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# NONLINEAR RESTRAINTS IN SEISMIC ISOLATION OF BUILDINGS

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This paper presents some remarks regarding the applicability limits of seismic base isolation technology. First, is make mention about restraints give by building and site natural periods and then some considerations regarding the nonlinear calculus necessity are exposed.

*Key-Words*: - seismic base-isolation, non-linear dynamics, numerical simulation

## 1. INTRODUCTION

The fundamental concept of the base isolation systems is to isolate a structure from ground motion by introducing a flexible interface between the foundation and the base of structure, thus limiting the amount of forces that can be transferred to the superstructure and thus diminish the structural demand.

These attractive capacities make from base-isolation method a mainstream design option for seismic hazard mitigation over the past few decades [10], [13]. However, this excellent strategy is not suitable for all buildings and for all emplacements.

Because of lateral flexibilization, the natural period of the past fixed-base structure undergo a jump towards large values and the new base-isolation structure has a new and bigger natural period. This positive period-shift can extract the structure away from the characteristic period contents of earthquake ground motions or, per contra, can throw the structural ensemble into resonant conditions. Therefore, a first restraint is connected to the natural periods of structure and emplacement site. A succinct analysis of these period conditions make the object of chapter two.

The period-shift depends on strength and damping characteristics of materials or devices from isolation and site layers that exhibit nonlinear behavior which imposes a new restraint category linked by nonlinear shift determination. In chapter 3 for necessary period-shift evaluation a double step method based on the dynamic magnification functions is presented. First, assuming linear behavior for isolated-base structure one obtain an estimation of necessary period jump and then in the second step a nonlinear sdof (NKV model[3]) is used in order to evaluate the influence of the non-linear properties on linear period-shift assessment.

## 2. NATURAL SITE CONDITIONS

By introducing an isolant layer with reduced stiffness the structural base isolate ensemble, acquire expanded lateral flexibility. As a result of flexibilization, the natural period of the past fixed-base structure undergoes a jump and the base-isolation structure acquire a new natural period. The flexibility of the interposing layers between structure and its foundation lead to a bigger fundamental period for structural ensemble. Due to this augmentation trend – only towards large periods - base isolation technology is not suitable for all buildings. In terms of structure and site natural periods the isolation technology can avoid the dangerous effects of resonant conditions or contrary, can place the structure in the resonant conditions.

If the natural period of a fixed-base structure is too close to the dominant site period and the resonant danger exists, the period shift provoked by isolation layer can extract the structure from resonant zone. But in this case must avoid the excessive flexibilisation. If the natural period of the fixed-base structure is inferior to dominant site period the shift period due to isolation layer can bring the structure into resonant conditions.

From this reasons, most suitable candidates for base isolation technology are low to medium-rise structures rested on hard soil underneath. High-rise buildings or buildings rested on soft soil are not suitable for base isolation. But, a general rule is not exists. Only a careful analysis of building and site natural periods can show if the base-isolated technology are proper or not.

## 3. NONLINEAR CONSTAINTS

### 3.1 Linear assessment of necessary period-shift

For a qualitative evaluation of the dynamic behavior, one can consider the structure with fixed base as a linear single-degree-of-freedom (sdof) subjected to harmonic abutment accelerations:

$$\ddot{x}_g(t) = \ddot{x}_g^0 \sin \omega t \tag{1}$$

where is the acceleration amplitude and ω is the excitation pulsation.

In linear dynamics, a usual description of such sdof behavior is given by the Kelvin-Voigt model consisting of a mass *m* supported by a spring (with a stiffness *k*) and a dashpot (with a viscosity *c*) connected in parallel. The governing equation of this system is [11]:

$$\ddot{x} + 2\zeta\omega_0 \dot{x} + \omega_0^2 x = -\omega_0^2 \ddot{x}_g^0 \tag{3}$$

where $\omega_0$ is undamped natural pulsation $\varsigma$ and is the damping ratio:

$$\omega_0 = \sqrt{\frac{k}{m}} \quad ; \quad \zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_0} \tag{4}$$

By using the change of variable $\tau = \omega_0 t$ and by introducing a new "time" function $\varphi(\tau) = x(t) = x(\tau/\omega_0)$ [3] one obtains from eq. (3) a dimensionless form of the equation of motion:

$$\varphi'' + C\varphi' + K\varphi = \mu \sin \upsilon\tau \tag{5}$$

where the superscript accent denotes the time derivative with respect to τ and:

$$C = \frac{c}{m\omega_0} = 2\zeta \quad ; \quad K = \frac{k}{m\omega_0^2} = 1 \quad ; \quad \mu = \frac{\ddot{x}_g^0}{\omega_0^2} \quad ; \quad \upsilon = \frac{\omega}{\omega_0} \tag{6}$$

Steady-state solution of the equation (5) read as:

$$\varphi(\tau, \upsilon, \zeta) = \mu\Phi(\upsilon, \zeta)\sin(\upsilon\tau - \psi) \tag{7}$$

where is *the magnification function:*

$$\Phi(\upsilon;\zeta) = \frac{1}{\sqrt{\left(1-\upsilon^2\right)^2 + \left(2\zeta\upsilon\right)^2}} \tag{8}$$

a ratio of maximum dynamic amplitude $\varphi_{\max} \equiv x_{dynamic}$ to static displacement $\mu = x_{static}$.

Usually, in the structural dynamics is used the natural $T_0$ (or impute $T$) period instead of natural $\omega_0$ (or excitation pulsation ω). Because $T = 2\pi/\omega$ the dimensionless pulsation is $\upsilon = \omega/\omega_0 = T_0/T$ and the magnification function (8) become:

$$\Phi(T;T_0,\zeta) = \frac{1}{\sqrt{\left[1-\left(\frac{T_0}{T}\right)^2\right]^2 + \left[2\zeta\left(\frac{T_0}{T}\right)\right]^2}} \tag{9}$$

The magnification functions (8) and (9) for $\zeta = ct.$ have the typical aspect depicted in fig. 1.

The magnification function can now be used to illustrate the structural behavior before and after period jump, that is to illustrate the behavior differences between structure with fixed base and the same structure but with isolator layer. If the natural period of the structure is too close to the dominant period of the seismic input and resonant danger exist, the structural natural period must change until the dynamic magnification amount became tolerable.

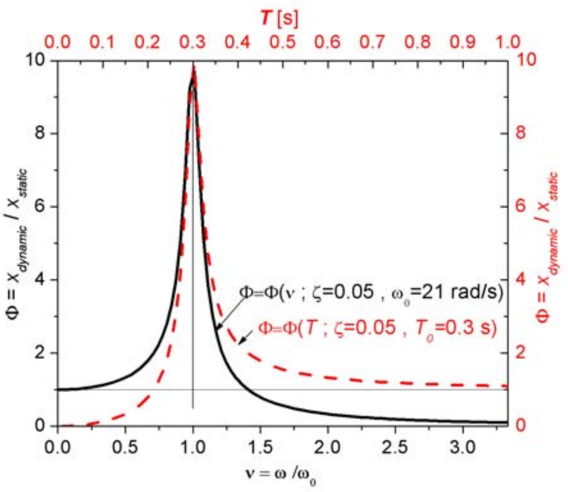


Fig. 1 Magnification function in terms of frequency or period

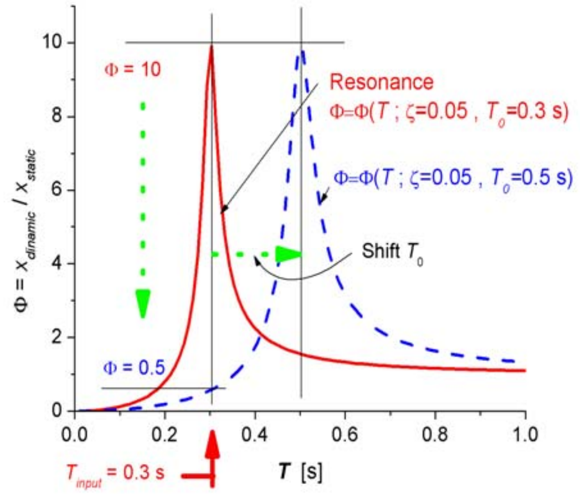


Fig. 2 Period-shift example

To illustrate this magnification functions ability a case study is presented in fig.2. Let a structure with following mechanical characteristics: $m = 30000$ kg ; $T_0 = 0.3$ s ; $\zeta = 5\%$ [18]. If such structure is located on a usual site, composed, for example, by consolidated aluvionary deposits, this structure become a proper candidate for isolated base technology. In this case, an period-shift from resonant period $T_0 = 0.3$ s to the $T_0 = 0.5$ s takes out the structure from dangerous resonant zone and leads to a great reduction of the dynamic magnification amount.

### 3.2 Effects of the isolator layer nonlinearity on period-shift

In chapter section 3.1, a first approximation of the period-shift amount was based on linear magnification functions, therefore on linear behavior hypothesis for the base-isolated structure. However, an isolated-base structure has nonlinear behavior due to the nonlinearity of the materials and devices from his isolator layer. See, for example, in fig. 3 a resonant column test performed upon rubber sample that exhibits an evident softening nonlinearity [4] and in fig.4 the hardening behavior of a rubber-pendular isolator [16].

For make apparent the linear-nonlinear differences another numerical nonlinear simulation study was performed using the same structure from chapter section 3.1. The nonlinear behavior was modeled by nonlinear Kelvin-Voigt model based on replacement of the dynamic linear characteristics – the spring stiffness $k$ and the damper viscosity $c$, by functions in terms of displacements: $k = k(x)$ and $c = c(x)$, experimentally obtained [3].

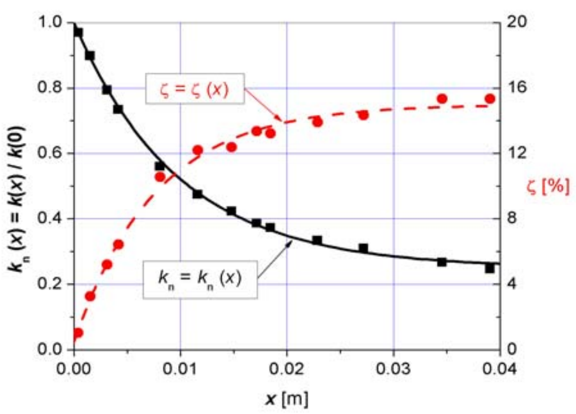


Fig. 3 Softening behavior of a rubber sample

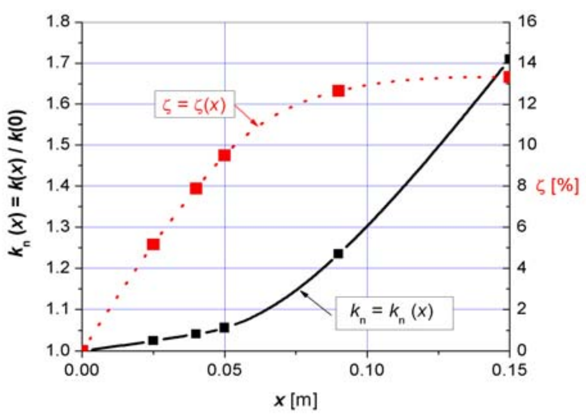


Fig. 4 Hardening behavior of an isolator

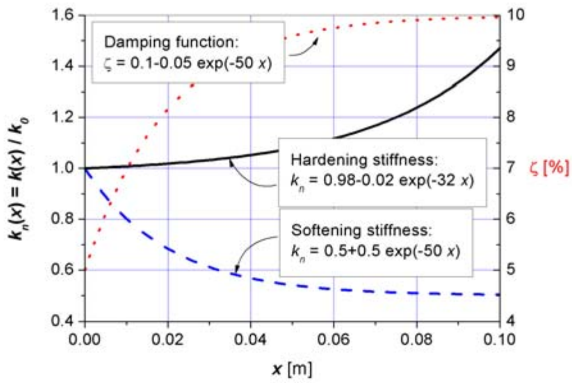


Fig. 5 Material functions used in simulation

For this non-linear simulation the non-linear aspect of the material functions $k = k(x)$ and $c = c(x)$ was build by extension starting to own test performed upon rubber [4] and experimental data performed upon different isolators given in [1], [7], [12], [16], [19] (fig. 5).

In order to compare the nonlinear results with the linear calculus from chapter section 3.1 the initial values of the nonlinear damping function were scaled for coincide with the constant values of the linear behavior hypothesis $\varsigma_0 = \varsigma(0) = 0.05$ and the initial value of stiffness function $k_0 = k(0)$ was put in correspondence with the natural period of the isolated-base structure: $T_0 = 0.5 \text{ s} \Rightarrow \omega_0 = \dfrac{2\pi}{T_0} \simeq 12.56 \text{ rad/s} \Rightarrow k_0 = m\omega_0^2 = 4733 \text{ kN/m}$.

Using nonlinear dynamic material functions $k = k(x)$ and $\varsigma = \varsigma(x)$, as like in fig.5, the differential equation of the nonlinear sdof system can be write as an extension of eq. (5):

$$\varphi'' + C(\varphi)\cdot\varphi' + K(\varphi)\cdot\varphi = \mu \sin \upsilon\tau \tag{10}$$

where $\mu$ and $\upsilon$ has the same expression as in eq.(6) and:

$$C(\varphi) \equiv C(x) = \frac{c(x)}{m\omega_0} = 2\zeta(x) \quad ; \quad K(\varphi) \equiv K(x) = \frac{k(x)}{m\omega_0^2} = \frac{k(x)}{k(0)} = k_n(x) \tag{11}$$

The solution of the equation (10) can be writing like eq. (7):

$$\varphi(\tau,\upsilon,\mu,\zeta) = \mu\Phi(\upsilon,\mu,\zeta)\sin(\upsilon\tau - \psi) \tag{12}$$

where $\Phi(\upsilon,\mu,\zeta)$ is *the non-linear magnification function*, a loads dependent material function through transformed amplitude $\mu$.

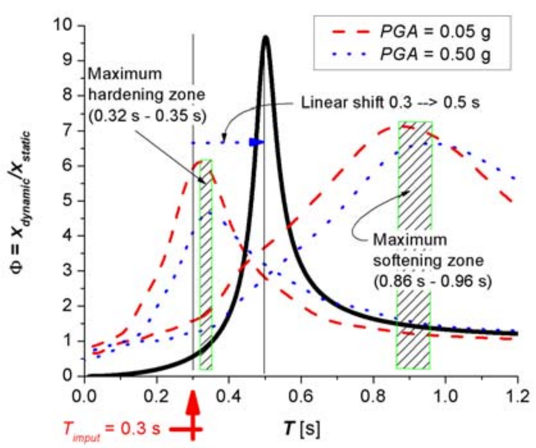


Fig. 6 Nonlinear period-shift dispersion

For given amplitude $\mu$ and relative pulsation $\upsilon$, the nonlinear equation (12) can be numerically solved using a computer program based on Newmark algorithm [14]. After this, from known solution and known excitation input, the non-linear magnification functions can be first obtained in term of normalized pulsation $\upsilon$ and then in term of period $T$.

The simulation results are summarized in fig.6. In the foregoing chapter, assuming a linear material behavior a period jump from 0.3 s to 0.5 s seems to be enough for take out the structure from dangerous resonance zone. But, as can see in figure 6, the nonlinear characteristics of the isolator layer change the linear period-shift estimation and can lead either to dangerous shortening shift in case of hardening nonlinearity or to unnecessary shift enlargement in case of softening nonlinearity.

This simulation prove that by neglecting this period-shift dispersion the main purpose of the base-isolation technology – the drawing out of structure from dangerous resonant zone - can be compromise.

### 3.3 Effects of the soils nonlinearity on period-shift

The strong dependence of the soils dynamic properties on strain or stress level produced by external loads is very well known. In the previous author's papers [2], [3], [5] this nonlinear behavior was modeled assuming that the geological materials are nonlinear viscoelastic materials. The dynamic model obtained was built upon two dynamic nonlinear functions – one for material strength modeling and another including material damping , both in terms of strain level generated by external loading conditions and both functions being completely determined from resonant column test data.

This nonlinear behavior is meeting, more or less, at all site materials – more pronounced at soft degradated materials (soils) and more reduced at rocks materials. However, is experimentally observed that all the site materials have the softening nonlinearity type. For example, in the next, some nonlinear dynamic material functions for two extremely different site materials, one for a soft material (clay) in fig. 7 and another for rock material (limestone) in fig.8, is presented.

Using these material functions, by nonlinear simulations the nonlinear magnification functions are obtained as can see in figs. 9 for clay and in fig.10 for limestone. From these figures results that the softening nonlinearity type of all site materials leads to the enlargement tendency of the period-shift, more pronounced at soils and more reduced at rocks.

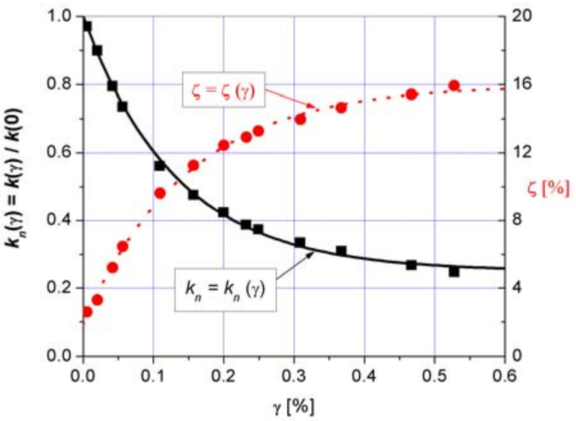


Fig. 7 Material functions for a clay sample

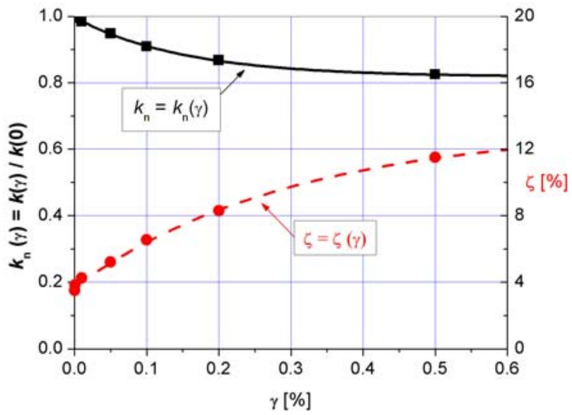


Fig. 8 Material functions for a limestone sample

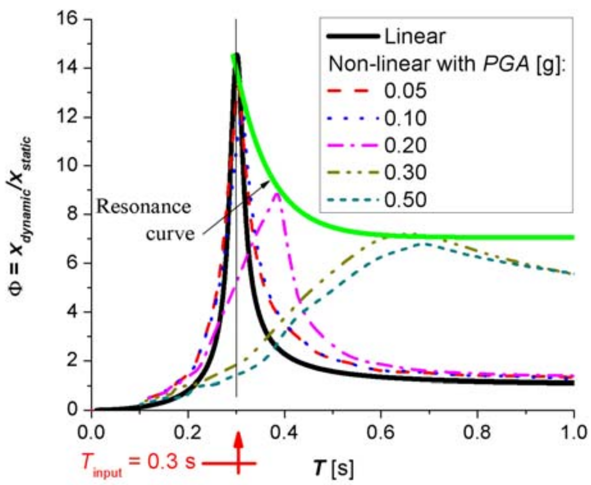


Fig. 9 Typical nonlinear magnification functions for clay

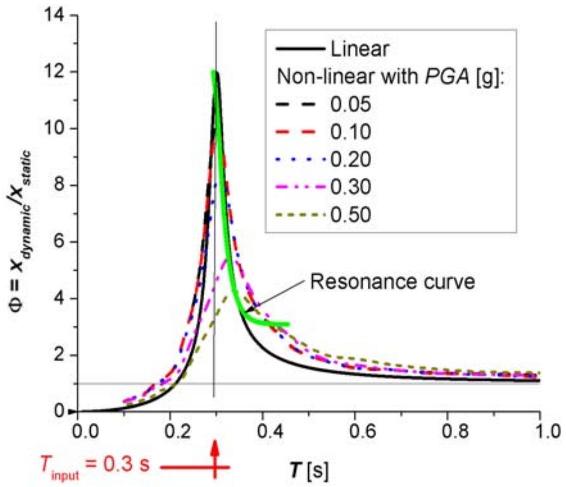


Fig. 10 Typical nonlinear magnification functions for limestone

## CONCLUDING REMARKS

Based on the aforementioned simulations results the following remarks can be drawn:

- From the beginning, we must keep in mind that ***not any structure can be correct located on any emplacement*** and certain emplacements get along only with certain buildings, depending on their natural periods.
- The linear and nonlinear magnification functions prove to be proper tools for necessary period-shift assessment.
- The period-shift from fixed-base to isolated-base of the same superstructure depends on nonlinear characteristics of the isolator and site layers. Thus, a first linear shift estimation must to be corrected in terms of nonlinear characteristics of isolator layer and site materials.
- The nonlinear magnification functions have different shapes in comparison with the linear one. The resonant amplitude peaks are displaced towards high periods for softening stiffness and towards low periods for hardening stiffness and thus can leads to dangerous shortening or unnecessary lengthening of the linear shift evaluation.
- Neglecting these nonlinear period-shift variations by using only linear assessment of necessary period-shift, the main purpose of the base-isolation technology – the drawing out of structure from dangerous resonant zone - can be compromise.

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