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THE KINEMATIC OPTIMIZATION OF THE MULTI-LINK SUSPENSION MECHANISMS USED FOR THE REAR AXLE OF THE MOTOR VEHICLES

Cătălin ALEXANDRU

University "Transilvania" of Braşov, e-mail: calex@unitbv.ro

In relative motion to the car body, the rear axle of the motor vehicles (usually, for the off-road and the commercial vehicles) is guided by spatial mechanisms, on which between axle and chassis a number of binary links or kinematics chains are interposed (multi-link mechanisms). From kinematic point of view, the suspension mechanisms of the rear axles have to assure the main motions of the axle, namely the vertical displacement and the roll rotation, with minimum values for the other spatial motions (the transversal & longitudinal displacements, and the yaw & pitch rotations). In this paper, an analytical method for the kinematic optimization of the multi-link mechanisms used for the guidance of the rear wheel is presented. According to the proposed method, the coordinates of the guiding points on axle are established by constructive criteria. Considering the geometric constraints, for imposed positions of the rear axle, the global coordinates of the joints on car body are determined.

Key words: Motor vehicle; Rear axle; Suspension mechanism; Kinematics; Optimization.

1. INTRODUCTION

For the guidance of the rear axle of the motor vehicles, there are two ways: independent guidance of the wheels (case in which each wheel is guided by its own mechanism), and dependent guidance of the wheels (rigid axle), respectively [3, 12, 15]. The rigid axle of the motor vehicles is guided by spatial linkage mechanisms on which between axle and chassis a number of binary links or kinematic chains are interposed. The connections of the guiding arms to axle and chassis are made by using elastic joints of rubber (i.e. bushings). To the suspension displacement, the bushings undergo elastic restricted linear and angular deformations, the "joint" having in fact six degrees of freedom (compliant joint). Usually, the theoretic study of the guiding linkages has at base the modelling of the bushing by a spherical joint, neglecting in this way the linear deformations [7, 8, 9, 13, 14]. By this supposition, there are simple models with one or two degrees of mobility (DOM), and the study can be made with classic "in-house made" methods/programs.

The structural systematization of the guiding mechanisms is made considering the simplified representation bushing → spherical joint. In this way, the guidance of the axle is performed by the guidance of a number of its points around suitably chosen surfaces & curves (Fig. 1). By a binary link with spherical joints in both ends, the guidance on sphere (S) is obtained. For the triangular arms, with two joints to car body, the guidance of an axle point on circle (C) is achieved. The guidance on coupler curve (CC) is performed by a spherical joint between axle and coupler. In the last case, Watt mechanism configuration is frequently used, but Roberts, Chebyshev or Evans straight - line linkages can also be used [4, 6, 11].

Joining in parallel the basic types of guidance shown in figure 1, by their combination, all possible guiding mechanisms with DOM=1 and DOM=2 are obtained, as follows (n - the number of bodies, including the guiding arms and the axle; c - the number of joints; Σf_i - the sum of the mobilities in joints; k - the number of kinematic chains):

- suspension mechanisms with DOM=2: 4S (n=5, c=8, Σf_i=24, k=3); 2S 1C (n=4, c=6, Σf_i=16, k=2); 2S 1CC (n=6, c=9, Σf_i=19, k=3);
- suspension mechanisms with DOM=1: 5S (n=6, c=10, Σf_i=30, k=4); 3S 1C (n=5, c=8, Σf_i=22, k=3); 3S 1CC (n=7, c=11, Σf_i=29, k=4); 1S 2C (n=4, c=6, Σf_i=14, k=2); 1S 2CC (n=8, c=12, Σf_i=20, k=4); 1S 1C 1CC (n=6, c=9, Σf_i=17, k=3).

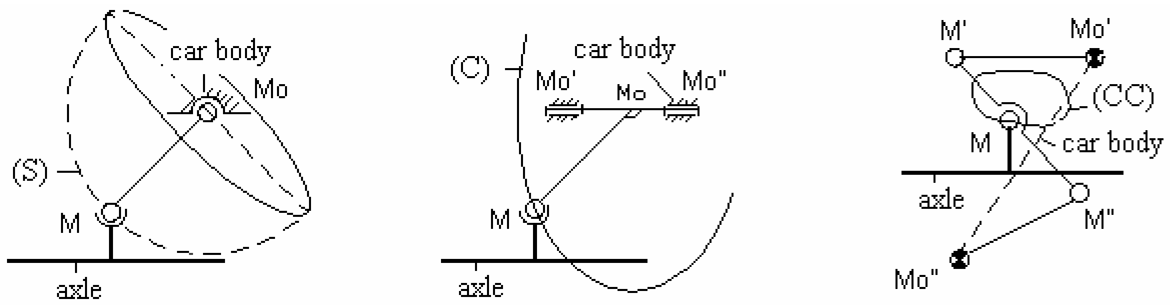

Fig.1. The basic types of guidance of the rear axle.

For optimizing the axle guiding mechanisms, from kinematic point of view, the disposing of the guiding arms has to be taken into consideration. Usually, in literature the optimization is based on design sensibility analyses, using as design variables the global coordinate of the points in which the guiding arms are connected to car body, and to axle, respectively [1, 8]. The idea is to determine the influence of the design variables on the design objectives that define the kinematic behaviour of the guiding mechanism, and to realize the optimization study by modifying the main design variables in pre-defined fields. These methods have to be particularized for each type of axle guiding linkage, representing in fact a multi-run analysis with different input data that gives the feedback on the effects of the changes.

On the other hand, the literature presents the optimization algorithms that are included in the commercial MBS (Multi-Body Systems) programs, such as ADAMS of MSC Software, NISA-DYMES of EMRC, or PAM-MEDYSA of ESI Group. The MBS optimization is based on the parameterization of the virtual model, by using design points, expressions and design variables, selecting the main design variables, through design studies and design of experiments, and minimizing or maximizing the objective function over a selection of design variables, while satisfying various constraints on the design and state variables of the system. Various algorithms are available for finding a solution to an optimization problem, for example OPTDES algorithms (Generalized Reduced Gradient, Sequential Quadratic Programming), or DOT algorithms (Modified Method of Feasible Directions, Sequential Linear Programming, Sequential Quadratic Programming) [5, 10, 16, 17]. Unfortunately, the commercial MBS programs are very expensive, even in academic license configuration.

In these terms, the paper presents a general analytical method for the optimal geometric synthesis of the axle guiding mechanisms, in a unitary approaching. The method can be applied for all suspension mechanisms of the rear axle (with DOM=1 and DOM=2), as well as for the guiding mechanisms of the front & rear wheels (independent suspension). The general characteristic of the method comes from the decomposition of the guiding mechanism in the elementary binary chains (ex. sphere-sphere, sphere-rotation), which are separately studied; with these chains, we have the possibility to build any type of guiding mechanism. In the mechanism context, the "connection" between the elementary chains is made through the global coordinates of the joints on axle, whose spatial positions (trajectories) are imposed. The idea is to determine the global coordinates of the joints on car body for generating the imposed trajectory of the axle. The optimal values of the objective functions, which define the kinematics of the suspension mechanism, are proportionally reduced relative to the initial variations, the reduction coefficients being established by constructive criteria; in this way, the mechanism remains in rational constructive limits. For realizing numeric simulations, the method was transposed on computer using the DELHI programming language.

2. OPTIMIZATION CRITERIA

In kinematics (analysis and synthesis), the geometrical model of the axle guiding mechanism (Fig. 2) is defined by: the global coordinates of the joints on car body, in the global reference frame (attached to car body, which is fixed in the kinematic study) - X_{M0i}, Y_{M0i}, Z_{M0i}; the local coordinates of the joints on axle, in the local reference frame - $X_{Mi(P)}$, $Y_{Mi(P)}$, $Z_{Mi(P)}$; the lengths of the guiding arms - $l_i = |M_iM_{0i}|$; the initial position of the axle in the global reference frame - X_P^0, Y_P^0, Z_P^0; the radius of the wheels; the distance between wheels. The global reference frame (OXYZ) has the axes parallel with longitudinal, transversal and vertical axes of the vehicle. The axle reference frame ($PX_PY_PZ_P$) has the origin P in the centre of the axle, the axes being parallel with the global axes in the initial modelling position.

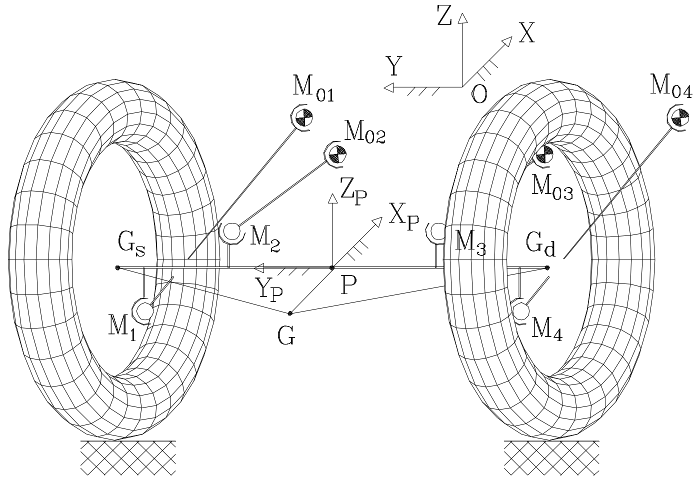

Fig. 2. The global and local reference frames for the rear suspension system.

According to figure 2, the spatial position of the rear axle, in OXYZ reference frame, is defined by three characteristic points, for all types of guiding mechanisms: the centres of the wheels G_s & G_d, and the point G from the transversal median plan of the axle, which is located on the technological axis X_P. The kinematical functions of the axle (Fig. 3) are established according to the coordinates of these characteristic points, as follows:

- the displacements of the axle's centre:

$$\Delta X_P = X_P - X_P^0, \Delta Y_P = Y_P - Y_P^0, \Delta Z_P = Z_P - Z_P^0 ; \qquad (1)$$

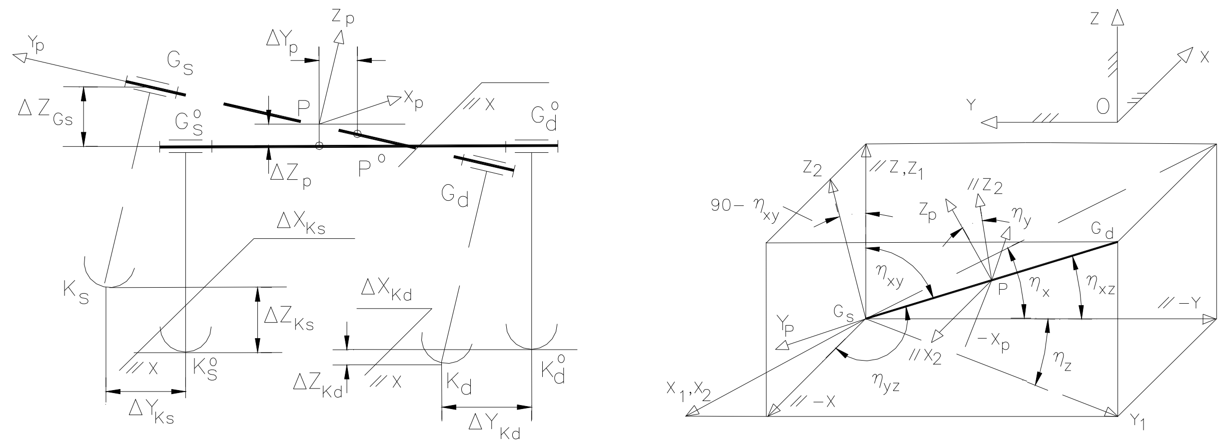

Fig. 3. The kinematic functions (parameters) of the rear axle.

- the roll rotation of the axle:

$$\eta_X = \text{arctg} \frac{Z_{Gd} - Z_{Gs}}{Y_{Gd} - Y_{Gs}} ; \qquad (2)$$

- the rotation of the axle around its own axis:

$$\eta_Y = \arcsin\frac{\left[Z_G - Z_G^0\right] - \left[Z_P - Z_P^0\right]}{\left|X_{G(P)}\right|}; \quad (3)$$

- the yaw rotation of the axle:

$$\eta_Z = \text{arctg}\frac{X_{Gd} - X_{Gs}}{Y_{Gd} - Y_{Gs}}. \quad (4)$$

Relative to the car body, the rear axle must have the possibility of vertical motion (ΔZ_P), as well as the roll rotation (η_X). When the car is in motion, the modification of the mechanism's position relative to the car body, determines (besides the above-described necessary motions) secondary undesirable motions: displacements of the axle's centre along longitudinal (ΔX_P) and transversal (ΔY_P) directions; rotations of the axle around the vertical (η_Z) and transversal (η_Y) axes. The minimization of the undesirable motions can be transposed into kinematical optimization criteria, as follows: $\Delta X_P \to 0$, $\Delta Y_P \to 0$, $\eta_Z \to 0$, $\eta_Y \to 0$. These criteria cannot be equally satisfied, and for this reason in optimal synthesis of the axle guiding mechanisms a certain criterion has priority, or a compromise will be accepted such as: $\Delta X_P \in [\Delta X_{P\,min}, \Delta X_{P\,max}]$, $\Delta Y_P \in [\Delta Y_{P\,min}, \Delta Y_{P\,max}]$, $\eta_Z \in [\eta_{Z\,min}, \eta_{Z\,max}]$, $\eta_Y \in [\eta_{Y\,min}, \eta_{Y\,max}]$, where the limits can be established depending on the top speed of the vehicle, the type of tires, and the type of vehicle (these characteristics are imposed by the automotive designer's requirements) [3, 11].

3. OPTIMIZATION ALGORITHM

Theoretically, all geometric parameters have influence on the kinematical behaviour of the guiding mechanism. By the proposed method, the local coordinates of the guiding points (i.e. the joints between the axle and the guiding arms), the initial position of the axle, the radius of the wheels and the distance between the wheels, are established by constructive criteria. Therefore, the global coordinates of the joints on car body remain as design variables that are available for the optimal synthesis of the axle guiding mechanisms. In this way, the proposed method involves three steps: imposing finite positions for axle, determining the coordinates of the joints between the car body and the guiding arms, and analyzing the obtained guiding mechanism.

According to the chapter 2, the characteristic points G_s, G_d and G define the spatial position of the rear axle. Between the nine coordinates of these points there are three dependent relationships (the axle is rigid, so that the distances between the characteristic points have constant values):

$$\begin{aligned}
&(X_G - X_{Gd})^2 + (Y_G - Y_{Gd})^2 + (Z_G - Z_{Gd})^2 - GG_d^{\,2} = 0,\\
&(X_G - X_{Gs})^2 + (Y_G - Y_{Gs})^2 + (Z_G - Z_{Gs})^2 - GG_s^{\,2} = 0,\\
&(X_{Gd} - X_{Gs})^2 + (Y_{Gd} - Y_{Gs})^2 + (Z_{Gd} - Z_{Gs})^2 - G_dG_s^{\,2} = 0.
\end{aligned} \quad (5)$$

In this way, only six coordinates are independent parameters for kinematics. Imposing "m" positions to the characteristic points, which define the origin and the orientation of the axle reference frame in relation to the global reference frame, the global coordinates of the guiding points M_i (the joints between axle and links) can be established as follows:

$$[r_M] = [r_P] + [M_{P0}] \cdot [r_M]_P, \quad (6)$$

where $[r_P]$ is the position vector of the axle's centre in the global reference frame, $[r_M]_P$ - the position vector of the guiding point in the axle reference frame, $[M_{P0}]$ - the matrix that defines the orientation of the axle reference frame relative to the global coordinate system.

The equation (6) can be re-written in the following expression (form):

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix}_P, \quad (6')$$

where the components $[a_{ij}]$ of the matrix M_{P0} are the directors cosines between the local (axle) reference frame and the global (car body) reference frame.

The finite positions that will be imposed to the axle (in fact, to the characteristic points) can be established according to the behaviour of a concrete mechanism (vehicle). A possibility to impose the positions of the characteristic points has at basis the conditions that, on the desired trajectory, the final variations $[\Delta X_P, \Delta Y_P, \eta_Z, \eta_Y]^f$ will be proportionally reduced relative to the initial mechanism $[\Delta X_P, \Delta Y_P, \eta_Z, \eta_Y]^i$, as follows ($q_{Xp}$, q_{Yp}, $q_{\eta z}$ and $q_{\eta y}$ are sub-unitary coefficients):

$$\Delta X_P^f = \Delta X_P^i \cdot q_{Xp}, \Delta Y_P^f = \Delta Y_P^i \cdot q_{Yp}, \eta_Z^f = \eta_Z^i \cdot q_{\eta z}, \eta_Y^f = \eta_Y^i \cdot q_{\eta y} . \tag{7}$$

The equations (5) and (7), coupled with the imposed vertical positions of the wheels centres (the independent kinematic parameters), form the system that is used to determine the positions of the characteristic points on the chosen trajectory. Then, using the equation (6/6'), the global positions of the guiding points M_i can be determined. The guiding points M_i are constrained to remain on the fixed surfaces or curves that have the centres in the joints M_{0i} between the car body and the guiding arms. The geometric constraint equations may be written by constant length equations.

For the guidance on sphere (case S in figure 1), the constraint equation has the general form:

$$(X_M - X_{M0})^2 + (Y_M - Y_{M0})^2 + (Z_M - Z_{M0})^2 - l^2 = 0 . \tag{8}$$

The coordinates of the guiding point M in OXYZ reference frame were determined in the previous stage. Equation (8) can be written [2]:

$$X_M^2 + Y_M^2 + Z_M^2 + X \cdot X_M + Y \cdot Y_M + Z \cdot Z_M + R = 0, \tag{9}$$

where:

$$X = -2 \cdot X_{M0}, Y = -2 \cdot Y_{M0}, Z = -2 \cdot Z_{M0}, R = X_{M0}^2 + Y_{M0}^2 + Z_{M0}^2 - l^2 . \tag{10}$$

Writing the equation (9) for "m" finite positions,

$$(X_M)_k^2 + (Y_M)_k^2 + (Z_M)_k^2 + X \cdot (X_M)_k + Y \cdot (Y_M)_k + Z \cdot (Z_M)_k + R = 0, \tag{11}$$

and subtracting the first relation (corresponding to k=1) from the others (k=2, ..., m), we obtain a system with "m-1" equations and three unknown factors (X, Y, and Z), as follows:

$$\begin{gathered} X \cdot \left[(X_M)_{k+1} - (X_M)_1\right] + Y \cdot \left[(Y_M)_{k+1} - (Y_M)_1\right] + Z \cdot \left[(Z_M)_{k+1} - (Z_M)_1\right] = (r)_1 - (r)_{k+1}, \\ (r)_k = \left(X_M^2 + Y_M^2 + Z_M^2\right)_k . \end{gathered} \tag{12}$$

In fact, the equation (12) has the following form,

$$F_k(X, Y, Z) = L_k, k = 1 ... m-1 . \tag{13}$$

For m=4 positions, the system is linear (3 equations with 3 unknowns). For m>4, an over-determined system is obtained; this system is solved with the least square's approach. Considering X', Y', and Z' the solution of the system for k = 1, 2 and 3, the system (13) can be written:

$$F_k(X', Y', Z') = L'_k, k = 1 ... m-1 . \tag{13'}$$

Subtracting the relations (13') from (13), the following relation is obtained:

$$F_k(X, Y, Z) - F_k(X', Y', Z') = L_k - L'_k = d_k . \tag{14}$$

Considering that the differences $\delta_x = X - X'$, $\delta_y = Y - Y'$, $\delta_z = Z - Z'$ are small, the system (14) has the following form:

$$\begin{gathered} a_k \cdot \delta_x + b_k \cdot \delta_y + c_k \cdot \delta_z = d_k, \\ a_k = (X_M)_{k+1} - (X_M)_1, b_k = (Y_M)_{k+1} - (Y_M)_1, c_k = (Z_M)_{k+1} - (Z_M)_1 . \end{gathered} \tag{15}$$

According to the least square's approach, the solution (15) has to verify the equations:

$$\frac{\upsilon f}{\upsilon \delta_x} = 0, \frac{\upsilon f}{\upsilon \delta_y} = 0, \frac{\upsilon f}{\upsilon \delta_z} = 0\,, \tag{16}$$

$$f(\delta_x, \delta_y, \delta_z) = \sum_{k=1}^{m-1} (a_k \cdot \delta_x + b_k \cdot \delta_y + c_k \cdot \delta_z - d_k)^2\,. \tag{17}$$

In this way, a linear system of 3 equations with 3 unknown factors (δ_x, δ_y, δ_z) is obtained, as follows:

$$\begin{aligned} &[aa]\cdot\delta_x + [ab]\cdot\delta_y + [ac]\cdot\delta_z = [ad],\\ &[ba]\cdot\delta_x + [bb]\cdot\delta_y + [bc]\cdot\delta_z = [bd],\\ &[ca]\cdot\delta_x + [cb]\cdot\delta_y + [cc]\cdot\delta_z = [cd]. \end{aligned} \tag{18}$$

where:

$$\begin{aligned} &[aa] = \sum_{k=1}^{m-1} a_k^2, [ab] = \sum_{k=1}^{m-1} a_k \cdot b_k, [ac] = \sum_{k=1}^{m-1} a_k \cdot c_k,\\ &[ba] = \sum_{k=1}^{m-1} b_k \cdot a_k, [bb] = \sum_{k=1}^{m-1} b_k^2, [bc] = \sum_{k=1}^{m-1} b_k \cdot c_k, \;.\\ &[ca] = \sum_{k=1}^{m-1} c_k \cdot a_k, [cb] = \sum_{k=1}^{m-1} c_k \cdot b_k, [cc] = \sum_{k=1}^{m-1} c_k^2. \end{aligned} \tag{19}$$

The best solution of the system (18) is the following:

$$X = X' + \delta_x, Y = Y' + \delta_y, Z = Z' + \delta_z\,. \tag{20}$$

In this way, the global coordinates of point M_0 will be:

$$X_{M0} = -\frac{X}{2}, Y_{M0} = -\frac{Y}{2}, Z_{M0} = -\frac{Z}{2}\,, \tag{21}$$

with the following medium square errors:

$$\mu_x = \varepsilon \cdot \sqrt{\frac{\Delta_A}{\Delta}}, \mu_y = \varepsilon \cdot \sqrt{\frac{\Delta_B}{\Delta}}, \mu_z = \varepsilon \cdot \sqrt{\frac{\Delta_C}{\Delta}}\,, \tag{22}$$

where Δ is the determinant of the system (18), Δ_A, Δ_B, Δ_C are the algebraic co-factors of the elements [aa], [bb], [cc] from the main diagonal of the system, and ε is the errors with which the values d_k are determined,

$$\varepsilon = \frac{\sqrt{\sum_{k=1}^{m-1} (a_k \cdot \delta_x + b_k \cdot \delta_y + c_k \cdot \delta_z - d_k)^2}}{m-4}\,. \tag{23}$$

Then, the radius of the sphere (i.e. the length of the link) can be determined from relation (8):

$$l = \sqrt{(X_M - X_{M0})^2 + (Y_M - Y_{M0})^2 + (Z_M - Z_{M0})^2}\,. \tag{24}$$

For the guidance on circle (case C in figure 1), two equations of form (8) can be written,

$$\begin{aligned} &(X_M - X_{M0'})^2 + (Y_M - Y_{M0'})^2 + (Z_M - Z_{M0'})^2 - l'^2 = 0,\\ &(X_M - X_{M0''})^2 + (Y_M - Y_{M0''})^2 + (Z_M - Z_{M0''})^2 - l''^2 = 0, \end{aligned} \tag{25}$$

and the coordinate of the joint M_0 (the pair M_0’ - M_0”) can be similarly determined.

For the guidance on coupler curve (case CC in figure 1) of the four-bar mechanism M_0'- M'- M"- M_0", which is disposed in the transversal - vertical plane YZ ($X_{M0'} = X_{M0''} = X_M$), the synthesis problem consists in the determination of the fixed joint M_0'- M_0" considering the trajectory of the point M from the coupler of the mechanism. This problem can be solved in the following steps:

▪ adopting the coordinates for a fixed joint (for example, $Y_{M0'}$, $Z_{M0'}$) and the lengths $l_1 = |M'M_0'|$, $m_1 = |M'M|$, $m_2 = |M''M|$;

▪ establishing the coordinates of the joint M' $(Y_{M'}, Z_{M'})_k$ by solving the system:

$$\begin{aligned}&(Y_{M'} - Y_{M0'})^2 + (Z_{M'} - Z_{M0'})^2 - l_1^2 = 0,\\&(Y_{M'} - Y_M)^2 + (Z_{M'} - Z_M)^2 - m_1^2 = 0;\end{aligned} \tag{26}$$

▪ establishing the coordinates of the joint M" $(Y_{M''}, Z_{M''})_k$,

$$\begin{aligned}Y_{M''} &= Y_M \cdot (1 + k_m) - Y_{M'} \cdot k_m,\\Z_{M''} &= Z_M \cdot (1 + k_m) - Z_{M'} \cdot k_m,\end{aligned}; \tag{27}$$

where $k_m = m_2/m_1$;

▪ establishing the coordinates of the centre M_0" ($Y_{M0''}$, $Z_{M0''}$) and the radius $l_2 = |M''M_0''|$ of the circle on which the point M" is guided (in fact, the position of the other fixed joint on car body), in a similar way with the guidance on sphere.

The kinematic analysis of the axle guiding mechanism is performed with the "characteristic point's method" [1, 14]. In the analysis method, there are the same three characteristic points that define the technological axle reference frame (see figure 2). Establishing the global position of the axle involves the determination of the nine coordinates of the characteristic points in the global reference frame, by using the following relations: three dependent relations between points (the constant distances - the axle is a rigid body), the geometric constraint equations (i.e. the mathematic modelling of the joints), and the variation equations of the independent parameters (the vertical positions of the wheels centres). In the constraint equations, the global coordinates of the guiding points are expressed with respect to the origin and orientation of the axle reference frame (see relation 6/6').

In this way, a nonlinear system is obtained, which is solved by the Newton-Kantorovici approach, the initial position of the system (guiding mechanism), corresponding to the vehicle in rest. In this position, we calculate the coordinates of the wheels centres and of the guiding points. The Jacobian of the system is formed by the analytical partial derivatives of the equations relative to the unknown coordinates of the characteristic points. In the solving procedure, we also use the Gauss-Jordan approach, for calculating the new solution of the system, which is compared with the previous solution. The iterative process is finished when the differences between values in two successive iterations satisfy the imposed precision. For a current position of the guiding mechanism, the initial solution of the system corresponds to the above-obtained position, and in this way the kinematics of the guiding mechanism is established for the whole displacement of the suspension.

Afterwards, the necessary position functions that describe the kinematic behaviour of the guiding mechanism are established using the equations (1) - (4). The velocity state can be determined by the analytical derivation of the position functions; similarly for the acceleration state, considering the velocity functions.

4. RESULTS AND CONCLUSIONS

Based on the above-described method, a computer program for the kinematic optimization of the axle guiding mechanisms was developed using the DELPHI programming language. The analysis and optimization methods are based on the same principle: defining the spatial position & orientation of the axle by three characteristic points, and considering the vertical displacement of the wheels centres as independent kinematic parameters. In this way, the integration in a unitary analysis & optimization software platform is assured.

The computer program has two specific modules: analysis module and optimization module. In the analysis module, the input data are: the global coordinates of the joints on car body, in the global reference frame; the local coordinates of the joints on axle, in the local reference frame; the lengths of the guiding arms; the initial (static) position of the mechanism, in the fixed reference frame. As independent kinematic parameters (generalized coordinates), the vertical positions of the wheels centres are considered. The variation functions of these parameters simulate the displacement of the wheels over bumps; the "zero" position corresponds to the car in rest (the static equilibrium position of the guiding mechanism). For post-processing, the analysis results are saved in tabular and plotting form. At the same time, the program contains specific graphic procedures for animating the mechanisms in different views (front/back, left/right, top/bottom, perspective).

The optimization module contains the above-described optimization algorithm. There are selected the basic kinematic chains that form the mechanism, for example sphere - sphere (S - see fig. 1), or sphere - circle (C). The global coordinates of the joints on axle are established by using the imposed operating conditions (7), depending on the initial values of the objective functions (obtained in the analysis module), and the reduction coefficients. Each basic binary chain is individually studied, and finally the whole guiding mechanism is analyzed for evaluating the kinematic behaviour after optimization. If there is a problem (for example, from constructive limits points of view), we can change the values of the reduction coefficients.

The method and the computer program have been tested on the guiding mechanisms of different motor vehicles. For instant, this paper presents the results obtained for the kinematic optimization of a 2S1C axle guiding linkage (fig. 4), which is similar with the guiding mechanism used for the rear axle suspension of the domestic vehicle DACIA 1300. In the initial mechanism (before optimization), there are the following input data (in mm): M_{0i} [(-2014.5, 536, 40);(-2362, 0, 168);(-2014.5, -536, 40)] - in OXYZ reference frame, $M_{i(P)}$ [(72.75, 536, -63.5); (-33.3, 0, 62.5); (72.75, -536, -63.5)] - in $PX_PY_PZ_P$ reference frame, P^0 [-2596, 0, 111] - in OXYZ, r = 259, E_a= 1376.

The mechanism has two degrees of mobility, practically being taken into consideration the following functional cases: $Z_{Gs}=Z_{Gd}$ (equal displacements of the wheels centres), and $Z_{Gs}=-Z_{Gd}$ (equal and opposite displacements). For this paper, we have considered the first functional case. The optimization has been performed considering the following criteria: $\Delta X_P \in [\Delta X_{P\,min}, \Delta X_{P\,max}]$, and $\eta_Y \in [\eta_{Y\,min}, \eta_{Y\,max}]$. The variations ΔY_P and η_Z have not been taken into consideration in the optimization process because, as result of the kinematic analysis, we have established that these variations are insignificant for the considered functional case (equal displacements of the wheels). The finite positions imposed to the axle have been established considering the conditions that the final variations ΔX_P^f and η_Y^f to be proportionally reduced relative to the initial values ΔX_P^i - η_Y^i, while ΔY_P and η_Z are null for the whole displacement of the suspension. In this way, we obtained the following system for determining the coordinates of the characteristic points that define the spatial position of the axle (input data in the optimization process):

$$
\begin{aligned}
X_{Gs} &= \Delta X_P^i \cdot q_{Xp} + X_P^0 = X_{Gd}\,, \\
Y_{Gs} &= Y_{Gs(P)} + Y_P^0 = -Y_{Gd}\,, \\
Z_{Gs} &= Z_{Gd} = Z_{Gs,d}^i\,, \\
Y_G &= Y_{G(P)}\,, \\
Z_G &= |X_{G(P)}| \cdot \sin(\eta_Y^i \cdot q_{\eta y}) + Z_P^i\,, \\
X_G &= X_{Gs} + \sqrt{GG_s^2 - (Y_G - Y_{Gs})^2 - (Z_G - Z_{Gs})^2}\,,
\end{aligned}
\tag{28}
$$

in which ΔX_P^i, η_Y^i şi Z_P^i are specific to the initial mechanism, and q_{Xp} and $q_{\eta y}$ are the sub-unitary coefficients for reducing the kinematic parameters ΔX_P and η_Y.

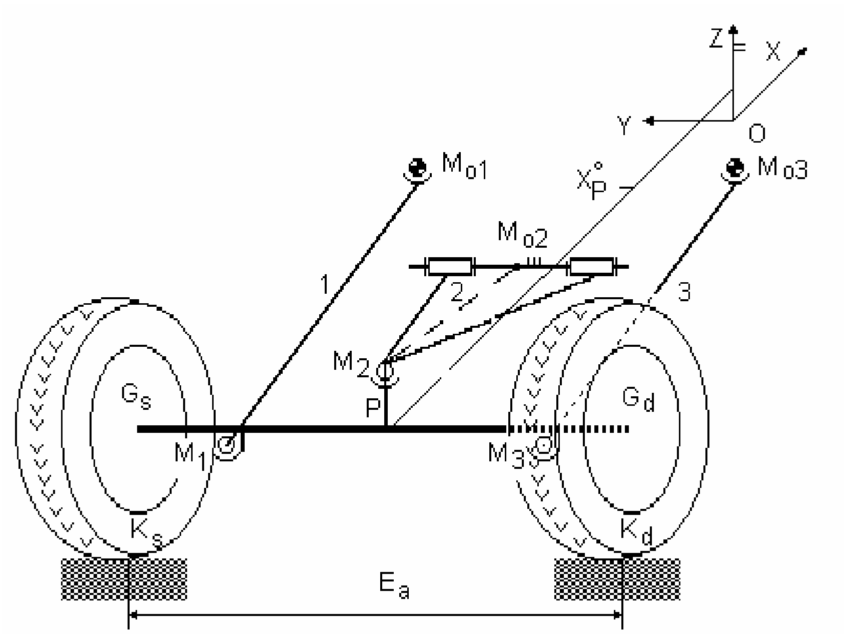

Fig. 4. The structural model for the 2S1C guiding mechanism of the rear axle.

The reduction coefficients have the values q_{Xp} =1/1.5 and $q_{\eta y}$ = 1/100, in order to keep the guiding mechanism in rational-constructive limits. The vertical position of the wheels corresponds to the passing over a bump, in the field $\Delta Z_{Gs,d} \in [-120, 120]$ mm, relative to the static position ($Z_{Gs,d}$=111 mm). After the integrated optimization & analysis study, as we can see in the diagrams shown in figure 5 (curve 1 - initial mechanism, curve 2 - final/optimum mechanism), the axle guiding mechanism respects the imposed reduction coefficients, and this demonstrates the viability of the optimization algorithm. The final values of the design parameters (the global coordinates of the fixed joints on car body), corresponding to the optimum mechanism, are: M_{0i} [(-2020.7, 536, 41.4); (-2131.25, 0, 167.5); (-2020.7, -536, 41.4)].

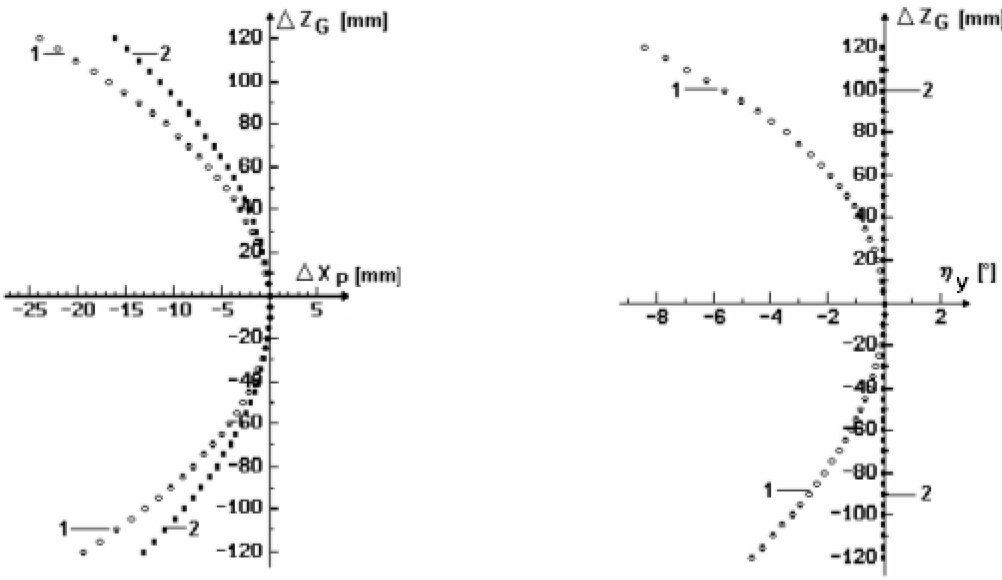

Fig. 5. The results of the optimization process.

The above-presented optimization method is characterized by generality, and allows us to perform the unitary study of the axle guiding mechanisms. By this method, the influences of the deviations of the dimensions can also be performed. The computer program, by its generality, allows a comparative study of different types of guiding mechanisms with particular values of the input data.

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