

THE COMPUTING OF THE DAMPING GENERALIZED COEFFICIENTS

Camelia CIOBANU, Arlette-Rabela ANGHEL

“Mircea cel Batran” Naval Academy Constanta

Corresponding author: Camelia CIOBANU, E-mail: cciobanu@anmb.ro

The ship hull forms have been described by the well-known classic Lewis transformation, Lewis in 1929, and by an Extended-Lewis transformation with three parameters, as given by Athanassoulis and Loukakis [4] in 1985, with practical applicability for any types of ships. We present here an algorithmic method solving directly the problems that appear in naval architecture domain concerning the contour of ship's cross-section. The application was made in Java language and creates both a text file and a graphical chart. With this algorithmic method in section 4 we propose a method for determining the adherent mass coefficients and in section 5 we compute the damping generalized coefficients.

Key words: hydrodynamic ship cross section, Lewis transformation, area of the cross section

1. INTRODUCTION

During the last 20 years, in the area of naval hydrodynamics as well as in other domains of mechanical engineering, a growing interest has occurred towards the algorithmic methods of solving some definite problems.

The shapes of the ships have been described around a point, as it is difficult to describe them according to analytical mappings.

It is necessary to approximate the ship's shape by continuous functions, in order to get some practical results. A method which has imposed itself during the last few years, is that of multi-parameter conformal mapping, with good results also in the case of extreme bulbous forms.

The advantage of conformal mapping is that the velocity potential of the fluid around an arbitrary shape of a cross section in a complex plane can be derived from the more convenient circular section in another complex plane. In this manner, hydrodynamic problems can be solved directly by using the coefficients of the mapping function.

The general transformation formula is given by:

$$f(Z) = \mu_s \sum_{k=0}^n a_{2k-1} Z^{-2k+1}$$

with $f(Z) = z$, $z = x + iy$ is the plane of the ship's cross section.

$Z = ie^\alpha e^{-i\varphi}$ is the plane of the unit circle, μ_s is the scale factor, $a_{-1} = 1$, a_{2k-1} are the conformal mapping coefficients ($k = 1, \dots, n$), n is the number of parameters.

Therefore we can write in turn:

$$x + iy = \mu_s \sum_{k=0}^n a_{2k-1} (ie^\alpha e^{-i\varphi})^{-(2k-1)},$$

$$x + iy = \mu_s \sum_{k=0}^n (-1)^k a_{2k-1} e^{-(2k-1)\alpha} [i \cos(2k-1)\varphi - \sin(2k-1)\varphi]$$

From the relation between the coordinates in the z - plane (the ship's cross section) and the variables in the Z - plane (the circular cross section), it follows:

$$x = -\mu_s \sum_{k=0}^n (-1)^k a_{2k-1} e^{-(2k-1)\alpha} \sin(2k-1)\varphi, \quad y = -\mu_s \sum_{k=0}^n (-1)^k a_{2k-1} e^{-(2k-1)\alpha} \cos(2k-1)\varphi.$$

Now by using conformal mapping approximations, the contour of the ship's cross section follows from putting $\alpha = 0$ in previous sections. We get:

$$x_0 = -\mu_s \sum_{k=0}^n (-1)^k a_{2k-1} \sin(2k-1)\varphi, \quad y_0 = \mu_s \sum_{k=0}^n (-1)^k a_{2k-1} \cos(2k-1)\varphi.$$

The breadth on the waterline of the approximate ship's cross section is defined by

$$B_0 = 2\mu_s \beta, \quad \text{with } \beta = \sum_{k=0}^n a_{2k-1},$$

and the draft is defined by

$$D_0 = 2\mu_s \delta, \quad \text{with } \delta = \sum_{k=0}^n (-1)^k a_{2k-1}.$$

The breadth on the waterline is obtained for $\varphi = \pi/2$, that means:

$$x_{\pi/2} = -\mu_s \sum_{k=0}^n (-1)^k a_{2k-1} \sin(2k-1)\pi/2, \quad \text{hence } x_{\pi/2} = \mu_s \sum_{k=0}^n a_{2k-1} \quad \text{and } B_0 = 2x_{\pi/2}.$$

The scale factor is $\mu_s = \frac{B_0}{2\beta}$ and the draft is obtained for $\varphi = 0$:

$$y_0 = \mu_s \sum_{k=0}^n (-1)^k a_{2k-1} \cos(2k-1) \cdot 0, \quad \text{hence } y_0 = \mu_s \sum_{k=0}^n (-1)^k a_{2k-1} \quad \text{and } D_0 = y_0$$

with $\mu_s = \frac{D_0}{\delta}$.

2. LEWIS CONFORMAL MAPPING

A simple transformation of the cross sectional hull form will be obtained by choosing $n = 2$ in the Lewis transformation. An extended description of the representation of ship hull forms by Lewis two-parameter conformal mapping is given by Kerczek and Tuck [5] in 1969.

The two-parameter Lewis transformation of a cross section is defined by:

$$z = f(Z) = \mu_s a_{-1} Z + \mu_s a_1 Z^{-1} + \mu_s a_3 Z^{-3},$$

where $a_{-1} = 1$, μ_s is the scale factor and the conformal mapping coefficients a_1 and a_3 are called Lewis coefficients.

Then, for $z = x + iy$ and $Z = ie^\alpha e^{-i\varphi}$, that is $Z = ie^\alpha [\cos(-\varphi) + i \sin(-\varphi)]$, we have:

$$x = \mu_s (e^\alpha \sin \varphi + a_1 e^{-\alpha} \sin \varphi - a_3 e^{-3\alpha} \sin 3\varphi) \quad \text{and} \quad y = \mu_s (e^\alpha \cos \varphi - a_1 e^{-\alpha} \cos \varphi + a_3 e^{-3\alpha} \cos 3\varphi)$$

For $\alpha = 0$ we obtain the contour of the so- called Lewis form expressed as:

$$x_0 = \mu_s (\sin \varphi + a_1 \sin \varphi - a_3 \sin 3\varphi) \quad ; \quad y_0 = \mu_s (\cos \varphi - a_1 \cos \varphi + a_3 \cos 3\varphi),$$

where the scale factor μ_s is:

$$\mu_s = \frac{B_s}{2(1+a_1+a_3)} \quad \text{or} \quad \mu_s = \frac{D_s}{(1-a_1+a_3)}$$

in which B_s is the sectional breadth on the load waterline and D_s is the sectional draft.

The half breadth to draft ratio H_0 is given by:

$$H_0 = \frac{B_s}{2D_s} = \frac{1+a_1+a_3}{1-a_1+a_3}.$$

An integration of the Lewis form delivers the sectional area coefficient σ_s :

$$\sigma_s = \frac{A_s}{B_s D_s} = \frac{\pi}{4} \cdot \frac{1-a_1^2-3a_3^2}{(1+a_3)^2-a_1^2}$$

in which A_s is the area of the cross section, $A_s = \frac{\pi}{2} \mu_s^2 (1-a_1^2-3a_3^2)$ and $B_s \cdot D_s = 2[(1+a_3)^2-a_1^2]$.

Now the coefficients a_1 , a_3 and the scale factor μ_s will be determined in such a manner that the sectional breadth, the draft and the area of the approximate cross section and of the actual cross section are identical.

From H_0 expression, we obtain $a_1 = \frac{(H_0-1)(1+a_3)}{1+H_0}$ and, by putting that into the expression of σ_s , we

get a quadratic equation in a_3 : $\alpha a_3^2 + \beta a_3 + \gamma = 0$,

$$\text{where} \quad \alpha = 3 + 4 \cdot \frac{\sigma_s}{\pi} + \left(1 - 4 \frac{\sigma_s}{\pi}\right) \left(\frac{H_0-1}{H_0+1}\right)^2,$$

$$\beta = 8 \cdot \frac{\sigma_s}{\pi} + \left(2 - 8 \frac{\sigma_s}{\pi}\right) \left(\frac{H_0-1}{H_0+1}\right)^2 = 2\alpha - 6,$$

$$\gamma = 4 \cdot \frac{\sigma_s}{\pi} + \left(1 - 4 \frac{\sigma_s}{\pi}\right) \left(\frac{H_0-1}{H_0+1}\right)^2 - 1 = \alpha - 4.$$

For this equation we obtain: $a_3 = \frac{-\alpha + 3 + \sqrt{9 - 2\alpha}}{\alpha}$.

The Lewis form with the other solution of a_3 in the quadratic equation, namely:

$$a_3 = \frac{-\alpha + 3 - \sqrt{9 - 2\alpha}}{\alpha}$$

are looped. As ships are „better behaving”, this solution is not considered.

It is obvious that a transformation of a half immersed circle with radius r will result for $\mu_s = r$, $a_1 = 0$ and $a_3 = 0$.

Two typical and realistic forms are presented below. More precisely we have considered the “Mircea” school ship and a dry bulk carrier.

3. JAVA APPLICATION

The following application was made in Java language and creates both a text file and a graphical chart.

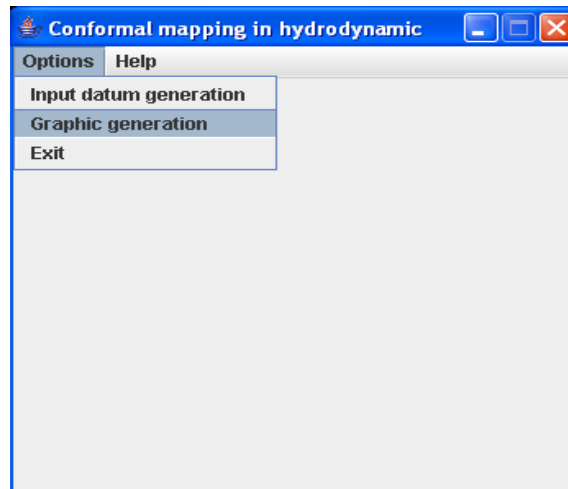


Fig. 1. The interface of Java application

The following code is associated with “Input datum generation” option of the menu and generates a file named “input.txt” that contains the coordinates of points that will be used for plotting the graph.

```

.....
if (command.equals("Input datum generation"))
{
    for(int i=0;i<x.length;i++)
    {bs[i]=2*x[i];
    ds[i]=y[i];
    if(bs[i]==0 || ds[i]==0) ss[i]=0;
      else ss[i]=as[i]/(bs[i]*ds[i]);
    if(ds[i]==0) h0[i]=0;
      else h0[i]=bs[i]/(2*ds[i]);

    c1[i]=3+4*ss[i]/Math.PI+(1-4*ss[i]/Math.PI)*((h0[i]-1)/(h0[i]+1))*((h0[i]-1)/(h0[i]+1));
    a3[i]=(-c1[i]+3+Math.sqrt(9-2*c1[i]))/c1[i];
    a1[i]=(h0[i]-1)*(1+a3[i])/(1+h0[i]);
    if((1+a1[i]+a3[i])==0) ms[i]=0;
      else ms[i]=bs[i]/(2*(1+a1[i]+a3[i]));

    for(int j=0;j<=18;j++)
    {
        x0[i][j]=ms[i]*((1+a1[i])*Math.sin(Math.PI*j/36)-a3[i]*Math.sin(3*Math.PI*j/36));
        y0[i][j]=ms[i]*((1-a1[i])*Math.cos(Math.PI*j/36)+a3[i]*Math.cos(3*Math.PI*j/36));
    }
}
}
.....

```

The pairs of coordinates for those two types of ships are obtained from the following initial values of the parameters: the half breadth (x), the draft (y), and the area of the cross section (as), for a dry bulk carrier. The half breadth (x), and the draft (y) have been measured from each cross section, and the area of each cross section (as) has been calculated with Trapezium Method.

a) the parameters for the ship’s cross- section of the “Mircea” school ship

as = {0, 35.092, 147.86, 243.61, 318.06, 368.77, 386.73, 395.16, 397.3, 397.3, 397.3, 397.3, 397.3, 396.74, 391.72, 368.86, 297, 184.04, 26.66};

0.0 - 0.0

0.0 - 0.0

0.0 - 0.0

.....
0.0 - 12.4

2.120788786475164 - 12.413306945880853

4.1941237694314575 - 12.44900403280683

6.175046248477997 - 12.494733093904317

8.023423315002322 - 12.530919273607507

9.7059608736487 - 12.532230280805823

.....
15.933578383823935 - 9.197767307164254

16.157461428952423 - 8.0154878266859

16.28968975675114 - 6.6553427416996485

16.358436399060594 - 5.139770068067918

16.388235044855662 - 3.4995310587637385

.....
15.093333294483939 - 10.985946576384604

.....
8.741474048339656 - 12.161310740744948

.....
15.823799334390396 - 4.586431083130775

15.926266444957927 - 3.1136258591201558

15.982268073343706 - 1.5739711583871545

.....
6.914431432713245 - 11.782315093924838

8.060481841842734 - 11.466454537623578

9.094315966063338 - 11.063327338656455

10.009799095881057 - 10.562835115686907

.....
13.49320077052138 - 2.7193182668963467

13.573683469567966 - 1.3714611686179892

.....
6.087270999830458 - 11.206858034610427

6.611085318313334 - 10.731407577247174

7.033548947818976 - 10.144818785039785

7.36081202752367 - 9.439350800934108

.....
7.994093096226718 - 2.80901380021834

7.999023428114188 - 1.417527341051886

.....
0.0 - 0.0

1.0

The graphical representation of the points shows the contour of the ship's cross- section of the "Mircea" school ship.

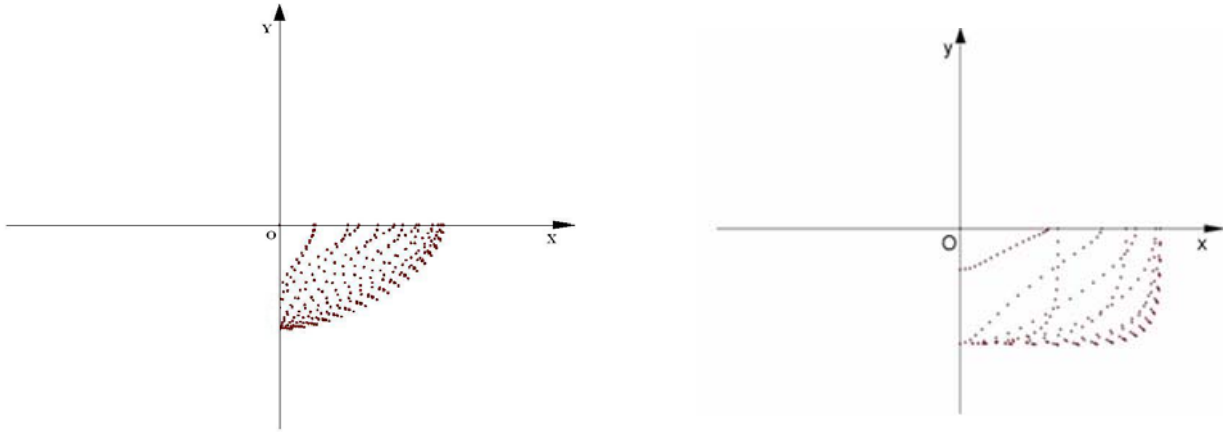


Fig.2. The graphical representation
 a) the contour of the ship's cross- section of the "Mircea" school ship
 b) the contour of the ship's cross section of the dry bulk carrier

4. THE COMPUTING OF THE ADHERENT MASS COEFFICIENTS

Take the case of a ship's hull on progressive regular waves; there are many forces acting, but we are interested in the inertial force, which is far larger than mass multiplied by acceleration. Because this increased inertial force can be defined as the ship's acceleration multiplied by a quantity having the same dimensions like the mass, it was called adherent mass.

In 1929, Lewis proposed a method of determining the adherent mass on the mid-ship, which is highly used in ship's oscillations, a method assumed afterwards by others like Grim, Motora and Tasai. Taking into account the Lewis transforming coefficients, the λ_{ij} , in order to determine the equations of the ship's motion on progressive regular waves, we need μ_{22} , μ_{24} , μ_{33} , μ_{44} , which are from [1] and [2]:

$$\begin{aligned}\mu_{22} &= \frac{\pi\rho}{2} \mu_s^2 [(1 - a_1)^2 + 3a_3^2] \\ \mu_{24} &= 4\rho\mu_s^3 \left\{ \frac{1}{3} a_1(1 - a_1)(1 + a_3) + a_3 \left[\frac{3}{5}(1 + a_3) + \frac{4}{15} a_3(1 - a_1) - \frac{6}{7} a_3 \right] \right\} \\ \mu_{33} &= \frac{\pi\rho}{2} \mu_s^2 [(1 + a_1)^2 + 3a_3^2] \\ \mu_{44} &= \pi\rho\mu_s^4 [a_1^2(1 + a_3)^2 + 2a_3^2]\end{aligned}$$

5. THE COMPUTING OF THE DAMPING GENERALIZED COEFFICIENTS.

Damping – the fact that water resists on ships oscillations can be explained by that an oscillating ship uses energy for annihilating the friction forces and the formation of vortexes and waves. The generalized damping coefficients for the mid-ship sections based on linear theory of the generated wave by a Lewis contour in oscillation, which floats in a ideal liquid can be computed [1] with formulas

$$\lambda_{22} = \rho\omega I_{22} D_S^2$$

$$\lambda_{24} = \rho\omega I_{24} D_S^3$$

$$\lambda_{33} = \rho\omega I_{33} B_S^2$$

$$\lambda_{44} = \rho\omega I_{44} D_S^4$$

I.N. Paschenko and I.M. Kogan gave numerical tabular values using [2] monogram for I_{ij} coefficients ω is the oscillations angular frequency.

Assuming that on the progressive regular waves' action the ship motion is similar with that of a rigid solid with six degrees of freedom and oscillate harmonically with small amplitude. These oscillation motions can be expressed using a six linear equations system:

$$\sum_{k=1}^6 [(M_{kj} + A_{kj})\ddot{u}_k + B_{kj}\dot{u}_k + C_k u_k] = F_j \exp i\omega_k t \quad ; \quad j = \overline{1,6}$$

with:

- M_{kj} - generalized element matrices of the ships width;
- A_{kj} - sum of the generalized adherent masses – see table 1;
- B_{kj} - damping generalized coefficients;
- C_{kj} - redress forces coefficients,
- F_j - disturbed forces and moments;
- ω_k - meeting waves frequency;
- k, j - directions motion index of the ship appearing in disturbing forces;
- u_1 - moving of the gravity centre along longitudinal-horizontal direction reported to the bench being translation motion[1];
- u_2 - moving of the gravity centre along transversal-horizontal direction reported to the bench being translation motion[1];
- u_3 - moving of the gravity centre along vertical direction reported to the bench being translation motion[1];
- $u_4 = \varphi$ - roll angle;
- $u_5 = \theta$ - angle pitch angle;
- $u_6 = \psi$ - yaw angle .

The masses generalized matrix is given by:

$$M_{kj} = \begin{bmatrix} M & 0 & 0 & 0 & MZ_p & 0 \\ 0 & M & 0 & -MZ_p & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -MZ_p & 0 & I_4 & 0 & -I_{46} \\ MZ_p & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_6 \end{bmatrix}$$

with:

- M - ships mass;
- I_j - inertial moment of masses reported to j axe;
- I_{46} - centrifugal moment which can be zero if the mass is symmetrical reported to the mid-ship.

The other elements not situated on the diagonal become null if the origin of the coordinates coincides with the ship's masses centre.

The masses generalized matrix or the damping coefficients are determined by:

$$A_{jk} \text{ (or } B_{jk}, C_{jk} \text{)} = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{13} & 0 \\ 0 & A_{22} & 0 & A_{14} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix}$$

Computing the problem of hydrodynamics coefficients is highly difficult, from the mathematical point of view. If integrals are substituted with sums on each mid-ship section, we obtain formulas for determining the hydrodynamics coefficients, from equations like those in table 1.

In order to determine the A_{kj} , B_{kj} coefficients, we need to sum up 20 mid-ship sections, equally positioned, with a Δx distance between them.

Table 1. Equations hydrodynamics coefficient

kj	A_{kj}	B_{kj}	C_{kj}
33	$\sum \mu_{33} \Delta x$	$\sum \lambda_{33} \Delta x$	$\rho g S_0$
35	$-\sum x \mu_{33} \Delta x - u \omega_k^{-2} B_{33}$	$-\sum x \lambda_{33} \Delta x + u A_{33}$	$-\rho g x_f S_0$
53	$-\sum x \mu_{33} \Delta x + u \omega_k^{-2} B_{33}$	$-\sum x \lambda_{33} \Delta x - u A_{33}$	$-\rho g x_f S_0$
55	$\sum x^2 \mu_{33} \Delta x + u \omega_k^{-2} A_{33}$	$\sum x^2 \lambda_{33} \Delta x - u \omega_k^{-2} B_{33}$	$\rho g I_y$
22	$\sum \mu_{22} \Delta x$	$\sum \lambda_{22} \Delta x$	0
24	$\sum \mu_{24} \Delta x$	$\sum \lambda_{24} \Delta x$	0
26	$\sum x \mu_{22} \Delta x + u \omega_k^{-2} B_{22}$	$\sum x \lambda_{22} \Delta x - u A_{22}$	0
42	$\sum \mu_{24} \Delta x$	$\sum \lambda_{24} \Delta x$	0
44	$\sum \mu_{44} \Delta x$	$\sum \lambda_{44} \Delta x$	$g M h$
46	$\sum x \mu_{24} \Delta x + u \omega_k^{-2} B_{24}$	$\sum x \lambda_{24} \Delta x - u A_{24}$	0
62	$\sum x \mu_{22} \Delta x - u \omega_k^{-2} B_{22}$	$\sum x \lambda_{22} \Delta x + u A_{22}$	0
64	$\sum x \mu_{24} \Delta x - u \omega_k^{-2} B_{24}$	$\sum x \lambda_{24} \Delta x + u A_{24}$	0
66	$\sum x^2 \mu_{22} \Delta x + u \omega_k^{-2} A_{22}$	$\sum x^2 \lambda_{22} \Delta x + u \omega_k^{-2} B_{22}$	0

The computing is usually made for the 20 mid-ship and for 15 to 20 values of the oscillations' frequency, which highly influences not only the adherent masses, but also the dumping coefficients.

We assume that the fluid has no viscosity and the dumping is solely determined by the loss of energy, caused by the formation of waves on the surface. This assumption is often used in the case of longitudinal oscillations, because the viscous resistance is smaller in the case of vertical oscillations. However, on one hand, in order to analyse roll, it is necessary to consider the water viscosity, which can lead to a substantial increase of the oscillations' dumping. On the other hand, in the case of transversal-horizontal and yaw oscillations, it is compulsory to take into account the forces and the momenta of viscous nature on the horizontal plan.

REFERENCES

1. AMES, W.F., *Nonlinear Partial Differential Equations in Engineering*, Academic Press, New York, London, 1965.
2. ANGHEL, A., *Contribution Concerning the Limit Thin Theory the Convex Symmetrical areas*, Proceedings of The Scientific Symposium, TEHNONAV, Constanta, 1991.
3. ARSENIU, D.I., FLOREA, M., *Hidraulica*, Ovidius University Press, Constanta, 1998.
4. ATHANASSOULIS, G.A., LOUKAKIS, T.A., *An Extended-Lewis Form Family of Ship Sections and its Applications to Seakeeping Calculations*, International Shipbuilding Progress, **32**, 366, 1985.
5. IKEDA, Y., HIMENO, Y., TANAKA, N., *A Prediction Method for Ship Rolling. Technical Report 00405*, Department of Naval Architecture, University of Osaka Prefecture, Japan, 1978.
6. FANG, C.M., KIM, H.C., *Hydrodynamically coupled motions of two ships advancing in oblique waves*, Journal of Ship Research, September, 1996.
7. JIANBO Hua, *Wave load mechanism to avoid low damage*, the Naval Architect, Londra, January, 1996.
8. JOURNEE, J.M.J., *Verification and Validation of Ship Motions Program SEAWAY, Technical Report 1213a, 2001*, Delft University of Technology, Ship Hydromechanics Laboratory, The Netherlands, Internet: www.shipmotions.nl.
9. JOURNEE, J.M.J., MASSIE, W.W., *Offshore Hydromechanics*, Lecture Notes, First Edition, January 2001, Delft University of Technology, Ship Hydromechanics Laboratory, The Netherlands, Internet: www.shipmotions.nl.
10. KERCZECK, C.V., TUCK, E.O., *The Representation of Ship Hulls by Conformal Mapping Functions*. Journal of Ship Research, **13**, 4, 1969.
11. SIMONSEN, Claus, D., RUDER, *Propeller and Hull, Interaction by RANS*, Ph. D. tesis, Technical University of Denmark, 2000
12. TASAI, F., *Improvements in the Theory of Ship Motions in Longitudinal Waves*. Proceedings 12th I.T.T.C, 1969.
13. URSELL, F., *On the Heaving Motion of a Circular Cylinder on the Surface of a Fluid*. Quarterly Journal of Mechanics and Applied Mathematics, **II**, 1949.
14. VANTORRE, M., JOURNEE, J.M.J., *Validation of the Strip Theory Code SEAWAY by Model Tests in Very Shallow Water*, Colloquium on Numerical Modelling, 23-24 October 2003, Antwerp, Belgium, Internet: www.shipmotions.nl

Received May 25, 2008