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CONNECTIONS BETWEEN DIFFERENT CLASSES OF SOLUTIONS IN MULTI-OBJECTIVE PROGRAMMING, WITH APPLICATION TO STOCHASTIC PROGRAMMING

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We present some relations between the efficient sets of several problems of deterministic multi-objective programming. These results are then used to analyze concepts of efficient solution to multi-objective stochastic programming. Also, we consider the generation of efficient solutions using weighting factors.

1. INTRODUCTION

A whole series of production processes, economic systems of different types, and technical objectives is described by mathematical models which are multicriteria optimization problems (Steuer [17], Chankong and Haimes [4] and Stancu-Minasian [15]). This situation is quite usual, because it is frequently necessary to take simultaneously into account the influence of a number of contradictory extremal factors on the system.

The most intensive development of the theory and methods to be found in Miettinem [9], Zeleny [23] and Urli and Nadeau [20], consists of linear and non linear multicriteria optimization problems. Some classifications of the methods of this type, oriented to the specific user, are given in Salukavadze and Topchishvili [12]. In the same reference, multicriteria optimization problems with contradictory constraints were explored and very interesting results in the case of domination with respect to a cone were given.

Now, one of the widely developing fields in multicriteria optimization is its qualitative theory; the most important results are given in Salukavadze and Topchishvili [12] and Preda [11]. Well-known algoritms can be modified and new theoretical results derived.

The aim of this paper is to examine properties of different classes of multicriteria optimization problem solutions.

In many multi-criteria decision problems, some parameters take unknown values at the moment of making the decision. This uncertainty can be due to problems of observing the parameters themselves or to the fact that their values depend on such factors as nature, decisions of other agents, etc. If these parameters are random variables, the resulting problem is called a stochastic multi-objective programming problem.

There is much research dealing with such problems, among which we could mention the books by Goicoechea *et al*. [7], Stancu Minasian [15], Slowinski and Teghem [14] and the articles of Teghem *et al*. [19], Stancu Minasian and Tigan [16], Urli and Nadeau [20], Ben Abdelaziz *et al*. [1].

Stochastic programming models can lead to very large scale problems, and methods based on approximation and decomposition become paramount. In this paper, we will provide a road map for these methods, and point to fruitful research directions along the way. Stochastic programming models have been developed for a variety of applications including electrical power generation (Murphy [10]), financial planning (Carino *et al*. [5]), telecommunications network planning (Sen *et al*. [13]), and supply chain management (Fisher *et al*. [6]), to mention a few.

The paper is organized in the following manner. Section 2 describes the mathematical models for a general stochastic programming problem and a stochastic multi-objective programming problem. Section 3 presents some principles from stochastic optimization and relations between the efficient sets of several problems of deterministic multi-objective programming. These results will be used later to analyze concepts of efficient solutions to multi-objective stochastic programming. Section 4 considers the generation of efficient stochastic multi-objective solutions. In Section 5, we present the multi-objective weighting factor auxiliary optimization problems for some non-convex auxiliary function. Moreover, we introduce our class of auxiliary functions and give some standard results. For some proofs see [18].

2. SOME PRINCIPLES FROM STOCHASTIC OPTIMIZATION

Let us consider the stochastic multi-objective programming problem (Caballero *et al.* [3])

$$\min_{x\in D}\left(z_1(x,c),\ldots,z_q(x,c)\right) \tag{2.1}$$

where: (i1) $x\in\mathbf{R}^n$ is a vector of decision variables of the problem and c a random vector whose components are continuous random variables, defined on a subset of C a finite dimensional Euclidean space. Here, the family $\Im$ of events, that is, the subsets of C, and the distribution of probability P defined on $\Im$ are known. Also, P is independent of the decision variables $x_1,\ldots,x_n$; (i2) the functions $z_1(x,c),\ldots,z_q(x,c)$ are defined on $\mathbf{R}^n\times C$; (i3) the set of feasible solutions $D\subset\mathbf{R}^n$ is nonempty, compact and convex.

Let $\bar{z}_k(x)$ denote the expected value of the *k*th objective function, and let $\sigma_k(x)$ be its standard deviation, $k\in\{1,\ldots,q\}$. Let us assume that the standard deviation $\sigma_k(x)$ is finite for every $k\in\{1,\ldots,q\}$ and for every feasible vector *x* of the stochastic multi-objective programming problem.

<u>Principle of the Expected-Value Efficient Solution</u>. A point $x\in D$ is an expected-value efficient solution of the stochastic multi-objective problem if it is Pareto efficient to the problem

$$PE:\quad \min_{x\in D}\left(\bar{z}_1(x),\ldots,\bar{z}_q(x)\right).$$

Let E_{PE} be the set of expected-value efficient solution to the stochastic multi-objective problem.

<u>Principle of the Minimum-Variance Efficient Solution</u>. A point $x\in D$ is a minimum-variance efficient solution to the stochastic multi-objective problem if it is a Pareto efficient solution for the problem

$$P\sigma^2:\quad \min_{x\in D}\left(\sigma_1^2(x),\ldots,\sigma_q^2(x)\right).$$

Let $E_{P\sigma^2}$ be the set of efficient solutions to the problem $(P\sigma^2)$.

Next, we define the principle of expected value standard-deviation efficient solution. In this case, the concept of efficiency arises from the construction of a problem with $2q$ objective involving the expected value and the standard deviation of each stochastic objective.

<u>Principle of the Expected-Value Standard-Deviation Efficient Solution or *E*σ Efficient Solution</u>. A point $x\in D$ is an expected-value standard-deviation efficient solution to the stochastic multi-objective programming problem if it is a Pareto efficient solution to the problem

$$PE\sigma:\quad \min_{x\in D}\left(\bar{z}_1(x),\ldots,\bar{z}_q(x),\sigma_1(x),\ldots,\sigma_q(x)\right).$$

Let $E_{PE\sigma}$ be the set of expected-value standard-deviation efficient solutions to the stochastic multi-objective programming problem.

3. CONNECTIONS BETWEEN DIFFERENT CLASSES OF SOLUTIONS

First, we consider some relations between the efficient sets of some problems of deterministic multi-objective programming with application to stochastic optimization.

Let f and g be vectorial functions defined on the same set $H \subseteq \mathbf{R}^n$, with $f : H \subset \mathbf{R}^n \to \mathbf{R}^q$ and $g : H \subset \mathbf{R}^n \to \mathbf{R}^q$. Let us consider the multi-objective problems

$$\min_{x \in D} \left(f_1,..., f_q, u_1(g_1),..., u_q(g_q) \right) \tag{3.1}$$

$$\min_{x \in D} \left(f_1,..., f_q \right) \tag{3.2}$$

$$\min_{x \in D} \left(u_1(g_1^{s_0}),..., u_q(g_q^{s_0}) \right) \tag{3.3}$$

with $D \subseteq H$, $u : \mathbf{R}_+ \to \mathbf{R}^q$, $u = (u_1,..., u_q)$ and $s_0 > 0$ a real number.

Let E_1, E_2, E_3 be the set of efficient points of problems (3.1), (3.2) and (3.3) respectively. The following theorem relates problems (3.1)-(3.3) to each other. The superscripts *w* and *p* stand for *weak* and *proper efficiency*, respectively.

Theorem 3.1. Assume that $g(x) > 0$ for every $x \in D$ and that $u_k(t_1) \leq (<) u_k(t_2)$ for $t_1, t_2 > 0$ and $k = \overline{1, q}$ implies $u_k(t_1^{s_0}) \leq (<) u_k(t_2^{s_0})$. Then

(i) $E_2 \cap E_3 \subset E_1$,

(ii) $E_2 \cup E_3^w \subset E_1$,

(iii) $E_2^w \cup E_3^w \subset E_1^w$.

Remark 3.1. Clearly, (ii) can be deduced from (iii). It is obvious from (iii) that $E_2^w \cap E_3^w \subset E_1$. If $E_2 \subset E_2^w$ and $E_3 \subset E_3^w$, then $E_1 \cup E_3 \subset E_1^w \cup E_3^w$.

Remark 3.2. $u : \mathbf{R} \to \mathbf{R}$ with $u(t) = \lambda t$ or $u(t) = \lambda t^p$, $p > 0$ and $\lambda > 0$ satisfy the condition from Theorem 3.1.

Let us consider the functions *f* and *g* as above and the problem

$$\min_{x \in D} \left(f_1(x) + \tilde{u}_1(g_1(x)),..., f_q(x) + \tilde{u}_q(g_q(x)) \right), \tag{3.4}$$

where $\tilde{u} = (\tilde{u}_1,..., \tilde{u}_q) : \mathbf{R}_+ \to \mathbf{R}^q$.

Let $E_4(\tilde{u})$ and $E_4^p(\tilde{u})$ denote the efficient solutions set and the properly efficient solutions set to problem (3.4), respectively. We will now present some relations between these sets and the set of efficient solutions and properly efficient solutions to problem (3.3).

Theorem 3.2 For $u = (u_1,..., u_q) : \mathbf{R}_+ \to \mathbf{R}^q$ and $\tilde{u} = (\tilde{u}_1,..., \tilde{u}_q) : \mathbf{R}_+ \to \mathbf{R}^q$ such that $u_k(t_1) \leq (<) u_k(t_2)$ implies $\tilde{u}_k(t_1) \leq (<) \tilde{u}_k(t_2)$, $k = \overline{1, q}$, we have $E_4(\tilde{u}) \subset E_1$.

We note that this theorem also holds for the set of properly efficient solutions. In order to obtain this result it is necessary to define problems $P_{fug}(\lambda, \mu)$ and $P_{\tilde{u}}(\xi)$ that result by applying the weighting method to problems (3.1) and (3.4), respectively, namely,

$$P_{fug}(\lambda, \mu): \quad \min_{x \in D} \sum_{k=1}^{q} \left(\lambda_k f_k(x) + \mu_k u_k(g_k(x)) \right).$$

$$P_{\tilde{u}}(\xi): \quad \min_{x \in D} \sum_{k=1}^{q} \xi_k \left(f_k(x) + \tilde{u}_k(g_k(x)) \right).$$

We can use the results available in the literature about the relationships between the optimal solutions to the weighting problem and the efficient solutions to the multi-objective problem. Some results, see

Chankong and Haimes [4], applied to problem (3.1) and its associated weighted problem $P_{fug}(\lambda,\mu)$ are as follows.

(a) If f and $(u_1(g_1),...,u_q(g_q))$ are convex functions, D is convex, and x^* is a properly efficient solution for the multi-objective problem (3.1), then there exist some weight vectors λ,μ with strictly positive components such that x^* is the optimal solution to the weighted problem $P_{fug}(\lambda,\mu)$.

(b) For each weight vector with strictly positive components, the optimal solution to the weighted problem $P_{fug}(\lambda,\mu)$ is properly efficient to the multi-objective problem (3.1).

Now, using (a) and (b) for problems $P_{fug}(\lambda,\mu)$ and $P_{\tilde{u}}(\xi)$, we get the result below.

Proposition 3.1 If f and $(u_1(g_1),...,u_q(g_q))$ are convex functions, D is a convex set and there exists $\tilde{u} = (\tilde{u}_1,...,\tilde{u}_q) : \mathbf{R} \to \mathbf{R}^q$ with $u_k(t)\cdot\tilde{u}_k(t) > 0$ for every $t \geq 0$ and $k \in \{1,...,q\}$ then $E_4^p(\tilde{u}) \subset E_1^p$.

Proposition 3.2 If f and $(u_1(g_1),...,u_q(g_q))$ are convex functions, then

$$E_1^p \subset \bigcup_{\tilde{u}\in\Omega} E_4^p(\tilde{u}),$$

with $\Omega = \{\tilde{u} = (\tilde{u}_1,...,\tilde{u}_q) : \mathbb{R}_+ \to \mathbb{R}^q | \tilde{u}_k(t)u_k(t) > 0,\ \forall k,t\}$.

By Propositions 3.1 and 3.2, if f and $(u_1(g_1),...,u_q(g_q))$ are convex functions and $\tilde{u} : \mathbb{R}_+ \to \mathbb{R}^q$, $\tilde{u}_k(t)u_k(t) > 0$, $\forall k,t$, then the sets of properly efficient solutions to problem (3.1) and (3.4) have the properties below.

a. Every properly efficient solution to problem (3.4) is properly efficient to problem (3.1).

b. Setting $u : \mathbf{R}_+ \to \mathbf{R}^q \setminus \{0\}$, the set of properly efficient solutions to problem (3.1) is a subset of the union in $\tilde{u}$ of the set of properly efficient solutions to problem (3.4).

By combining these results, we obtain

Corollary 3.1 If f and $(u_1(g_1),...,u_q(g_q))$ are convex functions, then $E_1^p = \bigcup_{\tilde{u}\in\Omega} E_4^p(\tilde{u})$.

4. THE CASE OF MULTI-OBJECTIVE WEIGHTING FACTOR AUXILIARY OPTIMIZATION PROBLEMS

We now consider the generation of efficient stochastic multi-objective solution using the weighting $\overline{q}$ th-power factor approach to some non-convex auxiliary function optimization problem. We will introduce our class of auxiliary functions, give some known standard results, and show three classes of non-convex auxiliary optimization problem, giving a concavity-preserving transformation for the $\overline{q}$ th power of a concave function. For classes 2 and 3 we will give an algorithm for finding ε-optimal solutions to the auxiliary optimization problems.

The most common of these auxiliary forms is

$$\nu(\cdot;\delta,F) = \sum_{k=1}^{q} \delta_k F_k \text{ , (i.e, } \nu(x;\delta,F) = \sum_{k=1}^{q} \delta_k F_k(x)\text{),} \tag{4.1}$$

with $\delta \in \Delta_+ = \left\{\delta \in \mathbb{R}_+^q : \sum_{k=1}^{q} \delta_k = 1\right\}$ or $\delta \in \Delta_{++} = \{\delta \in \Delta_+ : \delta > 0\}$.

Define

$$M(\delta, F) = \arg \min_{x \in D} \nu(x; \delta, F).$$

Then using well known results in multi-objective programming (see, for example Karlin [8]), we get

Proposition 4.1. a) $\bigcup_{\delta \in \Delta_{++}} M(\delta, F) \subseteq E$.

b) If F is convex vector function and D is convex, then (see Karlin [8] and White [21]) $E \subseteq \bigcup_{\delta \in \Delta_{++}} M(\delta, F)$.

There are two central issues arising from this class of auxiliary functions, namely

a. without preconditions (e.g. those of Karlin [8]), this class of auxiliary function may be unable to generate enough points in *E*;

b. if $F(\cdot)$ is not a convex vector function or *D* is not convex, then there may be no current method for finding $M(\delta, F)$.

5. SOME AUXILIARY OPTIMIZATION PROBLEMS

Consider a transformation

$$\theta = (\theta_1, ..., \theta_q) : \mathbf{R}_+ \to \mathbf{R}_+^q.$$

Relative to (3.1), (3.2), and (3.3), we define for problem (3.1) an auxiliary optimization problem $AP_1(\bar{q}, \lambda, \mu, \theta)$ with $\bar{q} \in \mathbf{Z}_+ \setminus \{0\}$, $(\lambda, \mu) \in \Delta_+^1$, where $\Delta_+^1 = \left\{ (\lambda, \mu) \in \mathbf{R}_+^q \times \mathbf{R}_+^q \,/\, \sum_{i=1}^q \lambda_i + \sum_{i=1}^q \mu_i = 1 \right\}$, as follows. Find

$$M_1(\bar{q}, \lambda, \mu, \theta) = \arg \min_{x \in D} \psi_1(x, \bar{q}, \lambda, \mu, \theta),$$

where $\psi_1(x; \bar{q}, \lambda, \mu, \theta) : D \to \mathbf{R}_+$ is given by

$$\psi_1(x; \bar{q}, \lambda, \mu, \theta) = \sum_{k=1}^q \lambda_k \theta_k^{\bar{q}}(f_k(x)) + \sum_{k=1}^q \mu_k \theta_k^{\bar{q}}(u_k(g_k(x))) \tag{5.1}$$

the set of optimal solutions for (3.1).

As a remark, for $\bar{q} = \infty$ equation (5.1) should be replaced by

$$\psi_1(x; \infty, \lambda, \mu, \theta) = \max\left\{ \max_k \lambda_k \theta_k(f_k(x)),\ \max_k \mu_k \theta_k(u_k(g_k(x))) \right\}$$

For problem (3.2) an auxiliary optimization problem $AP_2(\bar{q}, \lambda, \theta)$, with $\bar{q} \in \mathbf{Z}_+ \setminus \{0\}$, $\lambda \in \Delta_+^2$, where $\Delta_+^2 = \left\{ \lambda \in \mathbb{R}_+^q \,/\, \sum_{i=1}^q \lambda_i = 1 \right\}$, is as follows. Find

$$M_2(\bar{q}, \lambda, \theta) = \arg \min_{x \in D} \psi_2(x, \bar{q}, \lambda, \theta),$$

where $\psi_2(x; \bar{q}, \lambda, \theta) : D \to \mathbf{R}_+$ is given by

$$\psi_2(x; \bar{q}, \lambda, \theta) = \sum_{k=1}^q \lambda_k \theta_k^{\bar{q}}(f_k(x)) \tag{5.2}$$

For $\bar{q} = \infty$ equation (5.2) should be replaced by

$$\psi_2(x;\overline{q},\lambda,\theta) = \max_k \lambda_k \theta_k(f_k(x))$$

For problem (3.3) an auxiliary optimization problem $AP_3(\overline{q},\lambda,\theta)$ with $\overline{q} \in \mathbf{Z}_+ \setminus \{0\}$, $\lambda \in \Delta_+^2$, is as follows. Find

$$M_3(\overline{q},\mu,\theta) = \arg\min_{x\in D} \psi_3(x,\overline{q},\mu,\theta),$$

where $\psi_3(x;\overline{q},\mu,\theta) : D \to \mathbf{R}_+$ is given by

$$\psi_3(x;\overline{q},\mu,\theta) = \sum \mu_k \theta_k^{\overline{q}}(u_k(g_k^{s_0})). \quad (5.3)$$

For $q = \infty$ equation (5.3) should be replaced by

$$\psi_3(\infty) = \max_k(\mu_k \theta_k(u_k(g_k^{s_0}(x))). \quad (5.3)$$

On the same lines as in White [21, 22], Bowman [2], Karlin [8], relative to the auxiliary problems, we obtain the results below.

Theorem 5.1. a_1) If $\overline{q} \neq \infty$ then $M_1(\overline{q},\lambda,\mu,\theta) \subseteq E_1$ if $(\lambda,\mu) \in \Delta_{++}^1$

a_2) If a certain uniform dominance condition hold (Bowman [2]), then $M_1(\infty,\lambda,\mu,\theta) \subseteq E_1$ for $(\lambda,\mu) \in \Delta_{++}^1$.

a_3) If $\overline{q} \neq \infty$, $\{\theta_k^{\overline{q}}(f_k(\cdot))\}$, $k = \overline{1,q}$ are all convex on D and D is convex, then $E_1 \subseteq \bigcup_{(\lambda,\mu)\in\Delta_{++}^1} M_1(\overline{q},\lambda,\mu,\theta)$.

a_4) $E_1 \subseteq \bigcup_{(\lambda,\mu)\in\Delta_{++}^1} M_1(\infty,\lambda,\mu,\theta)$.

a_5) If D is finite, then there exists $\overline{q}^* \in Z_+ \setminus \{0\}$, such that $E_1 = \bigcup_{(\lambda,\mu)\in\Delta_{++}^1} M_1(\overline{q},\lambda,\mu,\theta)\ \forall \overline{q} \geq \overline{q}^*$.

Theorem 5.2 a_1) If $\overline{q} \neq \infty$ then $M_2(\overline{q},\lambda,\theta) \subseteq E_2$ for $\lambda \in \Delta_{++}^2$.

a_2) If a certain uniform dominance condition hold (Bowman [2]), then $M_2(\infty,\lambda,\theta) \subseteq E_2$ for $\lambda \in \Delta_{++}^2$.

a_3) If $\overline{q} \neq \infty$, $\{\theta_k^{\overline{q}}(f_k(\cdot))\}$, $k = \overline{1,q}$ are all convex on D and D is convex, then $E_2 \subseteq \bigcup_{\lambda\in\Delta_{++}^2} M_2(\overline{q},\lambda,\theta)$.

a_4) $E_2 \subseteq \bigcup_{\lambda\in\Delta_{++}^2} M_2(\infty,\lambda,\theta)$.

a_5) If D is finite, then there exists $\overline{q}^* \in Z_+ \setminus \{0\}$, such that $E_2 = \bigcup_{\lambda\in\Delta_{++}^2} M_2(\overline{q},\lambda,\theta)\ \forall \overline{q} \geq \overline{q}^*$.

Theorem 5.3 a_1) If $\overline{q} \neq \infty$ then $M_3(\overline{q},\mu,\theta) \subseteq E_3$ for $\mu \in \Delta_{++}^2$.

a_2) If a certain uniform dominance condition hold (Bowman [2]), then $M_3(\infty,\mu,\theta) \subseteq E_3$ for $\mu \in \Delta_{++}^2$.

a_3) If $\overline{q} \neq \infty$, $\{\theta_k^{\overline{q}}(f_k(\cdot))\}$, $k = \overline{1,q}$, are all convex on D and D is convex, then $E_3 \subseteq \bigcup_{\mu\in\Delta_{++}^2} M_3(\overline{q},\mu,\theta)$.

a_4) $E_3 \subseteq \bigcup_{\mu\in\Delta^2_{++}} M_3(\infty, \mu, \theta)$.

a_5) If D is finite, then there exists $\overline{q}^* \in \mathbf{Z}_+ \setminus \{0\}$, such that $E_3 = \bigcup_{\mu\in\Delta^2_{++}} M_3(\overline{q}, \mu, \theta)\ \forall \overline{q} \geq \overline{q}^*$.

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