# SOILS NONLINEARITY EFFECTS ON DOMINANT SITE PERIOD EVALUATION

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By numerical simulation using a nonlinear Kelvin-Voigt model this paper propose a modeling method for resonant peaks dispersion provoked by nonlinear properties of the site materials. This method leads to a better approximation of the nonlinear dependence of dominant site periods, dependence with an important impact about site-structure resonance avoidance.

*Key-Words*:  Nonlinear soil dynamics, Nonlinear natural periods, Site-structural resonance, Numerical simulation.

## 1.  INTRODUCTION

The dynamic response of a certain structure is strongly dependent of the ratio between the natural period of the structure and the dominant period of the emplacement site. It is very well known that the major damages arise at resonance, when the natural period of structure is equal or very close to the dominant period of the site. For this reason, a correct evaluation of site dominant period has a special importance.

The site materials, soils or rocks, are nonlinear materials with a dynamic behavior strongly dependent of loading level and this behavior affects the whole dynamic response including the system natural period values [2], [4]. The neglect of this nonlinear behavior by using the computational methods of the linear dynamics when evaluating the site period, may be a source of errors for predicting the structural-site response.

The present paper intends to discuss the dynamic effects given by nonlinear material properties on system natural period evaluation. In this intent we will use the magnification functions of the nonlinear Kelvin-Voigt model [2, 3]. By numerical simulation with different loading level, we can able to model the effects of the soils nonlinearity on the shape and resonant magnitude of the magnification functions. Thus, we shall obtain a proper tool for modeling the resonant peak dispersion, which is a very important condition for a correct natural periods evaluation and therefore for site-structure resonance avoidance.

## 2.  NATURAL PERIOD – RIGIDITY RELATIONSHIP



$F = F_0 \sin \omega t$

Fig.1.1 Linear Kelvin-Voigt model

In linear dynamics, a usual description of a solid single-degree-of-freedom behavior is given by the Kelvin-Voigt model consisting of a mass $m$ supported by a spring (with a stiffness $k$) and a dashpot (with a viscosity $c$) connected in parallel (fig. 1.1). The governing equation of this system under harmonic excitation (with the amplitude $F_0$ and pulsation $\omega$) is [15]:

$$\ddot{x} + 2\zeta\dot{x} + \omega_0^2 x = q \sin \omega t \qquad (2.1)$$

where:

$$2\zeta = \frac{c}{m} \quad ; \quad q = \frac{F_0}{m} \quad ; \quad \omega_0 = \sqrt{\frac{k}{m}} \qquad (2.2)$$
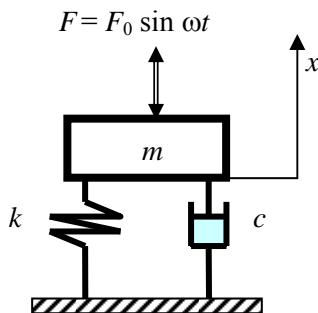
$\zeta$ being a usual damping measure (damping ratio $\zeta = c/c_{cr}$ ), and $\omega_0$ is the natural pulsation, connected by natural frequency $f_0$ through the relationship: $\omega_0 = 2\pi f_0$ .

The natural period of this system $T_0$ is:

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}} \qquad (2.3)$$

Thus, from eq. (2.3) it follows that, for a constant mass, the natural period and rigidity are in inverse ratio – the rigidity increasing leads to the natural period shortening and a diminishing rigidity leads to the oscillation with larger natural periods.

## 3. RIGIDITY OF THE SITE MATERIALS

The site materials are nonlinear materials with a dynamic behavior strongly dependent of loading level and this behavior affects the whole dynamic response including the system natural period values [2], [4]. The nonlinear behavior means the loading dependence of the material properties and the dynamic material characteristics $G$ and $\zeta$ becomes functions in terms of strain level $\gamma$. Therefore, another dynamic characteristic – the rigidity $k$ - become strain dependent function $k = k(\gamma)$ too. Because the rigidity is in direct proportional ratio with modulus $G$ we can write:

$$k(\gamma) = A \cdot G(\gamma) \qquad (3.1)$$

where $A$ are the proportional factor.

From eqs. (2.3) and (3.1) results that the natural period $T_0$ of the systems which include nonlinear materials becomes strain dependent, too:

$$T_0(\gamma) = \frac{2\pi\sqrt{m}}{\sqrt{k(\gamma)}} = \frac{2\pi\sqrt{m}}{\sqrt{A}} \frac{1}{\sqrt{G(\gamma)}} = \frac{B}{\sqrt{G(\gamma)}} \qquad (3.2)$$

where $B = 2\pi\sqrt{m/A}$ is another material constant.

The modulus-function $G = G(\gamma)$ can be written in the normalized form $G(\gamma) = G_0 \cdot G_n(\gamma)$ where $G_n(\gamma)$ is the modulus-function normalized in terms of his initial value $G_0 = G\big|_{\gamma=0}$ . Thus, the strain dependence of the natural period (3.2) can be written in the similar form:

$$T_0(\gamma) = \frac{B}{\sqrt{G_0}} \frac{1}{\sqrt{G_n(\gamma)}} = T_{init} \cdot T_n(\gamma) \qquad (3.3)$$

where $T_{init}$ represent the initial value of the natural period:

$$T_{init} = T_0(\gamma)\big|_{\gamma=0} = 2\pi\sqrt{\frac{m}{AG_0}} = 2\pi\sqrt{\frac{m}{k_0}} \qquad (3.4)$$

and the term $T_n(\gamma)$ represents the strain dependence of the natural period:

$$T_n(\gamma) = \frac{1}{\sqrt{G_n(\gamma)}} = \frac{1}{\sqrt{k_n(\gamma)}} \qquad (3.5)$$

where $k_n(\gamma)$ is the rigidity function $k(\gamma)$ normalized in terms of his initial value $k_0 = k(\gamma)\big|_{\gamma=0}$ .

## 4. EVALUATION OF THE SOILS NATURAL PERIOD CHANGES BY RESONANT COLUMN TESTS

In principle, from resonant column test a certain modulus-function value $G$ for a certain strain level $\gamma$ is obtained by mediation of shear vawe velocity $v_s$ using the well-known relationship:

$$G = \rho v_s^2 \tag{4.1}$$

where $\rho$ is the mass density of specimen [4], [14]. The shear wave velocity provided by resonant column test is obtained in the form:

$$v_s = \frac{\omega_0 h}{\Psi} = \frac{\omega_0 h}{\sqrt{R - \frac{1}{3}R^2 + \frac{4}{45}R^3}} \tag{4.2}$$

where $\omega_0$ is the specimen natural pulsation, $h$ is the specimen height and $\Psi$ is the root of torsional frequency equation with analytical form in terms of the ratio between torsional inertia of the specimen and the torsional inertia of the top cap system: $R = J / J_{top}$ [4], [15].

From eqs. (2.3) and (4.2) the expression for the specimen natural results:

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi h \sqrt{\rho}}{\Psi} \cdot \frac{1}{\sqrt{G}} \tag{4.3}$$

and thus the proportionality constant $A$ from eq. (3.4) can be now full determines:

$$A = \frac{2\pi h \sqrt{\rho}}{\Psi} \tag{4.4}$$

After several tests with different strain level $\gamma$ we can obtain the modulus function $G = G(\gamma) = G_0 \cdot G_n(\gamma)$ and the natural period functions (3.3) $T_0(\gamma) = T_{init} \cdot T_n(\gamma)$ where the initial value $T_{init}$ has (3.4) expression and normalized form $T_n(\gamma)$ is given in eq. (3.5).

The natural periods obtained by resonant column test is the natural periods of the s.d.o.f. oscillating system composed by a single mass (the vibration device) supported by a spring and a damper represented by the specimen [2]. The natural period value depends on mechanical properties of the specimen material but it is a system characteristic and not a material characteristics.

Fortunately, as can see from eqs. (3.4) and (3.5), the physical and geometrical properties are included only in the initial value $T_{init}$ of the nonlinear natural period function and the normalized form (3.5) depends only on normalized material characteristics.

Thus, the resonant column test can offer accurate data for obtaining the nonlinear dependence of the natural period. Certainly, the initial value of the natural period function must be obtained from another way by any test performed on investigated nonlinear system. As can see in the next, for the site nonlinear systems which contains soils nonlinear materials can be use the data recording during seismic events.

Relationship (3.5) allows estimating the nonlinear variation of the natural periods if the modulus-function $G_n = G_n(\gamma)$ is previously determined. For example, in fig. 4.1 there are given some normalized modulus-functions obtained from resonant column data and in fig. 4.2 the corresponding normalized natural period functions obtained from eq. (3.5) are given.

As we can see from this experimental data processing, the reduction of the modulus-function values due to dynamic degradation leads to the increasing period values, the same inverse variation between rigidity and natural period as in the linear material case.
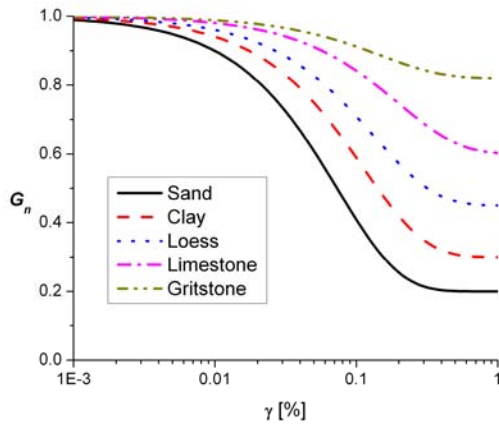
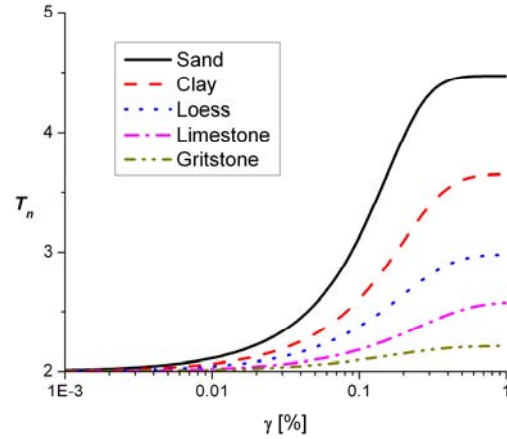Fig. 4.1 Some normalized modulus-functions



Fig. 4.2 Nonlinear variation of the natural periods

## 5. NONLINEAR NATURAL PERIOD FUNCTIONS IN TERMS OF LOADING

For practical applications it is necessary to determine the normalized natural period in terms of loading amplitude and because in the seismic calculus the usual loading amplitude is *PGA* (peak ground acceleration) we must modify the functions $T_n = T_n(\gamma)$ in $T_n = T_n(PGA)$ form.

For this conversion, we can use the nonlinear Kelvin-Voigt model [3], [4] subjected to support motion $\ddot{x}_g(t) = \ddot{x}_g^0 \sin \omega t$ with different acceleration amplitude $\ddot{x}_g^0$. In this loading case, the motion equation reads as:

$$\ddot{x} + 2\omega_0 \zeta(x) \cdot \dot{x} + \omega_0^2 k(x) \cdot x = -\ddot{x}_g \tag{5.1}$$

Using the change of variable $\tau = \omega_0 t$ and introducing the new time function $\varphi(\tau) = x(t) = x(\tau/\omega_0)$ we can obtain for eq. (5.1) another form:

$$\varphi'' + C(\varphi) \cdot \varphi' + K(\varphi) \cdot \varphi = \mu \sin \upsilon \tau \tag{5.2}$$

where the superscript accent denotes the time derivative with respect to $\tau$, and:

$$C(\varphi) \equiv C(x) = \frac{c(x)}{m\omega_0} = 2\zeta(x) \quad ; \quad K(\varphi) \equiv K(x) = \frac{k(x)}{m\omega_0^2} = \frac{k(x)}{k(0)} = k_n(x)$$

$$\mu = \frac{\ddot{x}_g^0}{\omega_0^2} \quad ; \quad \upsilon = \frac{\omega}{\omega_0} \tag{5.3}$$

The steady-state solution of the equation (5.2) can be written in the form:

$$\varphi(\tau, \upsilon, \zeta) = \mu \Phi(\upsilon; \zeta) \sin(\upsilon \tau - \psi) \tag{5.4}$$

where $\Phi(\upsilon; \zeta)$ is *the nonlinear magnification function:*

$$\Phi(\upsilon; \zeta) = \frac{\max_\tau[\varphi(\tau, \upsilon, \zeta)]}{\mu} = \frac{x_{dynamic}}{x_{static}} \tag{5.5}$$

a ratio of maximum dynamic amplitude $\varphi_{max} \equiv x_{dynamic}$ to static displacement $\mu = x_{static}$. For given amplitude $\mu$ the nonlinear equation (5.2) can be numerically solved using a computer program based on Newmark algorithm [4], and the nonlinear magnification function $\Phi(\upsilon;\mu,\zeta) = \Phi(\upsilon;\zeta)\big|_{\substack{\zeta=ct.\\ \mu=ct.}}$ can be obtained.

To illustrate this method in fig.5.1 such a numerical simulation result is presented. For this example the material dynamic function for clay was used [6, 9]. By this simulation one can point out a softening type behavior with resonant picks at pulsation $\upsilon$ located before the linear resonant pulsation $\upsilon = 1$.
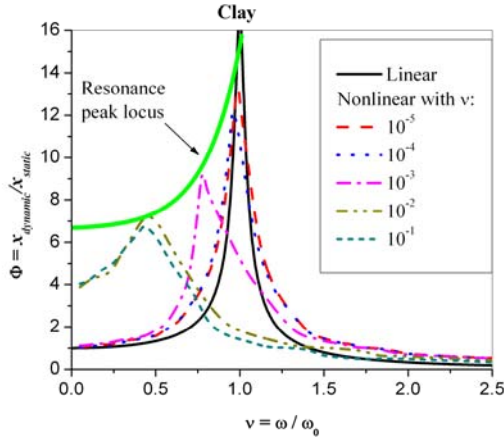


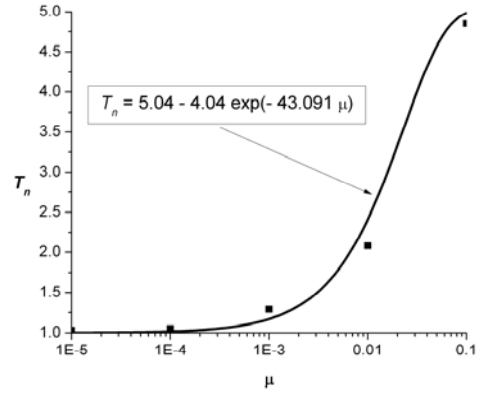Fig. 5.1 Nonlinear magnification function



Fig. 5.2  Natural period variation in terms of  $\mu$

Because  $\upsilon = \omega/\omega_0 = 1/T_n$  from the simulation results given in fig.5.1 we can obtain a relationship $T_n = T_n(\mu)$ (fig.5.2). The normalized amplitude $\mu$ is connected by abutment acceleration amplitude $\ddot{x}_g^0$ through the eq. (5.3$_3$) $\mu = \ddot{x}_g^0/\omega_0^2$ (fig. 5.3) and together with the relation $T_n = T_n(\mu)$ enables us to obtain a relationship (fig. 5.4):

$$\left.\begin{array}{l} T_n = T_n(\mu) \\[2mm] \mu = \mu(PGA) = \dfrac{\ddot{x}_g^0}{\omega_p^2} = \dfrac{g}{\omega_p^2}PGA \end{array}\right\} \Rightarrow T_n = -0.9 + 1.9\exp(3.521PGA) \qquad (4.12)$$

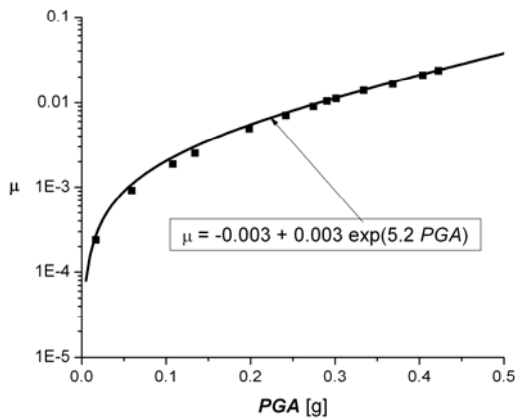where $g$ is gravity acceleration ( $g = 9.81$ m/s$^2$ ).
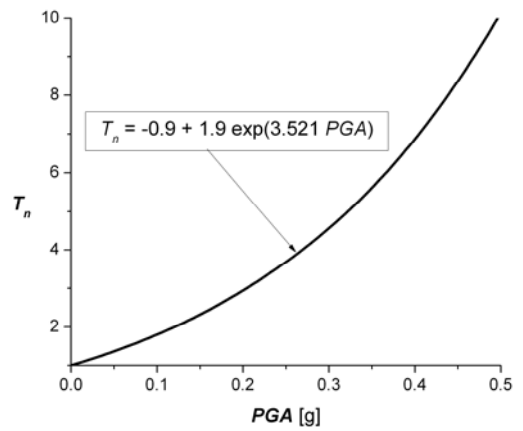


Fig. 5.3 Relationship $\mu$ - *PGA*



Fig. 5.4 Natural period variation in terms of *PGA*

## 6. EVALUATION OF THE SITE NATURAL PERIOD

Knowledge of the normalized natural period for each site materials as it was previously determined is not enough for the evaluation of the entire site normalized natural periods. Each layer has a contribution depending on its geometrical properties. For that, we accept the hypothesis of the normalized natural period dependence in terms of the layer thickness.

According to the thickness dependence hypothesis, the average natural period variation for the entire site layers $T_n^{av}$ will be composed as the average of the normalized natural period $T_n^i$ weighted with the thickness $h_i$ of each layer:

$$T_n^{av} = \frac{\sum T_n^i \cdot h_i}{\sum h_i} \tag{6.1}$$

To exemplify this method we choose the site emplacement of the seismic station INCERC with known stratification [1] (table 6.1).

Table 6.1

| Material | Thickness [m] | $v_S$ [m/s] |
|---|---|---|
| Silty clay | 4.5 | 460 |
| Sand and gravel | 28.5 | 460 |
| Clays | 17.0 | 385 |
| Fine sand and clay | 17.0 | 340 |
| Clays | 8.0 | 455 |
| Fine sand | 53.0 | 400 |



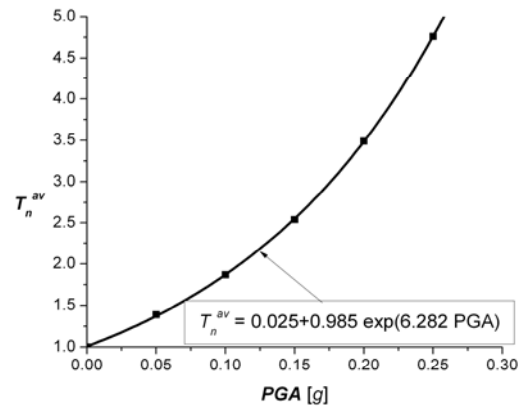$T_n^{av} = 0.025+0.985 \exp(6.282\ PGA)$

Fig. 6.1  Normalized natural period for INCERC emplacement using RC data

First, for each constituent layer the material modulus-function  was estimated in concordance with previous resonant column data [8] and thus by numerical simulation a function $T_n^i = T_n^i(PGA)$ was obtained. Then, for some *PGA* values (0.05, 0.10, 0.15, 0.20 and 0.25 g) using eq. (6.1) the site natural period averages was obtained. The results are given in fig. 6.1.

Table 6.2

| Event data | $M_w$ | Observed accelerations [cm/s²] | PGA [g] | $T_0$ [s] | $T_n$ |
|---|---|---|---|---|---|
| | | | 0 | 0.21 | 1.00 |
| 27.09.2004 | 4.6 | 7.97 | 0.008 | 0.23 | 1.07 |
| 18.06.2005 | 4.9 | 10.53 | 0.011 | 0.24 | 1.12 |
| 27.10.2004 | 6.0 | 13.69 | 0.014 | 0.25 | 1.16 |
| 30.05.1990 | 6.9 | 85.20 | 0.087 | 0.35 | 1.63 |
| 30.08.1386 | 7.1 | 95.72 | 0.098 | 0.50 | 2.33 |
| 04.03.1977 | 7.4 | | 0.250 | 1.20 | 5.58 |

This evaluation method of the natural period variation using resonant column data for a certain site can be validated by comparison with seismic data recording at the same site. The spectral analysis of seismic data recording during Vrancea earthquakes with different magnitudes shows a doubtless dependence of maximum accelerations and dominant periods on earthquake magnitude ( $M_w$ ) [11, 14]. Such data recording

at INCERC seismic station are given in table 6.2 (after [11, 12, 13, 14]). From these data one can obtain a numerical correlation of the type $T_0 = T_0(PGA)$ (fig. 6.2). As we can see in table 6.2 and fig. 6.2 the natural period $T_0$ is increasing function on seismic loading as it was observed in the case of processing resonant column data, too. Because in the both processing methods the information provided from the same site was used, a comparison between them is permissive. Such a comparison is given in fig. 6.3, which plots together the same function $T_n = T_n(PGA)$ provided by both column resonant data (fig.6.1) and seismic data (fig.6.2).
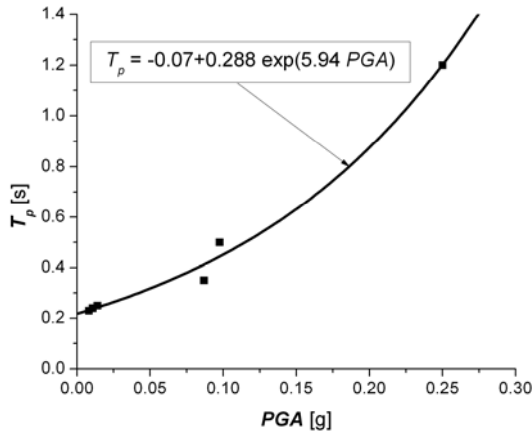


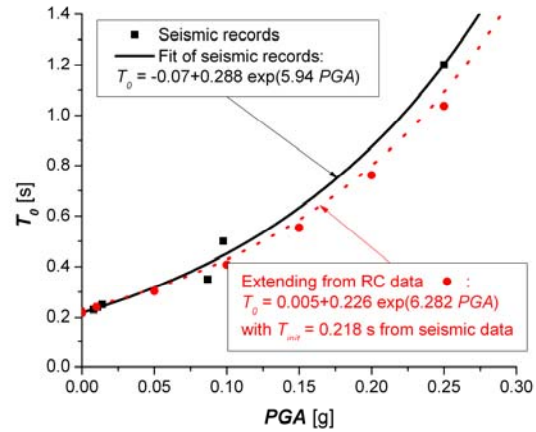Fig. 6.2  Dependence $T_0 - PGA$ from seismic records



Fig. 6.3 Dependence $T_0 - PGA$ provided both resonant column data and seismic record

As one can see from fig 6.3, the differences between these different acquisition methods are not too large.
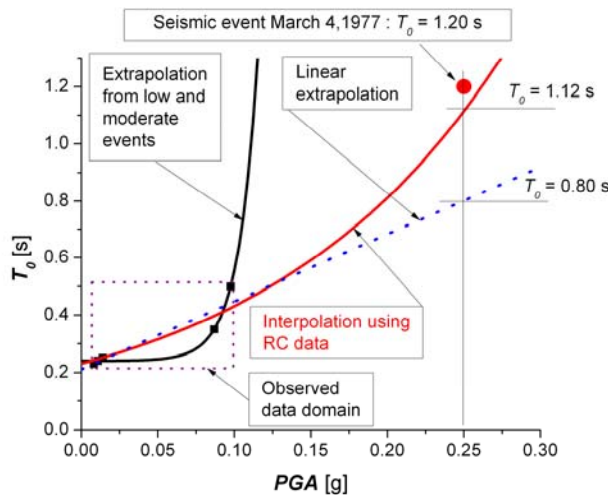


Fig. 6.4 Extrapolation from partial data and RC interpolation

In the previous, for the validation of the RC method the geological and seismic data of the INCERC site was used because for this site there are multiple seismic recording with different magnitudes beginning with low events until the strong March 4, 1977 earthquake. However, for a large majority of the usual sites only seismic recording of the low and moderate events are available. In these cases, the evaluation of the dominant period for strong earthquakes using only seismic low and moderate data presume an extrapolation procedure with inherent large errors. To exemplify, in fig. 6.4 are shown few numerical extending functions for INCERC site using only low and moderate data. As can see, the obtaining periods corresponding to strong March event are very differents.

In the resonant column device, the sample can be loaded with a large excitation domain equivalent to low until to strong earthquakes. The evaluation method of the nonlinear dependence of dominant periods using resonant column data presumes an interpolating process with a better approximation, which replace the uncertain extending process. From this reason, we think that the resonant column determination of the nonlinear variations in normalized form: $T_n = T_n(PGA)$ together with the determination of the normalization value $T_{init}$ from seismic recording can leads to a better approximation of the dominant periods for large $PGA$ values when only low and moderate seismic data are available.

## CONCLUDING REMARKS

- The natural period and rigidity are inversely proportional both for linear and nonlinear materials.
- The site geological materials have a degradable rigidity function of the strain level, and the increase excitation level leads to the rigidity decrease and the increase of the natural period values.
- The dependence of the dominant site period on the excitation level - $T_0 = T_0 (PGA)$ - can be obtained from recorded seismic data if these data cover the entire *PGA* value range.
- The dependence of the natural period on the excitation level in normalized form - $T_n = T_n (PGA)$ - can be modeled by interpolation of the resonant column data. This method described in this paper has been validated using recording earthquake data.
- When only low and moderate seismic data are available, the resonant column determination of the nonlinear variations in normalized form: $T_n = T_n (PGA)$ together with the determination of the normalization value $T_{init}$ from seismic recording can leads to a better approximation of the dominant periods for large *PGA* values.

## ACKNOWLEDGEMENTS

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