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SUSTAINABLE PRODUCTION PLANNING MODELS

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Several sustainable production planning models are formulated and studied. One of them is a discrete multiobjective programming model that takes into account conflicting goals as return and financial risk and environmental costs. Starting from it two single objective models are formulated: a maximum expected return model and a minimum financial risk model. In order to control the pollution several pollution and monetary penalties for overcoming them levels are considered. Based on the production planning models, a software application was designed and realized as a decision support tool. A numerical example is given.

Key words: software application, solver, environmental constraints, multiobjective programming models, production planning model, control, risk, pollution.

1. INTRODUCTION

Pollution prevention is one of the most serious challenges that are currently facing the industry. At present the majority of the industrial enterprises make production plans that pollute the environment. After that they apply different methods for cleaning up the environment. The pollution prevention seems to be the most appropriate policy the industrial enterprises must adopt. The clean up or control and the approach of source reduction may be a start to the prevention process. With increasingly stringent environmental regulations, there is a growing need for efficient production planning models that take into account the trade-off between return and environmental costs and therefore reduce the penalties paid for overcoming the pollution levels.

The importance of the problems connected with environment protection and pollution prevention represent a stimulus for the research in mathematical modeling of production processes and manufacturing systems. The idea of considering environmental constraints for the production of industrial plants is not new. In [9], [10] and [8], DISPATCHER, a decision support system (DSS), is described. The system was intended for the operative scheduling in industrial plants. The system takes into account environmental constraints. In [8] and [11] the environment is modeled as a finite dimensional reservoir in which the pollutant emissions accumulate. There are many papers which studied mathematical models connected with production systems and sustainable technologies for production [2]-[7], [12]-[20], [22]-[26]. Interesting references, environmental standards and several mathematical models can be found in [14]. A rich list of references concerning papers on sustainable production technologies is given in [21]. At present „ green manufacturing" is an objective which must be adopted by all industries with a view to reducing the environmental impact of product and production processes [1].

The enterprise managers become more and more aware of the potential benefits of the integrated production-planning decision support systems (DSS). Our research in the area of pollution prevention has been focusing on the formulation and study of some sustainable production planning models and the design of a software application that is a decision support tool for an efficient production planning. The remaining part of the paper is organized as it follows. First the multi-objective model of the sustainable production enterprise is proposed. Two single objective models such as minimum financial risk and maximum expected return are derived next. Then a software application is described. A practical example from the textile industry is given before presenting paper conclusions.

2. FORMULATION OF THE MULTIOBJECTIVE MODEL

We shall formulate several discrete production planning models that take into account several environmental constraints. A general multi-objective programming problem is formulated in which the objective functions are the expected return of the production plan and the penalties for the case when the cumulative effect of each emission overcome some environmental levels and the financial risk of the production plan. The manager tries to find a production plan that maximize the expected return of it, minimize the pollution penalties and satisfies the environmental constraints.

Suppose that an industrial enterprise has the possibility to manufacture products of types $T_1, T_2, \ldots, T_n$. For all *i*=1,2,...,*n*, denote by c_i the selling price of a product of type T_i. Note that all c_i are random variables.

The manufacture of a product generates none, one or several pollution emissions F_1, $F_2, \ldots, F_m$ and requires *p* resources $R_1, R_2, \ldots, R_p$.

Denote by b_{ij} the amount of pollution emission F_j when is manufactured a product of type T_i and by c_{ik} the amount of resource R_k required for manufacturing a product of type T_i. Denote by r_k the maximum availability of resource R_k. Note that b_{ij} and c_{ik} are nonnegative numbers. The enterprise manager wants to invest a sum *M* of money in the range $[M_1, M_2]$ in order to manufacture products of types $T_1, T_2, \ldots, T_n$. He desires to obtain a fabrication plan $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ that gives him a maximum expected return, a minimum risk for the environment pollution and a minimum financial risk.

In the present paper the pollution risk is measured by the penalties paid by the manager for the environment pollution. Denote by d_{j1} the desirable or target pollution level for the pollutant emission F_j. Denote by d_{j2} the alarm level of pollution for the pollutant emission F_j. Denote by d_{j3} the maximum acceptable limit of pollution for the pollutant emission F_j. Of course $0 \le d_{j1} \le d_{j2} \le d_{j3}$ for every *j* = 1, 2,..., *m*. A small overcome of the level d_{j1} represent no danger for the environment. It represents only a warning that the pollution process had already began. A small overcome of the level d_{j2} represent a warning that the pollution process may have consequences for the environment. An overcome of the level d_{j3} represents a warning that the pollution process had already produced bad consequences for the environment and urgent measures must be taken in order to stop the process.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be the fabrication plan of the manager. Here x_i represents the number of products of type T_i, *i* =1,2,...,*n*. Denote by p_i the production cost of a product of type T_i and by q_i a minimum quantity of products of type T_i that should be produced. Of course p_i are positive real numbers and q_i are natural numbers for all *i* =1,2,...,*n*. The production cost for the fabrication plan $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is equal to $\sum_{i=1}^{n} p_i x_i$. We shall call $\mathbf{q} = (q_1, q_2, \ldots, q_n)$ the vector of demand. If a is a real number we shall denote by a_+ the positive part of a, that is:

$$a_+ = \max(a, 0) = \frac{|a| + a}{2}$$

We shall consider that, the environmental penalty paid in the case the fabrication plan $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is applied is proportional to the amount of pollutant that overcomes the pollution level. Consequently in the case of pollutant emission F_j and pollution level d_{js} it is equal to

$$a_{js}\left(\sum_{i=1}^{n} b_{ij}x_i - d_{js}\right)_+$$

We denoted by a_{js} the proportionality factor from the environmental penalty. The overall environmental penalty will be in this case

$$f_1(\mathbf{x}) = \sum_{s=1}^{2}\sum_{j=1}^{m} a_{js}\left(\sum_{i=1}^{n} b_{ij}x_i - d_{js}\right)_+ \tag{1}$$

The idea of considering a desirable pollution level and environmental penalties proportional to the amount of pollutant that overcome the pollution level goes back to [20]. The manager must take into account environmental constraints. In our paper we shall consider constraints that impose some bounds on the expected amount of pollutant emissions:

$$\sum_{i=1}^{n} b_{ij}x_i \le d_{j4} \quad j = 1,2,\ldots,m \tag{2}$$

Here we denoted by d_{j4} a number smaller or equal than d_{j3}. It measures the aversion against a polluted environment. The smaller is d_{j4}, the cleaner will be the environment. We shall denote by E_1 the set of all nonnegative vectors $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ having integer components that satisfy: the inequalities $x_i \ge q_i$ for all *i*, the environmental constraints (2) and the resource constraints

$$\sum_{i=1}^{n} c_{ik}x_i \le r_k \qquad k = 1,2,\ldots, p \tag{3}$$

Denote by σ_{ij} the covariance of the random variables c_i and c_j. Let $\mathbf{C} = (\sigma_{ij})$ be the covariance matrix. We shall define the financial risk of the production plan **x** as the variance of the its return $\sum_{i=1}^{n} c_i x_i$.

One can easily see that

$$\mathrm{Var}\left(\sum_{i=1}^{n} c_i x_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij}x_i x_j = \mathbf{x}^T\mathbf{C}\mathbf{x}$$

In order to use efficiently the sum available, the manager tries to find a fabrication plan $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ such that it will bring a maximum return, it will minimize the overcome of the pollution levels and the financial risk and it will allow him to comply with environmental restrictions.

In order to find such a plan the manager must solve the following multiobjective programming problem:

$$(\mathrm{S})\begin{cases} \max\left[\sum_{i=1}^{n} (E[c_i] - p_i)x_i\right] \\ \min\left(\sum_{i=1}^{n} b_{ij}x_i - d_{js}\right)_+ \quad s = 1,2;\ j = 1,2,\ldots,m; \\ \min\left(\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij}x_i x_j\right) \\ M_1 \le \sum_{i=1}^{n} p_i x_i \le M_2,\ \mathbf{x} \in E_1 \end{cases}$$

There are several approaches for reducing the above problem to single objective programming problems. Two of them are presented in the following.

3. A MINIMUM FINANCIAL RISK MODEL

In the minimum financial risk problems the manager tries to minimize the financial risk taking into account the following restrictions:

- the production plans satisfy the environmental and resource conditions (2) and (3), that is $\mathbf{x} \in E_1$.
- the sum M invested in the fabrication plan is in the range $[M_1, M_2]$.
- the expected return of the production plan is greater than a given value W.

The model is the following:

$$\begin{cases} \min\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j\right) \\ f_1(\mathbf{x}) \le \nu \\ \sum_{i=1}^{n}\left(E[c_i] - p_i\right)x_i \ge W \\ M_1 \le \sum_{i=1}^{n} p_i x_i \le M_2,\ \mathbf{x} \in E_1 \end{cases}$$

Here W is the parameter that controls the expected return of the production plan and ν is the parameter that controls monetarily the penalties paid for pollution.

4. A MAXIMUM EXPECTED RETURN MODEL

In the maximum expected return problem the manager tries to maximize the expected net return taking into account the following restrictions:

- the production plans satisfy the environmental and resource conditions (2) and (3), that is $\mathbf{x} \in E_1$.
- the sum M invested in the fabrication plan is in the range $[M_1, M_2]$.
- the financial risk is smaller than a given value τ

$$(Q)\quad \begin{cases} \max\left[\sum_{i=1}^{n}\left(E[c_i] - p_i\right)x_i - f_1(\mathbf{x})\right] \\ \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \le \tau \\ M_1 \le \sum_{i=1}^{n} p_i x_i \le M_2,\ \mathbf{x} \in E_1 \end{cases}$$

The problem (Q) is a single objective parametric programming problem.

5. A BRIEF DESCRIPTION OF A SOFTWARE APPLICATION

Based on production planning models a software application was developed. It is composed from several modules such as: “user interface”, “model construction”, “model resolution”, and “sensitivity analysis”. The module “user interface” facilitates the construction of the input data collection and the selection of the user parameters. It gives a flexible way of working and is user oriented. The input data collection is validated in the module “model construction” following a set of rules. In order to make the validation of the user parameters the module solves several mathematical programming problems. As a result, ranges of variation of user parameters are determined. For example, for the determination of the range of variation of the user parameter *W*, the module solves two linear programming problems. The module “model resolution” facilitates the transition of the constructed model to the GAMS solver, solves the model and returns the obtained results back. The results are visualized and saved according to the user demand. The module “sensitivity analysis” allows by the variation of the user parameter *W*, the computation of the efficient frontier of the production planning models. The software application was realized in Visual Basic Net and the model resolution was realized with GAMS.

The system architecture is presented in figure 1.

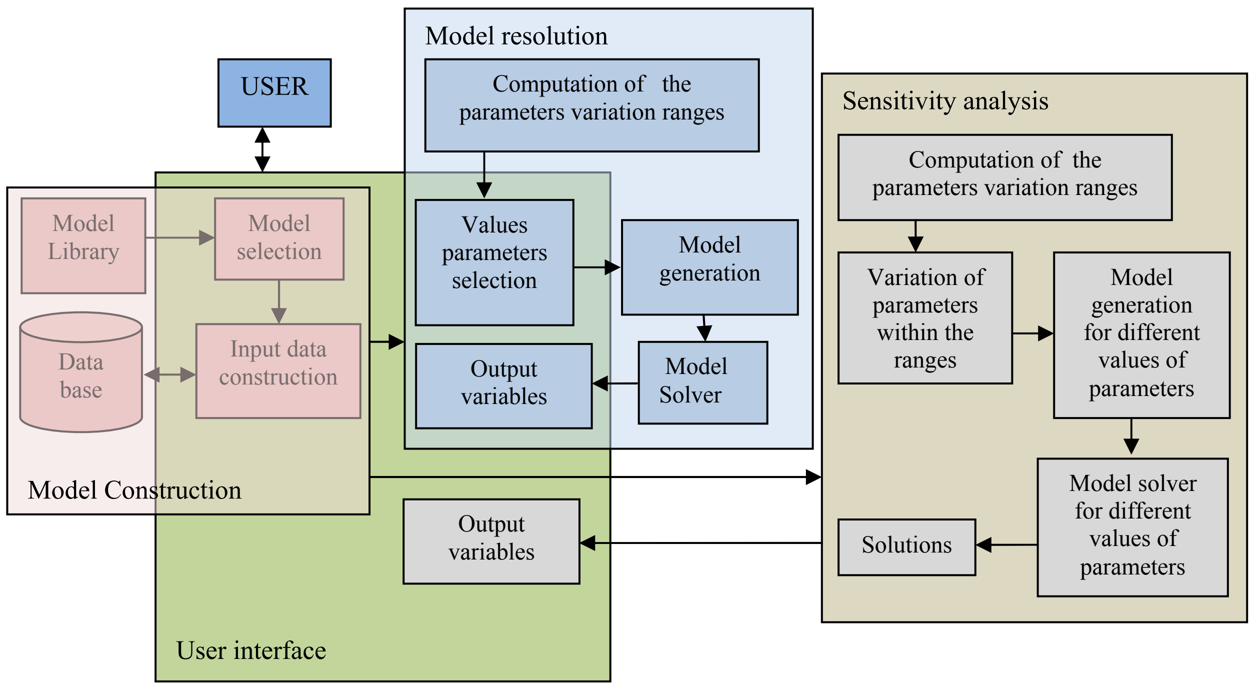

Figure 1. System architecture of the software application based on production planning models

The Data base contains input data of the models. Input data are:

- Product types
- Water pollution indicators
- Matrix of expected emission outputs
- Historical data on market product prices
- Fabrication costs of the products
- The demand for the product types
- Pollution levels (desirable pollution levels and maximum admissible pollution levels)
- Monetary penalizations for overcoming the desirable pollution levels

User parameters:

- Minimum and the maximum limits for the invested sum in the fabrication plan
- Control parameter for all penalties paid for overcoming the pollution levels
- Lower limit for the expected return

Output variables

- Number of products of each type that manufactured
- Minimum value of the risk
- Sum paid as penalties for overcoming the pollution levels
- Return obtained as the result of application of the optimal production plan
- Sum invested for the optimum production plan

6. NUMERICAL RESULTS

The textile industry uses vast amounts of water, energy and chemicals. It is one of the world's worst offenders in terms of pollution. About 2,000 different chemicals are used in the textile industry ranging from dyes to transfer agents. Dyes and auxiliary chemicals used in textile mills have hard environmental influences. Textile processing generates many waste streams, including water-based effluent as well as air emissions, solid wastes, and hazardous wastes. Textile manufacturing is one of the largest producers of wastewater. On average, approximately 160 liters of water are required to produce 1 kg of textile product. It takes about 2 cubic meters of water to produce enough fabric to cover one sofa. Textile is a chemically intensive industry since the waste wastewater from textile processing contains processing bath residues from preparation, dyeing, finishing, slashing and other operations. These residues can cause damage to the environment. In order to maximize pollution prevention it is necessary to apply production plans which maximize the return and minimize the pollution of the environment.

In the following we shall analyze a numerical example for the minimum financial risk problem.

Consider a textile firm. Its manager wants to find a fabrication plan for 12 types of products that will minimize the financial risk. The manager wants to control the amount of penalties paid for the pollution and the return. He wants to find a fabrication plan such that the sum of pollution penalties is lower than ν =500 euros and the expected return is greater than W=300 euros. The manager agrees that the sum invested in the fabrication plan lies between 1800 euros and 3500 euros. We shall consider that the amount of the resources available for the production plan is sufficiently large. For water consider the following pollution indicators:

Table 1. Pollution indicators

Nr. crt.	Pollution indicators	Unit measure	Target level	Alarm level	Maximum admissible level
1	CCO-Cr	mg O2/l	175	350	500
2	Suspended solid	mg/l	74	210	300
3	CBO5	mg O2/l	40	210	300
4	Ammonia nitrogen (NH4)	mg/l	1.2	21	30

Table 2. The matrix (b_{ij}) of expected emission outputs

	CCO-Cr	Suspended solid	CBO5	Ammonia nitrogen (NH4)
T01	1	0.02	0.02	0
T02	0.5	0.01	0	0.01
T03	0.04	0.03	0.02	0.01
T04	0.02	0.06	0.02	0.01
T05	0.01	0.03	0.05	0
T06	0.06	0.1	0.03	0.01
T07	0.1	0.1	0.7	0.01
T08	0.4	0	0.3	0.02
T09	0.3	0.1	0.1	0.01
T10	0.4	0	0.5	0
T11	0.04	0.03	0.02	0.01
T12	0.3	0.1	0.1	0.01

In the second column of table 3. is displayed the vector **p.** Its components represent the production cost for products of types displayed in the first column. In our example the unit measure for the components of vector **p** are euros/meter. The third column contains the vector **q**, the vector of demand. The fourth column of table 3 contains the optimal fabrication plan that is the components of the decision vector **x**.

Table 3. Vector of optimal fabrication plan versus vector of demand

	Vector **p**	Vector **q**	Optimal fabrication plan
T01	3,70	15	79
T02	3,30	25	27
T03	3,90	45	100
T04	1,50	15	15
T05	5,40	20	100
T06	2,80	55	55
T07	2,50	55	55
T08	5,40	37	37
T09	7,50	25	25
T10	2,70	15	15
T11	1,90	12	12
T12	3,50	10	10

7. CONCLUSIONS

Pollution prevention is a very actual problem. The paper presented several discrete production planning models under uncertainty that take into account both the economic and environmental problems. The models can be considered pollution prevention models. For each pollutant we defined three different contamination levels: a) the desirable or the target pollution level, b) the alarm (warning or critical) level and c) the maximum admissible (acceptable) level, and introduce penalties proportional to the amounts of pollutants that exceed these levels. The function which defines the monetary penalties for the pollution risk is not smooth since it contains positive parts of some affine functions. This implies mathematical difficulties, which can be solved by formulating an alternative linear programming model, which makes use of additional variables and has the same solutions as the initial problem.

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