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AN IMPROVED VERSION OF THE MODIFIED MAXIMUM FORCE CRITERION (MMFC) USED FOR PREDICTING THE LOCALIZED NECKING IN SHEET METALS*

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The improvement proposed in the frame of this paper consists in adding two material constants to the MMFC model. Their values are determined by enforcing a couple of experimental constraints on the model. In this way, the quality of the theoretical predictions can be improved. As proven by the numerical tests, the most appropriate constraints are related to the necking occurrence in plane strain and biaxial tension, respectively. By controlling these points of the FLD, the discrepancies between Hora's model and experiments can be minimised. If no experimental data is available, the constraints could be generated by using some other model for the FLD determination (e.g., empirical formulas or the model developed by Marciniak and Kuczynski). The performances of the new model are evaluated by comparing its predictions with Hora's necking criterion and experimental FLD's corresponding to DC01 steel.

Key words: Sheet metals, Forming Limit Diagram, MMFC model

1. INTRODUCTION

Failure prediction is one of the most important goals of the numerical simulation when used in the field of sheet metal forming processes. At present, the concept of Forming Limit Diagram (FLD) proposed by Keeler and Backofen [1] and Goodwin [2], respectively, is almost exclusively used for this purpose. FLD's are determined either by experiments or by computation. Unfortunately, the experimental approach is very time consuming and needs specialized laboratory equipment. On the other hand, the computational models currently used for the FLD determination are not in good agreement with the experimental data.

The first theoretical FLD models were based on the diffuse necking and localized necking theories proposed by Hill [3] and Swift [4], respectively. In the seventh decade of the previous century, Marciniak and Kuczynsky [5] proposed a theoretical model of strain localization based on the geometrical inhomogeneity already existing in the material. Later on, the Marciniak– Kuczynsky model has been extended by Hutchinson and Neale [6] in order to describe the left branch of the FLD. In 1975, Storen and Rice [7] proposed the so-called "vertex theory" to model localized necking under biaxial stretching conditions. Recently, Dudzinsky and Molinari [8] proposed "the small perturbation theory" to model the plastic instability. An exhaustive description of the FLD concept may be found in [9-11]. Hora [12] intended to improve Swift's criterion by taking into account the experimentally confirmed fact that the onset of necking depends significantly on the strain ratio. Due to the related stress change, there is an additional hardening effect, which 'hesitates' out the localized necking. Different strategies to determine the FLD's were approached in [13].

The aim of this paper is to propose an improvement of MMFC model in order to have a better prediction of FLD. By adding two material constants to the MMFC model, the predicted FLD is enforced to catch two experimental points. The experimental points have been chosen one in the plane strain range and the other in the biaxial range, respectively. Unfortunately, this approach does not avoid the singularity of MMFC model which emerges if the yield locus of the considered material exhibits straight lines. To solve the singularity issue of the model is not a topic of this paper. This singularity has been notified and presented in detail in [14, 15].

* Invited Paper

2. REVIEW OF THE MODIFIED MAXIMUM FORCE CRITERION (MMFC)

In order to predict the failure, Hora et al. [12] proposed the following mathematic formulation named “Modified Maximum Force Criterion” (abbr. MMFC):

$$\frac{\partial\sigma_{11}}{\partial\varepsilon_{11}}+\frac{\partial\sigma_{11}}{\partial\beta}\frac{\partial\beta}{\partial\varepsilon_{11}}=\sigma_{11} \tag{1}$$

where:

$$\beta\equiv\frac{\dot{\varepsilon}_{22}}{\dot{\varepsilon}_{11}}=\mathrm{const} \tag{2}$$

The model represents an improvement of the Swift’s model. The experiments confirm the fact that the onset of the failure significantly depends on the strain ratio defined by (Eqn. 2). Hora’s model took into account this issue. This model is based on the following assumptions [12]:

- The material under consideration is assumed to be rigid-plastic and orthotropic.
- A principal plane stress state is assumed, i.e. σ_{11}, σ_{22} are the only non-zero stresses acting in the sheet plane ($\sigma_{11}\geq\sigma_{22}$). During the plastic deformation the volume of the sheet metal remains constant ($\dot{\varepsilon}_{33}=-\dot{\varepsilon}_{11}-\dot{\varepsilon}_{22}$).
- The anisotropy axes coincide with the principal strain and stress axes.
- Isotropic hardening is considered.
- Only the case of linear strain paths is treated (β = const. - see Eqn. 2).

A detailed computational strategy may be found in [12, 14, 15].

3. IMPROVEMENT OF THE MODIFIED MAXIMUM FORCE CRITERION (MMFC)

In order to improve the prediction of limit strains using MMFC model, the authors chose to introduce two fitting coefficients. The new MMFC model has the following mathematical formulation:

$$A\frac{\partial\sigma_{11}}{\partial\varepsilon_{11}}+B\frac{\partial\sigma_{11}}{\partial\beta}\frac{\partial\beta}{\partial\varepsilon_{11}}=\sigma_{11} \tag{3}$$

where the parameter β is defined by Eqn. 2 and the coefficients *A* and *B* will be calculated using two experimental points representing different limit states.

The experimental points can be located anywhere along the forming limit curve. The authors recommend the plane strain and equibiaxial stretching points, as they lead to the best performances of the model.

Following the mathematical formulation developed by Aretz [14], Eqn (3) may be written as:

$$A\cdot Y'\left(\bar{\varepsilon}\right)\cdot f\left(\alpha\right)\cdot g\left(\alpha\right)-B\cdot Y\left(\bar{\varepsilon}\right)\cdot\frac{f'\left(\alpha\right)\cdot g\left(\alpha\right)\cdot\beta\left(\alpha\right)}{\beta'\left(\alpha\right)\cdot\bar{\varepsilon}}=f\left(\alpha\right)\cdot Y\left(\bar{\varepsilon}\right) \tag{4}$$

where:

$$\alpha\equiv\frac{\sigma_{22}}{\sigma_{11}},\quad \bar{\sigma}=\frac{1}{f\left(\alpha\right)}\cdot\sigma_{11},\quad \bar{\varepsilon}=g\left(\beta\right)\cdot\varepsilon_{11} \tag{5}$$

The values of the stress ratio α belong to the range $0\leq\alpha\leq1$, from uniaxial tension to equibiaxial stretching. The other notations used in Eqn. (5) have the following significance: $\bar{\sigma}$ - equivalent stress defined by the yield criterion; $\bar{\varepsilon}$ - equivalent plastic strain; $f\left(\alpha\right)$, $g\left(\beta\right)$ - functions depending on the particular formulation of the equivalent stress; $Y(\overline{\varepsilon})$ - hardening law; $Y'(\overline{\varepsilon})$ - hardening modulus (derivative of $Y(\overline{\varepsilon})$ with respect to $\overline{\varepsilon}$).

The parameter β can be rewritten as

$$\beta \equiv \left.\frac{\dfrac{\partial\overline{\sigma}}{\partial\sigma_{22}}}{\dfrac{\partial\overline{\sigma}}{\partial\sigma_{11}}}\right|_{\sigma_{22}=1,\sigma_{11}=\alpha} \tag{6}$$

Eqn. (6) shows that β depends on the stress ratio α. The functions $\beta'(\alpha)$ and $f'(\alpha)$ have the following formulation:

$$\beta'(\alpha)=\frac{d\beta}{d\alpha},\ f'(\alpha)=\frac{d f}{d\alpha} \tag{7}$$

One may see that, if $\beta'(\alpha)=0$, Eqn. (4) is indeterminate. Due to the fact that the strain ratio β do not suffer any changes regarding to α, the solution for $\overline{\varepsilon}$ cannot be found in such cases. They occur when the yield locus contains straight lines. This characteristic is often observed at the yield loci described by the non-quadratic functions. A large discussion on this theme may be found in [14].

4. BBC2003 YIELD CRITERION

The BBC2003 equivalent stress has the following formulation [16]:

$$\begin{aligned}&\overline{\sigma} := \left[a(\Gamma+\Psi)^{2k} + a(\Gamma-\Psi)^{2k} + (1-a)(2\Lambda)^{2k}\right]^{1/2k},\\ &a\in[0,1],\ k\in N^{*}\end{aligned} \tag{8}$$

where $0 \le a \le 1$ and $k \in N^{*}$ are material parameters. The parameter k depends on the crystallographic microstructure of the material: $k=3$ for BCC alloys and $k=4$ for FCC alloys. The functions Γ, Ψ and Λ depend on the non-zero components of the stress tensor:

$$\begin{aligned}&\Gamma=\frac{\sigma_{11}+M\sigma_{22}}{2},\ \Psi=\left[\left(\frac{N\sigma_{11}-P\sigma_{22}}{2}\right)^{2}+Q^{2}\sigma_{12}\sigma_{21}\right]^{1/2},\\ &\Lambda=\left[\left(\frac{R\sigma_{11}-S\sigma_{22}}{2}\right)^{2}+T^{2}\sigma_{12}\sigma_{21}\right]^{1/2}\end{aligned} \tag{9}$$

The coefficients M, N, P, Q, R, S and T are also material parameters. The stress components $\sigma_{\alpha\beta}(\alpha,\beta=1,2)$ are expressed in the system of plastic orthotropy axes (1 is the rolling direction – RD, 2 is the transverse direction – TD, and 3 is the normal direction – ND). The shape of the yield surface is defined by the coefficients a, M, N, P, Q, R, S and T. Their values are established in such a way that the constitutive equations reproduce as well as possible the following characteristics of the material: the uniaxial yield stresses and the r-coefficients associated to the 0^{o}, 45^{o} and 90^{o} directions. One must emphasize that the number of material parameters in Eqn (9) is bigger than the number of experimental values. In order to decrease the number of identification equations, we enforce the constraints N = P and M = R. The identification procedure is based on the Newton–Raphson method.

5. EXPERIMENTAL DETERMINATION OF FLD

5.1. MATERIAL CHARACTERIZATION

The DC01 steel has been used in the experimental work. This sort of steel is frequently used for obtaining deep-drawn parts in home-appliance industry (cookers, fridges, radiators). The chemical composition (expressed in percentage) of the DC01 steel is shown in Table 1.

Table 1. Chemical composition of DC01 steel alloy

Tipul	C	Mn	P	S	Ti	N	Al
	Max	Max	Max	Max	Max	Max	Max
DC01	0,12	0,60	0,045	0,045	-	0,003	0,04

The mechanical characteristics of the DC01 steel have been determined by uniaxial tensile tests performed on a Zwick/Roell 150kN testing machine. The specimens have been cut at 0^o, 45^o and 90^o angles with respect to the rolling direction. Five tensile tests have been performed for each direction. The program testeXpert II [17] has been used for data acquisition and post-processing. The nominal thickness of the sheet metal was 0.7 mm. The experimental results are listed in Table 2. Table 3 shows the mechanical coefficients of the hardening law. Table 4 shows the values of the coefficients in the BBC2003 yield criteria (a, M, N, P, Q, R, S, T) provided by the identification procedure.

Table 2. Mechanical parameters of the DC01 steel sheet (0.7 mm nominal thickness)

σ_0 [MPa]	σ_{45} [MPa]	σ_{90} [MPa]	r_0 [-]	r_{45} [-]	r_{90} [-]
211.6	223	216.7	1.41	1.16	1.91

A Swift hardening law has been adopted:

$$Y(\dot{\bar{\varepsilon}}) = K\left(\varepsilon_0 + \bar{\varepsilon}^p\right)^n \tag{10}$$

Table 3. Coefficients of the hardening law corresponding to the DC01 steel sheet (0.7 mm nominal thickness)

K[MPa]	ε_0	n	Y_{ref}[MPa]
615.8645	0.001	0.2095	211.6

Table 4. Coefficients of the BBC2003 yield criterion corresponding to the DC01 steel sheet (0.7 mm nominal thickness)

a	M	N	P	Q	R	S	T
0.526265	0.870113	0.935056	0.935056	0.919252	1.030797	1.034991	0.970411

5.2. EXPERIMENTAL DETERMINATION OF FLD

The limit strains can be determined according to different criteria: the occurrence of diffuse necking, localized necking or fracture. This paper presents an analysis of the strain localization process and, in connection with this aspect, the method used for establishing the limit strains. For the practical determination, the authors used an Aramis [18] device. In order to cover the domain of the strains usually met in industry, different load paths of the specimen must be obtained. Such paths should span between the uniaxial tension (ε_1 = -2ε_2) and the equibiaxial stretching (ε_1 = ε_2). They can be obtained by realizing different stress states, which in their turn may be obtained by modifying the shape or/and dimensions of the punch (Keeler, Marciniak), die (Jovignot), specimen geometry (Brozzo and de Luca, Sanz–Grumbach, Nakazima, Hasek, Azrin–Backofen) or the lubrication conditions (Hecker).

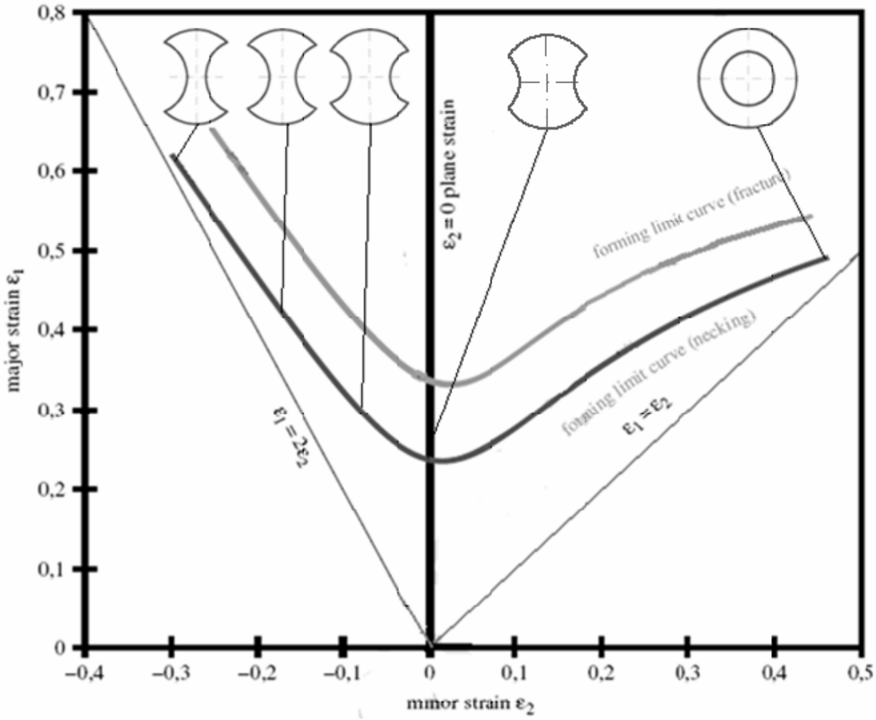

Figure 1. Shape of the specimens used in order to determine the FLD

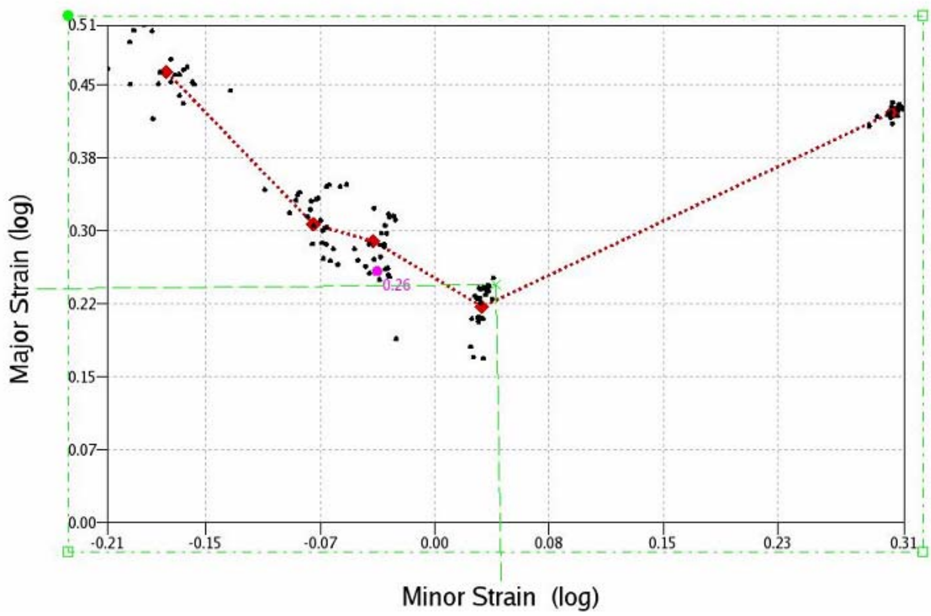

Figure 2. Experimental FLD determined using the Aramis system

The authors performed the following tests to determine the experimental FLD's: hydraulic bulge testing for the positive minor strains and punch stretching testing for the negative minor strain, respectively. The shape of the specimens is presented in Figure 1. Four types of specimens with different radii were used in order to cover the left side and de plane strain area of the FLD. On the right side of FLD, only one point has been determined by performing a bulge test. This point corresponds to the equibiaxial zone.

Figure 2 shows the experimental FLD for the DC01 steel sheet, in the case when the major strain is oriented along the rolling direction.

6. COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL DATA

Figure 3 presents a comparison between the FLD's computed using the classical MMFC model and the improved formulation proposed in this paper. Some experimental points are plotted on the same diagram. In order to compute the FLD's using the improved MMFC model, the authors select two pairs of experimental data. The first pair has been chosen in the plane strain (PS) and equibiaxial (EB) regions, respectively. The second pair of experimental data corresponds to the plane strain (PS) and uniaxial tension (UT) area. One may notice that the best prediction has been obtained by using the improved MMFC model and the first pair of experimental data (Hora 2 pts PS_EB). The predictions obtained with the second pair of experimental points (Hora 2 pts PS_UT) overestimate the experiments. In general, the classical formulation of the MMFC model underestimates the formability of the DC01 steel sheet.

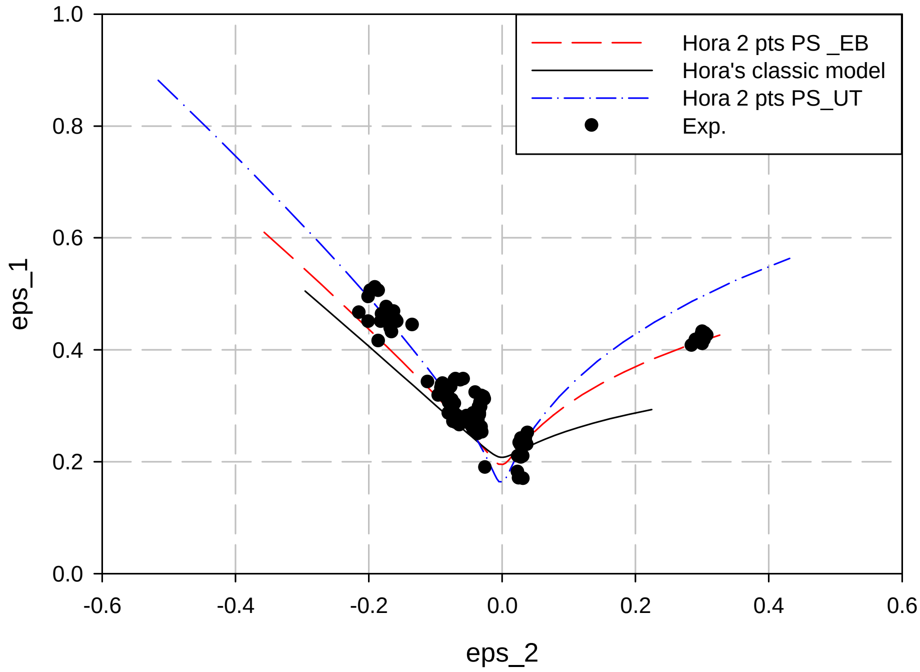

Figure 3. Comparison between numerical and experimental FLD's

7. CONCLUSIONS

The authors present an improved version of the Modified Maximum Force Criterion (MMFC) used for predicting the localized necking in sheet metals. The new model enforces the theoretical curve to catch two experimental points. The authors compare two forming limit curves obtained by imposing two pairs of experimental points located in different areas. The first curve is obtained by choosing experimental points located in the plane strain area and equibiaxial area, respectively. The second curve is determined by using the points located in the plane strain area and uniaxial tension area. As one may notice, the best prediction is given by the first curve, so the recommendation of the authors is to choose experimental points located in the plane strain and equibiaxial stretching areas. In order to determine the experimental points, a practical technique to determine the FLD has been described in the paper.

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