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MULTIOBJECTIVE SUBSET PROGRAMMING PROBLEMS INVOLVING GENERALIZED *D*-TYPE I UNIVEX FUNCTIONS

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We introduce new classes of generalized convex n-set functions that we call d-weak strictly pseudo-quasi type-I univex, d-strong pseudo-quasi type-I univex and d-weak strictly pseudo type-I univex functions. We focus on multiobjective subset programming problem. Sufficient optimality conditions are obtained under the assumptions involving such functions. Duality results are also established for Mond-Weir and general Mond-Weir type dual problems in which the functions involved satisfy appropriate generalized d-type-I univexity conditions. The results obtained in this paper present a refinement and improvement of previously known results in the literature.

Key words: Multiobjective subset programming problem; Efficient solution; D-type-I univex functions; Sufficient optimality conditions; Duality.

1. PRELIMINARIES

Let R^n be the n-dimensional Euclidean space and R_+^n its positive orthant. The following conventions for vectors in R^n will be followed throughout this paper: $x \geqq y \Leftrightarrow x_k \geqq y_k$, $k = 1,2,...,n$; $x \geq y \Leftrightarrow x_k \geq y_k$, $k = 1,2,...,n$ and $x \neq y$; $x > y \Leftrightarrow x_k > y_k$, $k = 1,2,...,n.$ We write $x \in R_+^n$ iff $x \geqq 0$. Let (X, A, μ) be a finite atomless measure space with $L_1(X, A, \mu)$ separable and let d be the pseudometric on A^n defined by

$$d(S,T) = \left[\sum_{k=1}^{n} \mu^2 (S_k \Delta T_k) \right]^{1/2}, \ S = (S_1, S_2, ..., S_n) \in A^n, \ \ T = (T_1, T_2, ..., T_n) \in A^n ,$$

where Δ stands for symmetric difference; thus, (A^n, d) is a pseudometric space. For $h \in L_1(X, A, \mu)$ and $Z \in A$ with characteristic function $\chi_Z \in L_\infty(X, A, \mu)$, the integral $\int_Z h d\mu$ will be denoted by $\langle h, \chi_Z \rangle$.

We next define the notions of differentiability for n-set functions. This was originally introduced by Morris [6] for set functions, and subsequently extended by Corley [1] to n-set functions.

A function $\phi : A \to R$ is said to be differentiable at $S^0 \in A$ if there exists $D\phi(S^0) \in L_1(X, A, \mu)$, called the derivative of ϕ at S^0 and $\psi : A \times A \to R$ such that $\phi(S) = \phi(S^0) + \langle D\phi(S^0), I_S - I_{S^0} \rangle + \psi(S, S^0)$ for each $S \in A$, where $\psi(S, S^0)$ is $o(d(S, S^0))$, that is, $\lim\limits_{d(S,S^0) \to 0} \dfrac{\psi(S, S^0)}{d(S, S^0)} = 0$.

A function $F : A^n \to R$ is said to have a partial derivative at $S^0 = (S_1^0, S_2^0, ..., S_n^0)$ with respect to its p^{th} argument if the function

$$\phi(S_k) = F(S_1^0, ..., S_{k-1}^0, S_k, S_{k+1}^0, ..., S_n^0)$$

has derivative $D\phi\left(S_k^0\right)$ and we define $D_k F\left(S^0\right) = D\phi\left(S_k^0\right)$. If $D_k F\left(S^0\right), k = 1,2,..., n,$ all exist, then we put $DF\left(S^0\right) = \left(D_1 F\left(S^0\right), D_2 F\left(S^0\right),..., D_n F\left(S^0\right)\right)$.

A function $F : A^n \to R$ is said to be differentiable at S^0 if there exist $DF\left(S^0\right)$ and $\psi : A^n \times A^n \to R$ such that

$$F\left(S\right) = F\left(S^0\right) + \sum_{k=1}^{n} \left\langle D_k F\left(S^0\right), I_{S_k} - I_{S_k^0} \right\rangle + \psi\left(S, S^0\right),$$

where $\psi\left(S, S^0\right)$ is $o\left(d\left(S, S^0\right)\right)$ for all $S \in A^n$.

Consider the nonlinear multiobjective subset programming problem

$$\text{(P) Minimize } F(S) = \left[F_1(S), F_2(S),..., F_p(S)\right]$$

$$\text{subject to } G_j(S) \leqq 0,\ j \in M, S = \left(S_1, S_2,..., S_n\right) \in A^n,$$

where A^n is the n-fold product of a σ-algebra A of subsets of a given set X, $F_i, i \in P = \{1,2,..., p\}$ and $G_j, j \in M = \{1,2,..., m\}$ are real-valued functions defined on A^n. Let $X_0 = \left\{S \in A^n : G_j(S) \leqq 0, j \in M\right\}$ be the set of all feasible solutions to (P).

Definition 1.1. A feasible solution S^0 to (P) is said to be an efficient solution to (P) if there exists no other feasible solution S to (P) such that $F(S) \leq F\left(S^0\right)$.

Definition 1.2. A feasible solution S^0 to (P) is said to be a weakly efficient solution to (P) if there exists no other feasible $S\left(S \neq S^0\right)$ to (P) such that $F(S) < F\left(S^0\right)$.

Along the lines of Jayswal and Kumar [2], we now define several classes of n-set functions, that we call d-weak strictly pseudo-quasi type-I univex, d-strong pseudo-quasi type-I univex and d-weak strictly pseudo type-I univex functions.

Definition 1.3. We say that the pair of functions (F, G) is d-weak strictly pseudo-quasi type-I univex at $S^0 \in A^n$ with respect to $b_0, b_1, \phi_0, \phi_1, \gamma = \left(\gamma_1, \gamma_2,..., \gamma_p\right)$, $\delta = \left(\delta_1, \delta_2,..., \delta_m\right)$ and $\eta = \left(\eta_1, \eta_2,..., \eta_n\right)$, if there exist $\eta : A^n \times A^n \to R^n$, $\gamma_i : A^n \times A^n \to R_+ \setminus \{0\}$, $i = 1,2,..., p$, $\delta_j : A^n \times A^n \to R_+ \setminus \{0\}$, $j = 1,2,..., m$, nonnegative functions b_0 and b_1, also defined on $A^n \times A^n$, and $\phi_0 : R \to R$, $\phi_1 : R \to R$, such that for all $S \in X_0$ the implications

$$b_0\left(S, S^0\right)\phi_0\left[\sum_{i=1}^{p} \gamma_i\left(S, S^0\right)F_i(S) - \sum_{i=1}^{p} \gamma_i\left(S, S^0\right)F_i\left(S^0\right)\right] \leq 0$$

$$\Rightarrow \sum_{i=1}^{p}\sum_{k=1}^{n} \eta_k\left(S, S^0\right)\left\langle D_k F_i\left(S^0\right), I_{S_k} - I_{S_k^0}\right\rangle < 0,$$

$$-b_1\left(S, S^o\right)\phi_1\left[\sum_{j=1}^{m} \delta_j\left(S, S^0\right)G_j\left(S^0\right)\right] \leqq 0$$

$$\Rightarrow \sum_{j=1}^{m}\sum_{k=1}^{n} \eta_k\left(S, S^0\right)\left\langle D_k G_j\left(S^0\right), I_{S_k} - I_{S_k^0}\right\rangle \leqq 0$$

do hold.

Definition 1.4. We say that the pair of functions (F, G) is d-strong pseudo-quasi type-I univex at $S^0 \in A^n$ with respect to $b_0, b_1, \phi_0, \phi_1, \gamma = \left(\gamma_1, \gamma_2,..., \gamma_p\right)$, $\delta = \left(\delta_1, \delta_2,..., \delta_m\right)$ and $\eta = \left(\eta_1, \eta_2,..., \eta_n\right)$, if there exist $\eta : A^n \times A^n \to R^n$, $\gamma_i : A^n \times A^n \to R_+ \setminus \{0\}$, $i = 1,2,..., p$, $\delta_j : A^n \times A^n \to R_+ \setminus \{0\}$, $j = 1,2,..., m$, nonnegative functions b_0 and b_1, also defined on $A^n \times A^n$, and $\phi_0 : R \to R$, $\phi_1 : R \to R$, such that for all $S \in X_0$ the implications

$$b_0\left(S,S^0\right)\phi_0\left[\sum_{i=1}^{p}\gamma_i\left(S,S^0\right)F_i\left(S\right)-\sum_{i=1}^{p}\gamma_i\left(S,S^0\right)F_i\left(S^0\right)\right]\leq 0$$

$$\Rightarrow \sum_{i=1}^{p}\sum_{k=1}^{n}\eta_k\left(S,S^0\right)\left\langle D_kF_i\left(S^0\right),I_{S_k}-I_{S_k^0}\right\rangle\leq 0,$$

$$-b_1\left(S,S^o\right)\phi_1\left[\sum_{j=1}^{m}\delta_j\left(S,S^0\right)G_j\left(S^0\right)\right]\leqq 0$$

$$\Rightarrow \sum_{j=1}^{m}\sum_{k=1}^{n}\eta_k\left(S,S^0\right)\left\langle D_kG_j\left(S^0\right),I_{S_k}-I_{S_k^0}\right\rangle\leqq 0$$

do hold.

Definition 1.5. We say that the pair of functions (F,G) is *d*-weak strictly pseudo type-I univex at $S^0 \in A^n$ with respect *to* $b_0, b_1, \phi_0, \phi_1, \gamma=\left(\gamma_1,\gamma_2,...,\gamma_p\right)$, $\delta=\left(\delta_1,\delta_2,...,\delta_m\right)$ and $\eta=\left(\eta_1,\eta_2,...,\eta_n\right)$, if there exists $\eta : A^n \times A^n \to R^n$, $\gamma_i : A^n \times A^n \to R_+ \setminus \{0\}$, $i=1,2,...,p$, $\delta_j : A^n \times A^n \to R_+ \setminus \{0\}$, $j=1,2,...,m$, nonnegative functions b_0 and b_1, also defined on $A^n \times A^n$, and $\phi_0 : R \to R$, $\phi_1 : R \to R$, such that for all $S \in X_0$ the implications

$$b_0\left(S,S^0\right)\phi_0\left[\sum_{i=1}^{p}\gamma_i\left(S,S^0\right)F_i\left(S\right)-\sum_{i=1}^{p}\gamma_i\left(S,S^0\right)F_i\left(S^0\right)\right]\leq 0$$

$$\Rightarrow \sum_{i=1}^{p}\sum_{k=1}^{n}\eta_k\left(S,S^0\right)\left\langle D_kF_i\left(S^0\right),I_{S_k}-I_{S_k^0}\right\rangle< 0,$$

$$-b_1\left(S,S^o\right)\phi_1\left[\sum_{j=1}^{m}\delta_j\left(S,S^0\right)G_j\left(S^0\right)\right]\leqq 0$$

$$\Rightarrow \sum_{j=1}^{m}\sum_{k=1}^{n}\eta_k\left(S,S^0\right)\left\langle D_kG_j\left(S^0\right),I_{S_k}-I_{S_k^0}\right\rangle< 0$$

do hold.

Remark 1.6. The above definitions extend to *n*-set functions the concept of weak strictly pseudo-quasi-*d*-*V*-type-I univex, strong pseudo-quasi-*d*-*V*-type-I univex and weak strictly pseudo-*d*-*V*-type-I univex of Jayswal and Kumar [2]. They also extend to univexity the concept of *d*-weak strictly-pseudoquasi-type-I, *d*-strong-pseudoquasi-type-I and *d*-weak strictly pseudo-type-I of Mishra *et al*. [5].

1. SUFFICIENT OPTIMALITY CONDITIONS

The theorem below gives sufficient optimality conditions for a weakly efficient solution to (P) under the assumptions of generalized *d*-type-I univexity introduced in Section 1.

Theorem 2.1. (Sufficient optimality conditions). *Let* S^0 *be a feasible solution to* (P). *Assume that there exist* $\lambda_i^0 \geqq 0$, $i \in P$, $\sum_{i=1}^{p}\lambda_i^0 = 1$ *and* $\mu_j^0 \geqq 0$, $j \in M_0 = \left\{j \in M : G_j\left(S^0\right)=0\right\}$, *such that*

$$\left\langle D_k\left(\lambda^0 F\right)\left(S^0\right)+D_k\left(\mu^0 G\right)\left(S^0\right),I_{S_k}-I_{S_k^0}\right\rangle\geqq 0$$

for all $S \in A^n$ *Moreover, assume any one of the conditions below.*

(S1) $\lambda > 0$ *and* $(F, \mu G)$ *is d-strong pseudo-quasi type-I univex at* S^0 *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$;

(S2) $(F, \mu G)$ *is d-weak strictly pseudo-quasi type-I univex at* S^0 *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$;

(S3) $(F, \mu G)$ *is d-weak strictly pseudo type-I univex at* S^0 *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$;

with η *satisfying* $\eta^T \alpha < 0 \Rightarrow \alpha_k < 0$ *for at least one* $k \in \{1, 2, ..., n\}$.

Further, assume that for $r \in R$ *we have*

$$r \leq 0 \Rightarrow \phi_0(r) \leq 0, \, r \leqq 0 \Rightarrow \phi_1(r) \leqq 0$$

and
$$b_0(S, S^0) > 0, \, b_1(S, S^0) \geqq 0, \; \forall S \in X_0.$$

Then S^0 *is a weakly efficient solution to* (P).

Definition 2.1. A feasible solution S^0 is said to be a regular feasible solution if there exists $\hat{S} \in A^n$ such that

$$G_j(S^0) + \sum_{k=1}^{n} \left\langle D_k G_j(S^0), I_{\hat{S}_k} - I_{S_k^0} \right\rangle < 0, \quad j \in M.$$

The following result below will be needed in the sequel.

Lemma 2.1 (Zalmai [7], Theorem 3.2). *Let* S^0 *be a regular efficient (or weakly efficient) solution to* (P) *and assume that* F_i, $i \in P$ *and* G_j, $j \in M$ *are differentiable at* S^0. *Then there exist* $\lambda \in R_+^p$, $\sum_{i=1}^{p} \lambda_i = 1$, *and* $\mu \in R_+^m$ *such that*

$$\sum_{k=1}^{n} \left\langle \sum_{i=1}^{p} \lambda_i D_k F_i(S^0) + \sum_{j=1}^{m} \mu_j D_k G_j(S^0), I_{S_k} - I_{S_k^0} \right\rangle \geqq 0 \quad \textit{for all } S \in A^n,$$

$$\mu_j G_j(S^0) = 0, \quad j \in M.$$

3. MOND-WEIR DUALITY

In this section, we associate the problem (P) with the Mond-Weir dual problem (MD):

(MD) maximize $F(T)$ subject to

$$\left\langle D_k(\lambda F)(T) + D_k(\mu G)(T), I_{S_k} - I_{T_k} \right\rangle \geqq 0, \quad \forall S \in A^n,$$

$$\sum_{j=1}^{m} \mu_j G_j(T) \geqq 0,$$

$$\lambda_i \geqq 0, \; i \in P \text{ and } \sum_{i=1}^{p} \lambda_i = 1,$$

$$\mu_j \geqq 0, \; j \in M \text{ and } T \in A^n.$$

Theorem 3.1 (Weak duality). *Let* S *and* (T, λ, μ) *be feasible solutions to* (P) *and* (MD), *respectively. Assume any one of the conditions below*

(WD1) $\lambda > 0$ *and* $(F, \mu G)$. *is d-strong pseudo-quasi type-I univex at* T *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$;

(WD2) $(F, \mu G)$ *is d-weak strictly pseudo-quasi type-I univex at* T *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$;

(WD3) $(F, \mu G)$ *is d-weak strictly pseudo type-I univex at* T *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$;

with η *satisfying* $\eta^T \alpha < 0 \Rightarrow \alpha_k < 0$ *for at least one* $k \in \{1,2,...,n\}$.

Further, assume that for $r \in R$ *we have*

$$r \leq 0 \Rightarrow \phi_0(r) \leq 0, r \leqq 0 \Rightarrow \phi_1(r) \leqq 0$$

and

$$b_0(S, S^0) > 0, b_1(S, S^0) \geqq 0, \ \forall S \in X_0.$$

Then $F(S) \leq F(T)$ *cannot holds.*

Theorem 3.2 (Strong duality). *Let* S^0 *be a regular weakly efficient solution to* (P). *Then there exist* $\lambda^0 \in R^p, \sum_{i=1}^{p} \lambda_i^0 = 1,$ *and* $\mu^0 \in R^m$ *such that* (S^0, λ^0, μ^0) *is a feasible solution to* (MD) *while the values of the objective functions of* (P) *and* (MD) *are equal at* S^0 and (S^0, λ^0, μ^0), *respectively. Furthermore, if the conditions of weak duality Theorem 3.1 also hold, for each feasible solution* (T, λ, μ) *to* (MD), *then* (S^0, λ^0, μ^0) *is a weakly efficient solution to* (MD).

4. GENERALIZED MOND-WEIR DUALITY

In this section, we associate the problem (P) with the generalized Mond-Weir dual problem (GMD):

(GMD) maximize $F(T) + \sum_{j \in J_0} \mu_j G_j(T) e$ subject to

$$\left\langle D_k(\lambda F)(T) + D_k(\mu G)(T), I_{S_k} - I_{T_k} \right\rangle \geqq 0, \qquad \forall S \in A^n,$$

$$\sum_{j \in J_\alpha} \mu_j G_j(T) \geqq 0 \quad \text{for } 1 \leqq \alpha \leqq r,$$

$$\lambda \geqq 0, \ \mu \geqq 0 \ \text{and} \ \sum_{i=1}^{p} \lambda_i = 1,$$

where $e = (1,1,...,1) \in R^p$ and J_α, $0 \leqq \alpha \leqq r$ is a partition of M, with $J_s \cap J_t = \phi$ for $s \neq t$ and $\bigcup_{s=0}^{r} J_s = M$.

Theorem 4.1 (Weak duality). *Let* S *and* (T, λ, μ) *be feasible solutions to* (P) *and* (GMD) *respectively. Assume any one of the conditions below.*

(GWD1) $\lambda > 0$ *and* $\left(F(\cdot) + \sum_{j \in J_0} \mu_j G_j(\cdot) e \ , \sum_{j \in J_\alpha} \mu_j G_j(\cdot) \right)$ *is d-strong pseudo-quasi type-I univex at* T *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p)$, $\delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$ *for any* α, $1 \leqq \alpha \leqq r$;

(GWD2) $\left(F(\cdot) + \sum_{j \in J_0} \mu_j G_j(\cdot) e, \sum_{j \in J_\alpha} \mu_j G_j(\cdot) \right)$ *is d-weak strictly pseudo-quasi type-I univex at* T *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p), \delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$ *for any* α, $1 \leqq \alpha \leqq r$;

(GWD3) $\left(F(\cdot) + \sum_{j \in J_0} \mu_j G_j(\cdot) e, \sum_{j \in J_\alpha} \mu_j G_j(\cdot) \right)$ *is d-weak strictly pseudo type-I univex at* T *with respect to* $b_0, b_1, \phi_0, \phi_1, \gamma = (\gamma_1, \gamma_2, ..., \gamma_p), \delta = (\delta_1, \delta_2, ..., \delta_m)$ *and* $\eta = (\eta_1, \eta_2, ..., \eta_n)$ *for any* α, $1 \leqq \alpha \leqq r$; *with* η *satisfying* $\eta^T \alpha < 0 \Rightarrow \alpha_k < 0$ *for at least one* $k \in \{1, 2, ..., n\}$.

Further, assume that for $r \in R$ *we have*

$$r \leq 0 \Rightarrow \phi_0(r) \leq 0, \, r \leqq 0 \Rightarrow \phi_1(r) \leqq 0$$

and

$$b_0(S, S^0) > 0, \, b_1(S, S^0) \geqq 0, \; \forall S \in X_0.$$

Then $F(S) \leq F(T) + \sum_{j \in J_0} \mu_j G_j(T) e$ *cannot holds.*

Theorem 4.2 (Strong duality). *Let* S^0 *be a regular weakly efficient solution to* (P). *Then there exist* $\lambda^0 \in R^p, \sum_{i=1}^{p} \lambda_i^0 = 1$ *and* $\mu^0 \in R^m$, *such that* (S^0, λ^0, μ^0) *is a feasible solution to* (GMD) *and* $\mu_{J_0} G_{J_0}(S^0) = 0$, *while the values of the objective functions of* (P) *and* (GMD) *are equal at* S^0 *and* (S^0, λ^0, μ^0), *respectively. Furthermore, if the conditions of weak duality Theorem 4.1 also hold for each feasible solution* (T, λ, μ) *to* (GMD), *then* (S^0, λ^0, μ^0) *is a weakly efficient solution to* (GMD).

The proofs will appear in [3].

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