THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 10, Number 1/2009, pp. 000–000

ENTROPY ANALYSIS OF NONEQUILIBRIUM PROCESSES IN ENERGY SYSTEMS

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We analyze the principle of entropy increment for analysis of the stability of operation of technical systems, including the case of presence of stochastic processes and fluctuation phenomena.

The structure of mechanical devices, as a rule, possesses spatial and temporal properties, which can hardly be analyzed by the well-known methods of mathematical formalization. The most important characteristics of modern technical systems are their nonlinearity, stochasticity, the presence of a large quantity of subsystems, and irreversibility. Some subsystems have a deterministic character, but the others are stochastic, and, hence, the optimization of each subsystem does not mean optimization of the entire system. The specific features of certain subsystems lead to permanent fluctuations, i.e., to random deviations of the quantities from their average values.

It should be emphasized that the combined action of deterministic and stochastic forces converts a technical system from its initial state into new, often unstable states.

We may isolate three types of systems: deterministic, chaotic, and deterministic-chaotic.

The concept of deterministic chaos is interpreted here as irregular (chaotic) motion generated by nonlinear systems, for which the dynamic laws determine unambiguously the time evolution of state of the system provided that the preceding processes are known.

One succeeds in constructing a theory of space-time chaos, in the first place, in those cases where the dynamics of a nonlinear field can be considered as the dynamics of a complex of interacting stable and unstable structures.

Note that one can consider the systems exploited for a long time and, due to this, having become less reliable as nonequilibrium and unstable.

By virtue of the extreme disorder and possible sharp variability of all parameters of nonequilibrium, nonstationary, and stochastic systems in time and space, one should apply new methods of investigations in studying such processes.

Important results in this direction were obtained in [1]: the randomness in time, as a rule, is connected with the action of external factors, whereas the random distribution of a field in space can be only a consequence of deterministic laws, which control the change in variables along spatial coordinates, and depends very weakly on random inhomogeneities.

For a mechanical conservative system, the condition of stability of its equilibrium is determined by the Lagrange – Dirichlet theorem, according to which equilibrium is stable if the potential energy of the system in the equilibrium state is minimal. For systems being in motion, the conditions of asymptotic or exponential stability are determined by the Lyapunov criterion [2].

For the solution of these problems, a method based on the principles of modern nonequilibrium thermodynamics and information theory can be efficient.

A significant contribution to the development of new approaches to the analysis of unstable systems was made by I. Prigogine and his school [3]. Prigogine has proved that irreversible, nonrepeatable processes represent a source of order. The combined actions of stochastic and deterministic forces convert the system from initial states to new, indicating here which exactly new configurations will be realized.

Despite the stability with respect to initial conditions, deterministic-stochastic systems possess a substantial stability to arising fluctuations.

Entropy represents a measure of uncertainty of the state of a system. It is determined as the mathematical expectation of logarithm of the probability of this state (Boltzmann formula)

$$S = -k \cdot \ln W, \tag{1}$$

where W is the so-called thermodynamic probability, which indicates the number of possible realizations of a given state.

Relation (1) is similar to the probability of information capacity (Hartley formula)

$$I = -k \cdot \ln P, \tag{2}$$

where P is the probability of state of the system.

As a result of a fluctuation, entropy changes from its equilibrium value S_0 to $S_0 + \Delta S$. Therefore, the probability of fluctuation under consideration is determined as (Einstein formula)

$$W = \left\{ \frac{\Delta S}{k} \right\}, \tag{3}$$

where k is the Boltzmann constant.

It should be emphasized that the concept of thermodynamic probability is distinguished formally and, as a rule, does not coincide with the usual concept of probability.

One can analyze the probability of some quantity a only in the case where this quantity changes within the limits of this system. This means that the function $W(a)da$ gives the actual probability of finding the a value in the interval from a to $a + da$. Within the framework of such presentations, the interrelation between probability and entropy is given by [3]

$$W(a)\,da = \frac{e^{(1/k)S(a)}\,da}{\int e^{(1/k)S(a)}\,da}. \tag{4}$$

Except the Boltzmann constant k, this equation contains only data on macroscopic quantities.

For stochastic processes, we may deal with fluctuation phenomena. In the case of fluctuations of several parameters, e.g., a and b, the probabilistic formula has the form

$$W(a,b)\,da\,db = const\; e^{\frac{1}{k}S(a,b)}\,da\,db. \tag{5}$$

These relations enable us to conclude that the probability of a fluctuation is proportional to the total entropy of the closed system. It follows from here that W can be expressed as

$$W \simeq \exp \Delta S$$

where ΔS is the change in entropy due to a fluctuation.

If the body under consideration is not in equilibrium with the environment, then

$$\Delta S = -R_{\min} \cdot T_0 \tag{6}$$

where $R_{\min}$ is the value of minimal work and T_0 is the temperature of the medium.

In this case,

$$W \sim \exp\left(-\frac{R_{\min}}{T_0} \right). \tag{7}$$

Taking into account that

$$R_{\min} = \Delta E - T_0 \Delta S + p_0 \Delta V,$$

where ΔE, ΔS, and ΔV are the changes in energy, entropy, and volume, respectively, of a given small part of the body induced by a fluctuation and T_0 и p_0 are the parameters of the environment, we have [4]

$$W \sim \exp\left(-\Delta E + T\Delta S - p_0 \Delta V / T\right). \quad (8)$$

In such form this relation is applicable to any fluctuations: both small and significant.

For any space of states of the system under consideration, the change in entropy can be represented as a sum of two components [5]

$$dS = dS_e + dS_i , \quad (9)$$

where dS_e is the change in entropy due to the action of the environment and dS_i is the change in entropy caused by internal processes taking place in the system.

The state of the system is determined by the relation between terms in formula (9). As follows from this formula, the equilibrium state of the system is realized if $dS_e = - dS_i$.

The stable stationary state will be maintained provided that the entropy increment per unit time, caused by irreversible internal processes, is compensated by the permanent inflow of negative entropy (negentropy) into the volume of the system. For this purpose, one should enhance the level of organization of the processes in a technical system, to increase the degree of its automation, etc.

To investigate the degree of perfection of deterministic-stochastic systems, it is reasonable to use the Kolmogorov entropy S_κ, which represents the most important characteristic of stochastic phenomena in a space of arbitrary dimension [6]. Showing the rate of loss of information, S_κ describes the change in the state of the system with time. This means that $S_\kappa\big|_{n+1} - S_\kappa\big|_n$ gives the loss of information on the system in the time interval from n to $n + 1$.

If certain equipment is worn out, and it is necessary to replace it, the exergy for the manufacture of new equipment is

$$E = \sum_{i=1}^{n} M_i \left(\frac{l_{pr}}{\eta_{pr}} + \frac{\Delta G}{\eta_r} + \frac{l_t}{\eta_t} \right) + W_{mt} , \quad (10)$$

where M_i is the mass of the i-th article, l_{pr} is the specific minimal work for the production of raw materials, ΔG is the Gibbs energy (specific) of the reduction reaction; l_t is the specific work of treatment of the article, η_i are the corresponding efficiencies of these processes, W_{mt} is the exergy consumption for the treatment, mounting, and transport of elements of the system, and n is the number of articles in the element of the system.

In recent years, the theoretical-graph method of optimal analysis and synthesis of technical systems of different functional destination is widely used in engineering practice. As to the energy transforming systems working by the direct and reverse cycles, work [7] is here of especial interest. Recently, this method was supplemented by analysis in terms of the present-day applied thermodynamics.

Current graphs with nodes reflecting the elements of the system and arcs describing physical fluxes are clear and convenient means for carrying out variant calculations and parametric optimization, which serves as a basis for developing a package of applied programs.

The aim of optimization is to minimize the value of the objective function

$$Z = \sum_i \sum_j Z_{ij} X_{ij} \quad (11)$$

for all ij belonging to the network, where Z_{ij} is the arc weight, i.e., the expenditures in a block (cycle) corresponding to the given arc with accepted boundary conditions. Here, $X_{ij} = 1$ if the arc ij is a part of the path under consideration, and $X_{ij} = 0$ otherwise.

It should be noted that the Kolmogorov entropy determines the average time for which one can predict the state of a system with dynamic chaos. This fact is evidence that S_κ is the quantity which characterizes chaotic motion and, hence, the degree of destruction of the object under study. In this case, we may define a strange attractor as an attractor with positive entropy.

It should be emphasized that a strange attractor possesses a complex structure. An attractor represents a contact invariant subset of the phase space that is asymptotically stable.

It is worth noting in conclusion that the high-performance work of a machine and the stability of its regime states correspond to the requirements of the minimum of energy dissipation. This principle can be formulated in the following way: for the set of states, including the state of the system as a whole, the stability of operation and the minimal energy dissipation are characterized by the minimal entropy increment. This postulate (Prigogine theorem) expresses the inertial property of nonequilibrium systems and represents the variational principle of irreversible processes [8].

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Received November 26, 2008