{l_2\sin\theta_A}{l_1 - l_2\cos\theta_A} + \mathrm{tg}^{-1}\frac{-\rho_{Ay}}{\rho_{Ax}} \tag{10}$$

or

$$\varphi_A = \mathrm{tg}^{-1}\left[\frac{\rho_{Ax}l_2\sin\theta_A - \rho_{Ay}(l_1 - l_2\cos\theta_A)}{\rho_{Ay}l_2\sin\theta_A + \rho_{Ax}(l_1 - l_2\cos\theta_A)}\right] \tag{11}$$

and

$$\varphi_A^* = \varphi_A + \beta, \tag{12}$$

if a variation of orientation β exists. The differential of position equations provides the necessary elements for building the inverse Jacobean matrix. This is useful in the incremental position control and in the case of velocity control. The kinematic differential model is:

$$\begin{bmatrix} \mathrm{d}\theta_A \\ \mathrm{d}\theta_B \\ \mathrm{d}\varphi_A \end{bmatrix} = J^{-1}\cdot\begin{bmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}\beta \end{bmatrix}, \tag{13}$$

in which:

$$\begin{bmatrix} \dfrac{-\rho_{Ax}}{l_1l_2\sin\theta_A} & \dfrac{-\rho_{Ay}}{l_1l_2\sin\theta_A} & 0 \\ \dfrac{-\rho_{Bx}}{l_1l_2\sin\theta_B} & \dfrac{-\rho_{By}}{l_1l_2\sin\theta_B} & 0 \\ D_x & D_y & 1 \end{bmatrix}, \tag{14}$$

where:

$$D_x = \frac{-1}{\rho^4 \sin\varphi_A}\left[2v\rho_x^2 + 2w\rho_x\rho_y - v\rho^2 - \frac{\rho^2}{l_1 w}\left(w\rho_x^2 + v^*\rho_x\rho_y\right)\right], \tag{15}$$

$$D_y = \frac{-1}{\rho^4 \sin\varphi_A}\left[2w\rho_y^2 + 2v\rho_x\rho_y - w\rho^2 - \frac{\rho^2}{l_2 w}\left(v^*\rho_y^2 + w\rho_x\rho_y\right)\right], \tag{16}$$

in which:

$$v = l_1 - l_2\cos\theta \ ; \ w = l_2\sin\theta \ ; \ v^* = l_1 - v - l_2\cos\theta, \tag{17}$$

for φ_A: $\theta = \theta_A$, and:

$$\rho^2 = \rho_x^2 + \rho_y^2, \tag{18}$$

where ρ_{Ax}, ρ_{Ay} and ρ_{Bx}, ρ_{By}, are given by equation (8).

When the chain MA′A and MB′B comprise prismatic joints in A′ and B′ (Fig. 8):

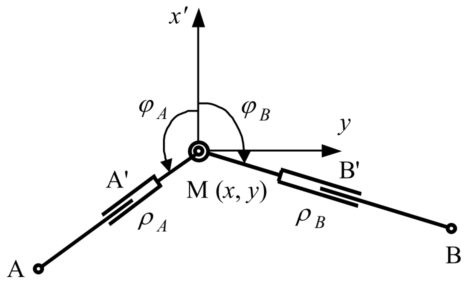

Fig. 8 – Kinematic propulsive chain, type RTRTR.

the direct geometric model is expressed by equations:

$$\begin{aligned} &\rho_{Ax} = x_A - x, \ \ \rho_{Bx} = x_B - x, \\ &\rho_{Ay} = y_A - y, \ \ \rho_{By} = y_B - y, \\ &\mathrm{tg}\varphi_A = \frac{-\rho_{Ay}}{\rho_{Ax}}, \ \ \mathrm{tg}\varphi_B = \frac{\rho_{By}}{\rho_{Bx}}. \end{aligned} \tag{19}$$

The inverse geometric model is:

$$\rho = \sqrt{\rho_x^2 + \rho_y^2}, \ \ \varphi_A = \mathrm{ATAN2}\frac{-\rho_{Ay}}{\rho_{Ax}}, \ \ \varphi_B = \mathrm{ATAN2}\frac{\rho_{By}}{\rho_{Bx}}. \tag{20}$$

The locomotion function for a circulating walker. The plane kinematic chain of propulsive mechanism may be considered with six elements and three actuated joints which ensure the position and orientation of the body. The legs configuration, the same as in the previous case, involves a similar solution. Dependent upon what is the intermediate joint of a leg, revolute or prismatic (like in Fig. 2), for the direct and inverse geometric model, the equations (7, 9, 11, 19, 20) can be used with some adaptations. The control of such walkers is more simplified.

4. THE USE OF FORCE SENSORS FOR FITTING THE FEET TO TERRAIN

The force control is a part of the general control algorithm for walking over irregular terrain. The concept of active compliance is used to accommodate the legs to terrain variations [9]. The commanded vertical component of velocity may be given by

$$\dot{z}_c = \dot{z}_d + k_p(z_d - z_c) + k_f(f_d - f_c) \tag{21}$$

where $\dot{z}_i$ is the desired velocity; z_d and z_c are the desired and actual positions; f_d and f_c are the desired and actual measured forces; k_p and k_f are gains. The real algorithm is a control with variable structure. The position z_d and velocity $\dot{z}_d$ are provided by feet trajectory planning and are modified by the attitude control of robot's body. The desired force f_d is obtained from force-setpoint calculation by a method given below. The measured force on the feet must be predicted.

If the robot's coordinate system is chosen so that the center of mass is the origin, the vertical forces on feet in support should satisfy

$$\sum f_i = G; \quad \sum f_i x_i = 0; \quad \sum f_i y_i = 0. \tag{22}$$

In the case of the robot taken in study, 5 or 4 feet are in support phase and the problem in statically indeterminate. The problem can be solved imposing the condition of minimum deformation energy. The pseudo inverse solution for force with the minimum norm, can be analytically calculated. Klein, Olson and Pugh, prove that the pseudo inverse solution for commanded forces changes linearly with robot motion with respect to a given set of feet in the ground. For a given leg stance the calculated force solution will have the form:

$$f_c(x_a, y_a) = f_{c0} + x_a f_{cx} + y_a f_{cy}, \tag{23}$$

where f_{c0} is the solution for force in the initial position; x_a, y_a are the translation of the body with respect to the supporting feet; f_{cx} and f_{cy} are constant forces for a given stance, and $f_c(x_a, y_a)$ is the force solution when the body has moved since the beginning of the stance. The equation (23), can be written as:

$$Af = \begin{bmatrix} G \\ 0 \\ 0 \end{bmatrix}, \tag{24}$$

where $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ & & x^{\mathrm{T}} & & \\ & & y^{\mathrm{T}} & & \end{bmatrix}$ and f is the vector of vertical forces for the supporting feet, G is the vehicle weight and x, y are the feet coordinates in the chosen frame.

For solving by a generalized inverse method of f, this must be entirely in the row space of A. One of the well known generalized inverse is the pseudo inverse (Moore Penrose) which is defined thus:

$$A^+ = A^{\mathrm{T}}\left(AA^{\mathrm{T}}\right)^{-1}. \tag{25}$$

A more simplified form may be applied using A^T as pseudo inverse. This can be written as:

$$f = A^{\mathrm{T}} \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \tag{26}$$

valid for some values of a, b and c. Substituting (28) into (25) we get

$$AA^{\mathrm{T}} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} G \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

and solving the linear system (29) the values of a, b and c are given. The force can be written as

$$f = a + bx + cy \tag{28}$$

and has 5 components in accordance with the number of the five considered supporting legs.

5. CONCLUSIONS

In this tackling of orthogonal walkers, some elements useful for their operative system building, are developed.

Starting from the premise that the position and orientation of the robot are given by the highest level of control, the navigation system, at the operative level the locomotion function is executed. This carry out the propulsion of the body, with respect to supporting legs and the stepping of the legs in transfer phase.

The algorithms take particular forms depending upon the configuration of the walker. The gait quality is improved if the control is assisted by sensors (force, attitude), providing accommodation to the unpredicted elements of the terrain. The force sensing was approached to conform the feet to terrain.

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