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ADDENDUM TO MY PAPER "THE SOLUTION TO W. MAY'S PROBLEM FOR ISOMORPHISM OF COMMUTATIVE GROUP ALGEBRAS OF MIXED GROUPS OVER FINITE FIELDS" (Proc.Ro.Acad. 2/2008 [3])

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The main purpose of this brief article is to give a more elementary, but however nontrivial, proof of the following fact appeared in Proc. Roman. Acad. Sci. – Ser. A, Math. (2008), namely: Let G be an abelian group whose p-component G_p is totally projective and F_p is a simple field with characteristic $p > 0$. Then for any group H the F_p – isomorphism of group algebras $F_pH \cong F_pG$ implies $H_p \cong G_p$, The idea is based on a new group criterion for total projectivity of p-primary abelian groups in arbitrary length.

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Suppose F_pG is the group algebra of an abelian group G over the finite p-element field F_p. In [4], Warren Lee May has asked the following, posed for an arbitrary field F of char(F) = $p \neq 0$:

Isomorphism Question (May, 1988). *If $F_pG \cong F_pH$ as F_p-algebras for some group H and G_p is totally projective, does it follow that $G_p \cong H_p$?*

Partial solutions to the above formulated problem are given by us in [1] and [2]. The complete settling of the May's query is stated in our investigation [3] as well.

Our major goal here is to simplify the method used in [3], but before do this, we shall argue one useful group necessary and sufficient condition for total projectivity of p-torsion groups in arbitrary lengths of some interest and importance. So, we proceed by proving

Group Criterion. *The reduced abelian p-group A is totally projective if and only if A/A^{p^λ} is a direct sum of groups with lengths strictly less than λ for some limit ordinal $\lambda \leq$ length(A) such that λ = length(A) provided length(A) is limit, and A/A^{p^α} is totally projective for all $\alpha < \lambda$ that is A is a C_λ-group.*

Proof. ***Case 1:*** Assume length(A) is limit. The necessity holds directly by [5].

For the sufficiency, we write down $A = \sum_{i\in I} A_i$, where length(A_i) $< \lambda$. Because $A^{p^\alpha} = \sum_{i\in I} A_i^{p^\alpha}$, the canonical isomorphism yields $A/A^{p^\alpha} \cong \sum_{i\in I} A_i/A_i^{p^\alpha}$. Moreover, by the supposition along with [5], $A_i/A_i^{p^\alpha}$ is totally projective for α = length(A_i) and every $i \in I$, i.e., so is A_i. Finally, taking into account [5], the same property has and A, as promised.

Case 2: Let length(A) be arbitrary, whence length(A) = $\lambda + n$ for some limit ordinal λ and natural number n.

The left implication is true by virtue of [5].

For the right one, bearing in mind that $(A^{p^\lambda})^{p^n} = 1$ together with [5], A is totally projective if and only if so is A/A^{p^λ}. But, it is obvious that length(A/A^{p^λ}) = λ. Thus, in view of the first step and by the text, the facts that $A/A^{p^{\lambda}} = \sum_{i\in I} G_i$ where length(G_i) $< \lambda$ and $A/A^{p^{\lambda}}/(A/A^{p^{\lambda}})^{p^{\alpha}} = A/A^{p^{\lambda}}/(A^{p^{\alpha}}/A^{p^{\lambda}}) \cong A/A^{p^{\alpha}}$ for each $\alpha<\lambda$ does imply that $A/A^{p^{\lambda}}$ must be totally projective, as required.

Well, we come to the central aim raising the

General Answer of the May's Isomorphism Question. We shall use a transfinite induction on lenfth(G_p). In this aspect, we presume that the theorem is valid for groups of lengths strictly less than length(G_p). Owing to the Main Proposition from [1], $F_p(G/G_p^{p^{\delta}}) \cong F_p(H/H_p^{p^{\delta}})$ as F_p-algebras for all ordinals δ. So, by hypothesis, $(H/H_p^{p^{\delta}})_p = H_p/H_p^{p^{\delta}}$ is totally projective for each $\delta <$ length(H_p) = length(G_p). According to the scheme of proof in [3], $(H/H_p^{p^{\gamma}})_p = H_p/H_p^{p^{\gamma}}$ is a direct sum of groups in lengths strictly not exceeding $\gamma \leq$ length(H_p) for an arbitrary limit ordinal γ with such a property. Next, the exploiting of the Group Criterion leads us to this that H_p is totally projective, as desired.

The proof is completed after all.

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