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THERMOSOLUTAL STABILITY OF A TWO-COMPONENT ROTATING PLASMA WITH FINITE LARMOR RADIUS

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The thermosolutal linear stability of a composite two-component plasma is studied in the presence of Coriolis forces, finite Larmor radius, taking into account the collisions between neutral and ionized particles. The thermosolutal instability appears due to a material convection (thermosolutal convection) in a two component fluid with different molecular diffusivities which contribute in an opposing sense to the locally vertical density gradient. The analysis shows that in the case of a stationary convection, the finite Larmor radius, stable solute gradients and rotation have stabilizing effects. Results also demonstrate that the mutual collisions between ionized and neutral particles does not affect the stationary convection.

Key words: Convection, MHD, Thermosolutal

1. INTRODUCTION

The properties of ionized space and laboratory magnetic fluids (plasmas) have been intensively investigated theoretically and experimentally in the past fifty years. One of the key aspects studied in this context is the stability of plasma structures. Usually, instabilities can be divided into two categories: macro- and microinstabilities. Macro-instabilities occur with low frequencies compared to the plasma and cyclotron frequency and they are studied within the framework of magnetohydrodynamics (MHD). Physicists have understood the behaviour of macro-instabilities and they showed how to avoid the most destructive of them, but small-scale gradient driven microinstabilities are still a serious obstacle for having a stable plasma for a large range of parameters. Micro-instabilities are described by models which include, e.g. finite Larmor radius (FLR) and collisionless dissipative effects in plasmas. Time and length scales of micro-instabilities are comparable to the turbulent length scales and the length scales of transport coefficients.

In general, the FLR effect is neglected. However, when the Larmor radius becomes comparable to the hydromagnetic length of the problem (e.g. wavelength) or the gyration frequency of ions in the magnetic field is of the same order as the wave frequency, finiteness of the Larmor radius must be taken into account.

Strictly speaking, the space and time scale for the breakdown of hydromagnetics are on the respective scales of ion gyration about the field, and the ion Larmor frequency. In the present paper, we explore the effect of FLR on the stability of a two-componenent (ions and neutrals) rotating plasma. Introduction of non-ideal effects such as the FLR is known to stabilize the plasma systems. In strong magnetic fields particles can gyrate many times around magnetic field lines before collisions. This gyration motion will add a supplementary term to collisions resulting in an additional viscosity called gyroviscosity. In the present paper we consider that the rate of collisions is changed by the modification in temperature and concentration. In such cases, the buoyancy forces can arise not only due to density differences induced by temperature variations, but also due to variations in solute concentration.

The stability of plasmas with the FLR effect taken into account was studied by]. M.N. Rosenbluth et al. (1962) and K.V. Roberts (1962) where the authors assumed that the non-ideal character of the plasma is provided by finite electrical conductivity. The combined effect of Hall currents, FLR and rotation on the thermal stability of plasmas has been studied by Sharma (1972). The effect of a thermal gradient on the thermal stability of a fluid layer heated from below has been discussed by S. Chandrasekhar (1961). Veronis (1965) has studied the problem of thermohaline convection in a layer of fluid heated and salted from below. The conditions under which convective motion is important in stellar atmosphere require consideration of a fluid acted on by a solute gradient and free boundaries. The problem of the onset of thermal and thermosolutal instabilities in the presence of concentration gradients is of a great importance in astrophysics and atmospheric physics applications, in the solar atmosphere and ionosphere, in particular. The model used in the present paper can be applied to stellar atmosphere where the helium acts like salt in increasing the density and in diffusing more slowly than heat. The effect of FLR and Hall currents on the thermosolutal instability of a two-component plasma has been investigated by K.C. Sharma (1991) where the authors derived the conditions for monotonic instability. This idea was further developed by R.C. Sharma and Sunil (1992) for porous media.

The goal of the present paper is to consider the effect of FLR on the thermosolutal stability of a two-component rotating plasma situated in a gravitational field when the concentration of the two components is changed. In Section II we introduce the basic physical concepts and equations used to study the thermosolutal stability for the two-component plasma mixture in the incompressible limit. In Section III we derive the dispersion relation and study the effect of change of various physical quantities on the stability of the system. Finally, our results are summarized in Section IV.

2. MODEL FORMULATION AND BASIC EQUATIONS

We consider a model of a composite incompressible plasma rotating with a uniform angular velocity $\vec{\Omega}(0,0,\Omega)$. The plasma is permeated by a homogeneous magnetic field $\vec{B}_0$ parallel to the z-axis and we suppose that the intensity of the magnetic field is large enough so that the effect of FLR is important. The plasma is confined into a horizontal layer of thickness l_0 infinitely extended in the x and y directions and is balanced by the vertically downward gravity (0,0,-g). The plasma layer has two incompressible components: ions and neutrals with densities ρ_i and ρ_n, respectively. Due to the friction between particles, we are going to consider non-ideal effects such as viscosity and magnetic diffusivity, with the viscosity of neutrals being neglected. The collisional frequency between ions and neutrals is denoted by ν_c and we neglect the influence of rotational motion on neutral plasma component. The effect of FLR on ionized particles implies that the pressure is a tensorial quantity depending on the gyration frequency of the ions. The layer is heated and soluted from below in such a way that a steady temperature $\beta = |dT/dz|$ and steady solute concentration gradient $\beta' = |dC/dz|$ are maintained. At equilibrium the plasma layer satisfies the conditions $T = T_0 - \beta z, C = C_0 - \beta' z$ and $\rho = \rho_0 [1 + \alpha\beta z - \alpha'\beta' z]$ where T_0, C_0 and C, C are the temperatures, concentrations at the bottom surface (z=0) and at an intermediate point between z=0 and z=l_0 , ρ_0 is the density at z=0, α and α' represent the thermal and solvent coefficients of expansion, respectively. In writing the perturbed equations we will use the Boussinesq approximation.

For the sake of simplicity and in order to make analytical progress we suppose that both incompressible viscous ionized fluid and incompressible neutral gas behave like continuum media and the neutral gas is not affected by the pressure gradient, gravitation, temperature gradient and stable solute gradient. The model is further simplified by considering that the magnetic effects on the neutral gas are negligible.

The linearized hydromagnetic equations describing the dynamics of the ions-neutrals mixture are given by

$$\frac{\partial \vec{v}_i}{\partial t} = -\frac{1}{\rho_i}\nabla \cdot \vec{P} + \nu_i \nabla^2 \vec{v}_i + \nu_c(\vec{v}_n - \vec{v}_i) - (\alpha\theta - \alpha'\gamma)\vec{g} + 2\vec{v}_i \times \vec{\Omega} + \frac{1}{\mu\rho_i}(\nabla \times \vec{b}) \times \vec{B}, \quad (1)$$

$$\frac{\partial \vec{v}_n}{\partial t} = -\frac{\nu_c}{\varepsilon}(\vec{v}_n - \vec{v}_i), \tag{2}$$

$$\nabla \cdot \vec{v}_i = 0, \nabla \cdot \vec{v}_n = 0, \tag{3}$$

$$\frac{\partial \theta}{\partial t} - \chi \Delta \theta = \beta w, \tag{4}$$

$$\frac{\partial \gamma}{\partial t} - \chi' \Delta \gamma = \beta' w, \tag{5}$$

$$\frac{\partial \vec{b}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \nu_m \Delta \vec{b}, \tag{6}$$

$$\nabla \cdot \vec{b} = 0, \tag{7}$$

where $\vec{v}_i(u_i, v_i, w_i)$ and $\vec{v}_n(u_n, v_n, w_n)$ describe the velocity perturbations for ions and neutrals, $\vec{P}$ is the stress tensor, $\vec{b}(b_x, b_y, b_z)$ is the perturbation of the magnetic field, $\chi, \chi', \nu_m, \nu_i, \varepsilon = \rho_n / \rho_i$ are the thermal diffusivity, solute diffusivity, magnetic diffusivity, coefficient of kinematic viscosity and the density ratio between neutrals and ions. In Eqs. (4) and (5), θ and γ denote the perturbations of the temperature, and concentration. For simplicity, we assume that all diffusive coefficients are constant quantities. The change in the density ρ caused by the perturbation in the temperature and concentration is given by

$$\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{8}$$

Employing a normal mode analysis, we write all perturbations in the form $f(x, y, z) = \tilde{f}(z)\exp[ik_x x + ik_y y + nt]$ where k_x and k_y are the x and y components of the wave number and n is the growth rate assumed to be a real quantity. In the forthcoming calculations the tilde will be omitted.
Substituting this Ansatz into Eqs. (1) and (2) , we obtain:

$$\Omega_{in}\vec{v} = -\frac{1}{\rho_i}\nabla\vec{P} - (\alpha\theta - \alpha'\gamma)\vec{g} + 2\vec{v} \times \vec{\Omega} + \frac{1}{\rho_i \mu}(\nabla \times \vec{b}) \times \vec{B}, \tag{9}$$

$$\Omega_n \vec{v}_n = \frac{1}{\varepsilon}\nu_c \vec{v}_i, \tag{10}$$

where $\Omega_n = n + \nu_c / \varepsilon$ and $\Omega_{in} = n^* + \nu_i \Delta$ with $n^* = n[1 + \varepsilon\nu_c / (n\varepsilon + \nu_c)]$ being the frequency of oscillations modified by the collisional frequency between the ion and neutrals. Applying the curl operator to Eq.(9) we can obtain the z-component in the form:

$$\Omega_{in}\zeta = \left[\nu_0\left(2D^2 + k^2\right) + 2\Omega\right]Dw + \frac{B_0}{\rho_i \mu}D\xi, \tag{11}$$

where $D = d/dz, D^2 = d^2/dz^2, \zeta = \left(\nabla \times \vec{v}_i\right)_z, \xi = \left(\nabla \times \vec{b}\right)_z$.
In a similar way, it is straightforward to show that the z-components of the momentum equation for ions and induction equation are

$$\Omega_{in}\Delta w_i = \alpha g\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\alpha' g\left(\frac{\partial^2\gamma}{\partial x^2}+\frac{\partial^2\gamma}{\partial y^2}\right)-\left[\nu_0\left(2D^2+k^2\right)+2\Omega\right]D\zeta+\frac{B_0}{\rho_i\mu}D\Delta b_z, \quad (12)$$

$$\Omega_m b_z = B_0 Dw, \quad (13)$$

or applying the curl operator

$$\Omega_m \xi = B_0 D\zeta, \quad (14)$$

where $\Omega_m = n + \nu_m k^2$.

3. THE DISPERSION RELATION

In what follows we are going to introduce dimensionless quantities. First, we suppose that using the normal mode analysis, the form of various physical quantities are given as

$$\{w, \theta, \gamma, b_z, \zeta, \xi\} = \{W(z), \Theta(z), \Gamma(z), K(z), Z(z), X(z)\}\exp\left[ik_x x + ik_y y + nt\right] \quad (15)$$

The quantities l_0 and l_0/ν are used to write lengths and times in dimensionless form. Accordingly, we can write $z^* = z/l_0, a = kl_0, \sigma = nl_0^2/\nu, \sigma^* = n^* l_0^2/\nu, p_1 = \nu/\chi, p_2 = \nu/\nu_m, p_3 = \nu/\chi'$, where ν is the viscosity due to ions. Using the above dimensionless quantities we define

$$C_0 = \frac{\beta l_0^2}{\chi}, \quad C_1 = \frac{\beta' l_0^2}{\chi'}, \quad C_2 = \frac{\sqrt{T}}{l_0}, \quad C_3 = \frac{B_0 l_0}{\mu\rho_i\nu}$$
$$C_4 = \frac{\sqrt{U}}{l_0}, \quad C_5 = \frac{B_0 l_0}{\nu_m}, \quad C_6 = \frac{\alpha g l_0^2 a^2}{\nu}, \quad C_7 = \frac{\alpha' g l_0 a^2}{\nu} \quad (15)$$
$$C_8 = l_0\sqrt{U}, \quad C_9 = l_0\sqrt{T_a}, \quad \sqrt{T_a} = a\Omega\frac{l_0^2}{\nu}, \quad \sqrt{U} = \frac{\nu_0}{\nu}$$

where T_a is the Taylor number and ν_0 is the coefficient of gyroviscosity.

In order to make the mathematics more transparent we choose to work in an operational form and we define the operators

$$\Xi = D^2 - a^2,\ D_\sigma^* = D^2 - a^2 - \sigma^*,\ D_1 = D^2 - a^2 - \sigma p_1,$$
$$D_2 = D^2 - a^2 - \sigma p_2,\ D_3 = D^2 - a^2 - \sigma p_3,\ D_a = 2\Xi^2 + a^2 \quad (17)$$

Using these notations, the equations (4),(5),(11),(12),(13) and (14) can be rewritten as

$$D_1\Theta = -C_0 W, \quad (18)$$

$$D_3\Gamma = -C_1 W, \quad (19)$$

$$D_\sigma^* Z = -(C_2 + C_4 D_a)DW - C_3 DX, \quad (20)$$

$$D_2 K = -C_5 DW, \quad (21)$$

$$D_\sigma^*\Xi W = C_6\Theta - C_7\Gamma + (C_8 D_a + C_9)DZ - C_3\Xi DK, \quad (22)$$

$$D_2 X = -C_5 DZ, \tag{23}$$

The above equations are used in what follows to derive the dispersion relation.

By eliminating $\Theta(z), \Gamma(z), K(z), X(z)$ and $Z(z)$ from the system of equations (18)-(24) and introducing The Chandrasekhar number $Q = C_3 C_5$ we obtain that the dispersion relation describing the evolution of W(z) is given by

$$\begin{aligned}&(D_\sigma^* D_2 - QD^2)[\Xi D_1 D_3 (D_\sigma^* D_2 - QD^2) + D_2 a^2 (RD_3 - SD_1)]W(z) = \\ &U(V+D)^2 D_1 D_3 D_2^2 D^2 W(z),\end{aligned} \tag{24}$$

where $R = \dfrac{g\alpha\beta l_0^4}{\chi\nu}$ is the Rayleigh number , $S = \dfrac{g\alpha'\beta' l_0^4}{\chi'\nu}$ is the solute Rayleigh number and $V = \dfrac{2\Omega l_0^2}{\nu_0}$ is a dimensionless number describing the importance of Coriolis rotation with respect to the gyrorotation. Neglecting the collisions between ions and neutral, we recover the result obtained by Gupta & Singh (1986), i.e. the operator $D^2 - a^2 - \sigma$ in their paper has been replaced by $D^2 - a^2 - \sigma^*$.In what follows we are studying the possible solutions of Eq.(24) in several particular cases:

(i) If the effects of FLR and rotation (V=0) are neglected, Eq.(24) reduces to

$$\left(D_\sigma^* D_2 - QD^2\right)\left[\Xi D_1 D_3 \left(D_\sigma^* D_2 - QD^2\right) + D_2 a^2 \left(RD_3 - SD_1\right)\right]W(z) = 0 \tag{25}$$

where we have taken into account that $UV^2 = T_a$. Eq. (25) recovers the result obtained by Sharma & Sharma (1981).

(ii) In the absence of FLR effect and neglecting the solute gradient (S=0), Eq. (24) becomes.

$$D_1\left[\Xi\left(D_\sigma^* D_2 - QD^2\right)^2 + TD_2^2 D^2\right]W(z) = -Ra^2\left(D_\sigma^* D_2 - QD^2\right)D_2 W(z) \tag{26}$$

(iii) For a single plasma component (pure magnetized plasma) in the absence of ion-neutral collisional frequency $(\nu_c = 0, \sigma^* = 0)$ the dispersion relation is of the form

(iv)

$$D_1\left[\Xi\left(D_\sigma D_2 - QD^2\right) + TD_2 D^2\right]W(z) = -Ra^2\left(D_\sigma D_2 - QD^2\right)D_2 W(z) \tag{27}$$

which is identical to the result obtained by Chandrasekhar.

4. RESULTS

In this section we discuss the stability of the mixture based on the analysis of the dispersion relation (24).

If both plasma boundaries are free and are perfect conductors (the most appropriate model for stellar atmospheres see, e.g. Veronis (1965)), the boundary conditions at z*=0 and z*=1 can be written as

$$W(z^*) = D^2 W(z^*) = DZ(z^*) = 0, \quad \Theta(z^*) = \Gamma(z^*) = X(z^*) = 0, \tag{28}$$

and b_x, b_y, b_z are continuous functions while the tangential components are zero outside the fluid. Having in mind the property of W(z*) at the two boundaries, the solution of the dispersion relation (24) characterizing the lowest mode (where all even order derivatives of W(z*) vanish) is

$$W(z^*) = W_0 \sin(\pi z^*), \tag{29}$$

where W_0 is a constant.

Substituting Eq. (29) into Eq. (24) we obtain the characteristic equation

$$R_1 = \frac{(1+x)\left[(1+x+\hat{\sigma})(1+x+b_1)\right]}{x} + \\ + \frac{S_1 x(1+x+b_1)(1+x+b_2)+Q_1(1+x)(1+x+b_1)(1+x+b_3)}{x(1+x+b_2)(1+x+b_3)} + \\ + \frac{U\left[V_1-(2-x)\right]^2(1+x+b_1)(1+x+b_2)}{x\left[(1+x+\hat{\sigma})(1+x+b_2)+Q_1\right]}, \tag{30}$$

where we have used

$$x = \frac{a^2}{\pi^2}, \quad R_1 = \frac{R}{\pi^4}, \quad S_1 = \frac{S}{\pi^4}, \quad Q_1 = \frac{Q}{\pi^2}, \quad V_1 = \frac{V}{\pi^2},$$
$$\hat{\sigma} = \frac{\sigma^*}{\pi^2}, \quad b_1 = \frac{\sigma p_1}{\pi^2}, \quad b_2 = \frac{\sigma p_2}{\pi^2}, \quad b_3 = \frac{\sigma p_3}{\pi^2}.$$

In the case of a stationary convection $(n = \sigma = n^* = \sigma^* = \hat{\sigma} = 0, b_1 = b_2 = b_3 = 0)$ Eq.(30) reduces to

$$R_1 = \frac{1+x}{x}\left[(1+x)^2 + Q_1\right] + U\frac{\left[V_1-(2-x)\right]^2(1+x)^2}{x\left[1+x)^2+Q_1\right]} + S_1, \tag{31}$$

Having in mind the definition of σ^* we can clearly see that considering a stationary convection means that the collision between neutrals and ions will not appear in our calculations, i.e. the collision between the two species does affect the stationary convection. Eq. (31) expresses the modified Rayleigh number as a function of the dimensionless wavenumber x and the parameters Q_1, V_1, S_1 and U.
If the corrections due to the FLR effect are neglected $(\nu_0 = 0)$, Eq. (31) can be further reduced to

$$R_1 = \frac{1+x}{x}\left[(1+x)^2 + Q_1\right] + T_{a_1}\frac{1+x)^2}{x\left[1+x)^2+Q_1\right]} + S_1, \tag{32}$$

where $T_{a_1} = \dfrac{T_a}{\pi^4}$.

If we neglect the solute gradient effect (S_1=0) then we recover the result obtained by Chandrasekhar. In general, the study of FLR, rotational motion and solute gradient is facilitated by the analytical study of the quantities dR_1/dU, dR_1/dV_1 and dR_1/dS_1. Using Eq. (30) it is straightforward to show that

$$\frac{dR_1}{dU} = \frac{\left[V_1-(2-x)\right]^2(1+x+b_1)(1+x+b_2)}{x\left[(1+x+\hat{\sigma})(1+x+b_2)+Q_1\right]},$$

for the study of the FLR effects,

$$\frac{dR_1}{dV_1} = U\frac{2\left[V_1-(2-x)\right](1+x+b_1)(1+x+b_2)}{x\left[(1+x+\hat{\sigma})(1+x+b_2)+Q_1\right]},$$

for the effect of the Coriolis force, and

$$\frac{dR_1}{dS_1} = \frac{1+x+b_1}{1+x+b_3}, \tag{33}$$

when studying the effect of the solute Rayleigh number.
When the instability sets in as a stationary convection, Eq.(31) yields

$$\frac{dR_1}{dU} = \frac{\left[V_1 - (2-x)\right]^2 (1+x)^2}{x\left[(1+x)^2 + Q_1\right]},$$

which is always positive, so the FLR has a stabilizing effect on the thermosolutal instability. Eq. (31) also means that

$$\frac{dR_1}{dV_1} = \frac{2U\left[V_1 - (2-x)\right](1+x)^2}{x\left[(1+x)^2 + Q_1\right]}.$$

The above quantity is positive provided $\Omega l_0^2 < \nu_0$ which is easily satisfied for large values of the magnetic field, so the rotation can induce a stabilizing effect on the plasma system. In fact, the above relation shows that, for x>2 (i.e.for wavelengths smaller than the thickness of the layer), the rotation has always a stabilizing effect on the system. For given rotational speed and gyroviscosity, the quantity which controls the stability of the system is the thickness of the plasma layer, l_0. Finally, for a stationary convection, from eq.(31) we have

$$\frac{dR_1}{dS_1} = 1,$$

which implies that stable solute gradient has always a stabilizing effect on the system considered in stationary limit.

Our treatment is limited only to stationary convection (see Eq.(31) where the modified Rayleigh number reaches its minimum when $dR_1/dx=0$. Accordingly, we obtain

$$x^7 + a_1 x^6 + a_2 x^5 + a_3 x^4 + a_4 x^3 + a_5 x^2 + a_6 x + a_7 = 0, \tag{34}$$

where the coefficients a_i (i =1..7) are given by

$$a_1 = 5.5 + 0.5U, \quad a_2 = 2(6 + Q_1 + U),$$
$$a_3 = 12.5 + 6.5Q_1 - 0.5T_{a_1} + U(1 + 1.5Q_1) + 2\sqrt{UT_{a_1}},$$
$$a_4 = 5 + 6Q_1 + Q_1^2 - 2T_{a_1} - 2U(3 + Q_1) + 2(4 + Q_1)\sqrt{UT_{a_1}},$$
$$a_5 = -1.5 + 0.5Q_1(Q_1 - 2) - 3T_{a_1} + 0.5Q_1 T_{a_1} - 0.5U(23 + 3Q_1) + 12\sqrt{UT_{a_1}},$$
$$a_6 = -2\left[(1 + Q_1)^2 + T_{a_1}\right] - 8U + 8\sqrt{UT_{a_1}},$$
$$a_7 = 0.5(1 + Q_1)\left[4\sqrt{UT_{a_1}} - 4U - T_{a_1} - (1 + Q_1)^2\right].$$

Using the solutions of Eq. (34) in conjunction with Eq. (31), we are able to determine a critical Rayleigh number, R_c, which will determine the stability of the system. According to the usual classification, the system is stable for $R<R_c$, and it is unstable for $R>R_c$.

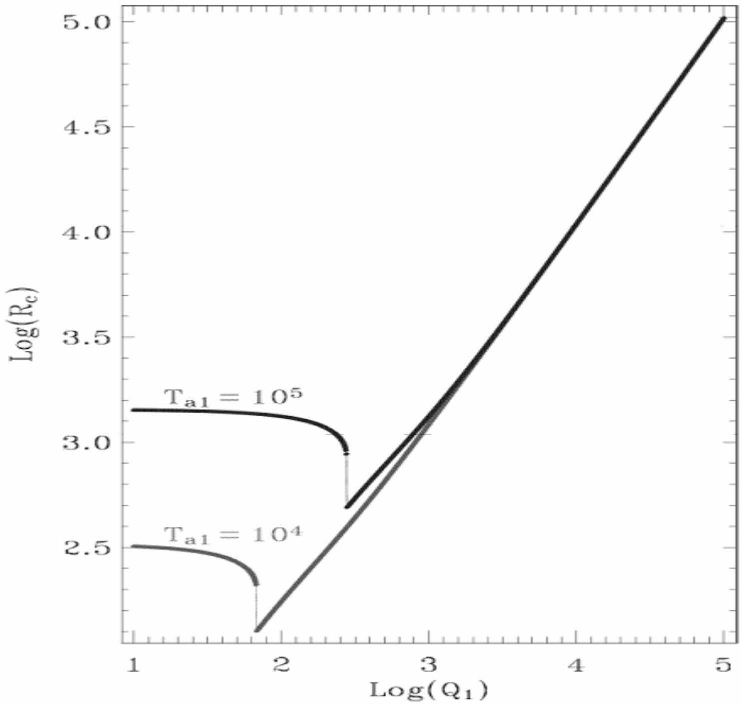

Figure 1. The variation of the Rayleigh number (R_1) with respect to the Chandrasekhar number when U=0 for two values of the Taylor number, T_{a1}:$T_{a1}=10^4$ and $T_{ai}=10^5$.

In what follows we are going to study the stability of the system using a numerical approach. Since we are interested in the minima of the relation (30) we solve the polynomial equation (34) and we choose the positive real solutions which then substitute in Eq. (30). Figure 1 shows the dependence of the critical Rayleigh number with respect to the Chandrasekhar number, Q (on logarithmic scale) in the absence of the FLR (U=0) and thermosolutal ($S_1=0$) effects. Curves are obtained for two values of the Taylor number: $T_1=10^4$ and $T_1=10^5$. As we can see, the critical Rayleigh number has a fairly constant behaviour, after which it decays. For both values, we find a vertical jump where we cannot obtain a stability criterion for the system since no solutions have been found. After this region, the dependence of the R_c becomes asymptotic. Increasing the effect of rotation, i.e. the Taylor number, the stability of the system is increased, so the rotation has a stabilizing effect.

If we introduce the FLR effect (Figure 2), the stability curves for all values of the Taylor number are moved upwards, i.e. the stability area under the curves is increased. In Figure 3, we have plotted the dependence of the critical Rayleigh number with respect to the Chandrasekhar number for a fixed Taylor number, $T_{a1}=10^5$, for three different values of U. For small values of Q, the stability threshold increases, but the area under the curve becomes smaller, i.e. the decrease in U has a stabilizing effect.

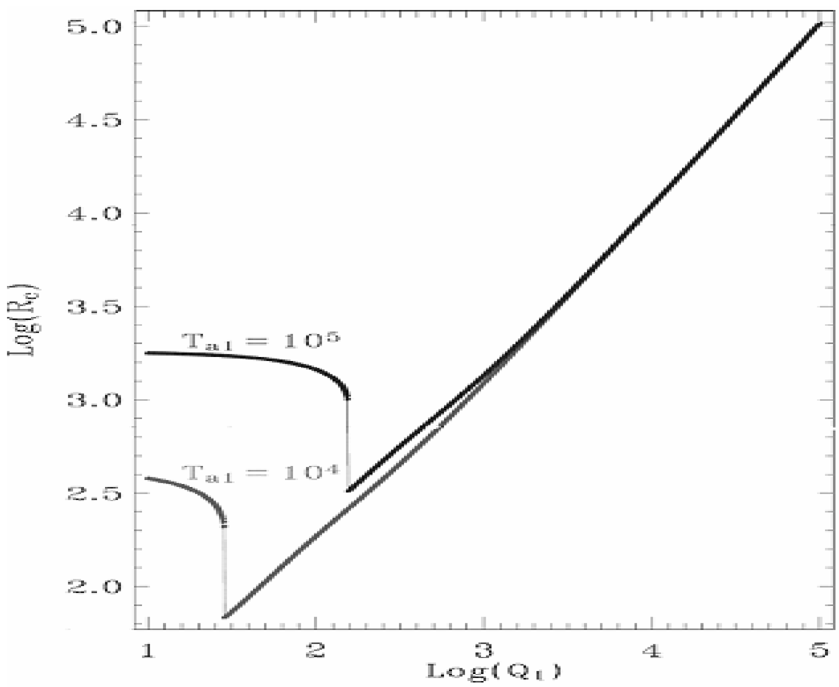

Figure 2. Similar to Figure 1, but with U=10

This results can be understood having in mind the definition of U, which is the ratio between the coefficients of kinematic viscosity and the gyroviscosity introduced by the FLR effect. Keeping the coefficient of kinematic viscosity constant, the decrease in U means an increase in the gyroviscosity. As the intensity of the magnetic field increases, the Larmor radius of gyrating ions becomes smaller, i.e. the collisions increase which will lead to an increase in gyroviscosity. Therefore, an increase in the FLR effect has a stabilizing effect.

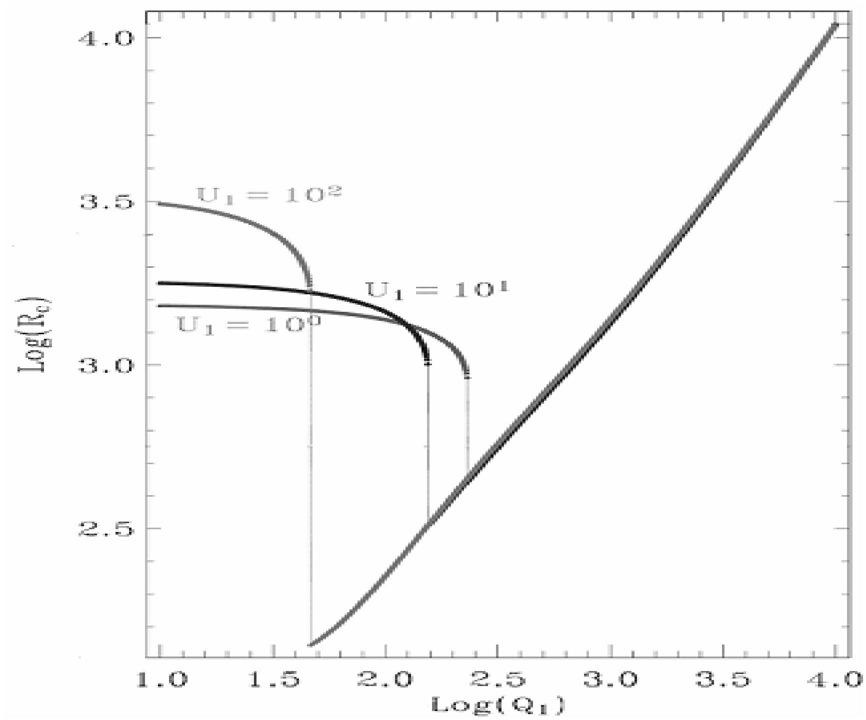

Figure 3. Variation of the Rayleigh number (R_1) with respect to the Chandrasekhar number for a fixed Taylor number ($T_{ai}=10^5$ and for three values of U: U=1, U=10, and U=100)

Finally, since the solute Rayleigh number appears only as an additive term to the expression giving the Rayleigh number (Eq.(30), the increase in the thermosolutal gradient will always increases the stability of the system (as predicted earlier).

SUMMARY

In the present paper we addressed the problem of linear stability of a thermosolutal mixture between ions and neutrals under the FLR effect, rotation and thermosolutal gradients. Analytical and numerical results proved that all three effects are able to stabilize the system. Our numerical analysis has been restricted to stationary cases but further analysis of non-stationary behaviour would be straightforward based on the present results

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