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ON THE MODELING OF HIBRID SYSTEMS

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The purpose of this paper is to analyse the motion of multibody hybrid systems characterized by switching between constraints, which are defined as different dynamical regimes. The dynamics of these systems is formulated within the framework of Lagrange formalism, based on the Lagrange equations, and on the symmetries by Noether's theorems. As an example, we consider the oscillations of a woodpecker model.

Key words: multibody systems, hybrid systems, Lagrange equations, the woodpecker.

1. THE LAGRANGE EQUATIONS

The motion of hybrid systems is characterized by switching between constraints, which are defined as different dynamical regimes. An important class of hybrid systems consists of systems with multiple elastic impacts (Brogliato [1]). The nonholonomic constraints with affine connections are analyzed by Bloch and Crouch [2], and Lewis [3]. Some controllability results for smooth mechanical systems and kinematically controllable systems are reported by Lewis and Murray [4], Sussmann [5], and Bullo and Lynch [6], Secară, Nițu and Cononovici [7]. The analysis leads to effective motion planning schemes for various classes of mechanical control systems, in closed connection to the Lie groups theory (Brockett [8], Leonard and Krishnaprasad [9], Lynch, Shiroma, Arai and Tanie [10], Teodorescu and Nicorovici [11]).

In this paper we present a model for a hybrid system S consisted of n rigid bodies that can move by switching between constraints. The spatial position of S is specified by $3N$ Cartesian coordinates. When the bodies are subjected to some constraints, the $3N$ coordinates have to satisfy certain relations, such that the number of independent coordinates becomes less than $3N$. Let us have $m<3N$ independent geometrical constraints (holonomic) of the form

$$f_i(r_1,r_2,...,r_n,t)=0\,,\ i=1,2,...,m\,, \tag{1.1}$$

where $r_i(x_i,y_i,z_i)$ denotes the position vector of the mass center of the body i. The matrix

$$\left[\frac{\partial(f_1,f_2,...,f_m}{\partial(x_1,y_1,z_1,x_2,...,z_n}\right],$$

contains at least one non-vanishing determinant of order m and it follows that m coordinates can be expressed as functions of $n=3N-m$ independent coordinates, denoted by generalized coordinates (Lagrange's coordinates) q_k, $k-1,2,...,n$. The set of all values q_k defines an n-dimensional space (configuration space, or Lagrange's space) denoted by Λ_n. To each representative center mass point $P(q_1,q_2,...,q_n)\in\Lambda_n$, it corresponds a position of S in E_3 in accordance with the holonomic constraints, and conversely by means of the one-to one mapping

$$r_i = r_i(q_1, q_2, ..., q_n, t),\ i = 1,2,...,m, \tag{1.2}$$

where t is regarded as a parameter. For $t \in [t_0, t_1]$, the system takes a sequence of positions in E_3, which corresponds to a sequence of points in the Lagrange's space. Consequently, the evolution of S is represented by equations

$$q_i = q_i(t),\ i = 1,2,...,n,\ t \in [t_0, t_1], \tag{1.3}$$

which defines a trajectory of the representative point P in Λ_n. The motion takes place in such a way that the Lagrangian action functional $A = \int_{t_0}^{t_1} L(q, \dot{q}, t)\mathrm{d}t$, is stationary, where $L = L(q, \dot{q}, t)$ is the Lagrange function or Lagrange's kinetic potential. From stationarity of action functional, the Lagrange equations are obtained

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0,\ k = 1,2,...,n, \tag{1.4}$$

where L is the sum between the total kinetic energy T, and the potential U, i.e. $L = T + U$. If $U = U(q)$ or $U = U(q,t)$, we have a simple potential, and a simple quasi-potential, respectively. The generalized force is a conservative forces and respectively a quasi-conservative force $Q_k = \frac{\partial U}{\partial q_k}$, $k = 1,2,...,n$. If $U = U(q, \dot{q})$ or $U = U(q, \dot{q}, t)$, we have a generalized potential, or a quasi-generalized potential, respectively. In this case, the generalized force is

$$Q_k = \frac{\partial U}{\partial q_k} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial U}{\partial \dot{q}_k}\right) = 0,\ k = 1,2,...,n,$$

and the motion equations (1.4) lead to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k,\ k = 1,2,...,n. \tag{1.5}$$

The equations (1.5) are the consequence of the principle of virtual work $\sum_{i=1}^{n}(m_i\ddot{r}_i - F_i)\cdot\delta r_i = 0$, where F_i is the force acting on the sub-body i, and m_i is the mass of the sub-body i. The generalized force is given in this case by

$$Q_k = \sum_{i=1}^{n} F_i \cdot \frac{\partial r_i}{\partial q_k},\ k = 1,2,...,n. \tag{1.6}$$

The general total kinetic is given by

$$T = \frac{1}{2} g_{jk}\dot{q}_j\dot{q}_k + g_j\dot{q}_j + g_0, \tag{1.7}$$

where g_{jk} are components of the metric tensor g

$$\begin{gathered} g_{jk} = \sum_{i=1}^{n} m_i \frac{\partial r_i}{\partial q_j}\cdot\frac{\partial r_i}{\partial q_k}, \\ g_j = \sum_{i=1}^{n} m_i \frac{\partial r_i}{\partial q_j}\cdot\frac{\partial r_i}{\partial t},\ g_0 = \sum_{i=1}^{n} m_i \left(\frac{\partial r_i}{\partial t}\right)^2. \end{gathered} \tag{1.8}$$

In the case of scleronomic constraints (which do not explicitly depend on time), the expression (1.7) becomes

$$T = \frac{1}{2} g_{jk} \dot{q}_j \dot{q}_k \,. \tag{1.9}$$

The matrix

$$\left[\frac{\partial(x_1, y_1, z_1, x_2, ..., x_n}{\partial(q_1, q_2, ..., q_n} \right],$$

is of rank n, and the generalized velocities $\frac{\partial r_k}{\partial q_j} \dot{q}_j = 0$, $k = 1,2,...,n$, have only trivial solutions. It results that the kinetic energy (1.9) is a positive definite quadratic form, which vanishes only when all the generalized velocities are zero. In the case of scleronomic constraints it follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_k} \right) = g_{jk} \ddot{q}_j + \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial q_l} + \frac{\partial g_{kl}}{\partial q_j} \right) \dot{q}_j \dot{q}_l \,,$$

$$\frac{\partial T}{\partial q_k} = \frac{\partial g_{jl}}{\partial q_k} \dot{q}_j \dot{q}_l \,, \; k = 1,2,...,n \,.$$

Thus, the Lagrange equations (1.5) can be written under the form

$$g_{jk} \ddot{q}_j + [jl,k] \dot{q}_j \dot{q}_l = Q_k \,, \; k = 1,2,...,n \,, \tag{1.10}$$

where $[jk,l]$ are the Christoffel symbols of the first kind for the metric g_{jk}

$$[jl.k] = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial q_l} + \frac{\partial g_{kl}}{\partial q_j} - \frac{\partial g_{jl}}{\partial q_k} \right), \; j,k,l = 1,2,...,n \tag{1.11}$$

Introducing the Christoffel symbols of the second kind

$$\left\{ \begin{matrix} m \\ j \;\; l \end{matrix} \right\} = g^{km} [jl,k] \,. \tag{1.12}$$

where g^{km} are the components of the inverse of g_{jk} ($g_{jk} g^{km} = \delta_j^m$), and the generalized forces

$$\tilde{Q}_m = Q_k g^{km} \,, \tag{1.13}$$

The normal form of the Lagrange equations (1.10) is

$$\ddot{q}_m + \left\{ \begin{matrix} m \\ j \;\; l \end{matrix} \right\} \dot{q}_j \dot{q}_l = \tilde{Q}_m \,, \; m = 1,2,...,n \,. \tag{1.14}$$

The equations (1.14) suggests that we can describe the motion of a hibrid system by a set $\Sigma = \{\Lambda_n, g, F, q\}$, where Λ_n is the configuration space (or Lagrange's space), g the metric tensor, F the vector of input forces, and q the generalized coordinate vector. The force F is related with the generalized force $\tilde{Q}$ by relations (1.13) and (1.6).

2. THE NOETHER'S THEOREMS

The Lagrange function $L(q,\dot{q},t)$ is a kinetic potential because it satisfies the Lagrange equations (1.4). Let introduce a transformation

$$L'(q,\dot{q},t) = L(q,\dot{q},t) + \tilde{L}(q,\dot{q},t)\,, \tag{2.1}$$

where $\tilde{L}$ has the form $\tilde{L} = a_k(q,t)\dot{q}_k + a_0(q,t)$. By substituting (2.1) in (1.4) we obtain similarLagrange equations for $\tilde{L}$. The function $\tilde{L}$ has the form

$$\tilde{L}(q_k,\dot{q}_k,t) = \frac{\partial\varphi}{\partial q_k}\dot{q}_k + \frac{\partial\varphi}{\partial t} = \frac{\mathrm{d}\varphi}{\mathrm{d}t}\,,\ \varphi\in C^2\,. \tag{2.2}$$

Thus, by applying a transformation of the form

$$.\,L'(q_k,\dot{q}_k,t) = L(q_k,\dot{q}_k,t) + \frac{\mathrm{d}\varphi}{\mathrm{d}t}\,, \tag{2.3}$$

called gauge transformation, to a Lagrangian L we obtain another Lagrangian L', which satisfies the same Lagrange equations (1.4). A general form of a transformation of independent variable is

$$t' = \varphi(t)\,, \tag{2.4}$$

and the changes of generalized coordinates are

$$q_i'(t') = Q_i(t,q_i)\,. \tag{2.5}$$

The equations of motion can be derived from the functional $\mathbb{F} = \int_{t_0}^{t_1} L(q_1,q_2,...,q_n,\dot{q}_1,\dot{q}_2,...,\dot{q}_n,t)\mathrm{d}t$, which is invariant with the infinitesimal transformation $t' = t + \delta t$ if

$$L'(q_i',\dot{q}_i',t')\mathrm{d}t' = L(q_i,\dot{q}_i,t)\mathrm{d}t\,. \tag{2.6}$$

The Lagrange equations are invariant if

$$L'(q_i',\dot{q}_i',t') = L(q_i',\dot{q}_i',t') + \frac{\mathrm{d}f}{\mathrm{d}t'}\,. \tag{2.7}$$

We can say that (2.4) is a symmetry transformation for a mechanical system if and only if the conditions (2.6) and (2.7) are satisfied. For the infinitesimal transformation $t' = t + \delta t$, these conditions yield to

$$\left(\delta t\frac{\partial}{\partial t_i} + \delta q_i\frac{\partial}{\partial q_i} + \delta\dot{q}_i\frac{\partial}{\partial t}\right)L = -\frac{\mathrm{d}}{\mathrm{d}t}\delta f(t,q_i)\,. \tag{2.8}$$

On this basis, a symmetry transformation of a mechanical system is associated with an equation of conservation. This result is proved by the Noether's theorem.

THEOREM 2.1. (Noether). *If the Lagrangian of a mechanical system is invariant with respect to a continuous group of transformation with* p *parameters, then exist* p *quantities, which are conserved during the evolution of the system.*

Let $C(q,p)$ be an integral of motion in the Hamilton formulation. In the Lagrange formulation, this integral becomes $F(q,\dot{q}) = C(q,p)$. The integral of motion along trajectories of the system satisfies the condition

$$\frac{\mathrm{d}}{\mathrm{d}t}F(q,\dot{q}) = 0\,. \tag{2.9}$$

Along the trajectories system, the variation of the Lagrangian due to the transformations $q_i' = q_i + \delta q_i$, $i = 1,2,...,n$, is given by

$$\delta L = \frac{\mathrm{d}}{\mathrm{d}t}(\frac{\partial L}{\partial \dot{q}_i}\delta q_i). \tag{2.10}$$

From (2.10) it follows certain conserved quantities.

Case 1. If $\delta L = 0$, the integral of motion is given by

$$\frac{\partial L}{\partial \dot{q}_i} = p_i \delta q_i. \tag{2.11}$$

Case 2. If there exists a function $f(q_i)$ such that $\delta L = \frac{\mathrm{d}}{\mathrm{d}t}\delta f(q_i)$, then the integral of motion is

$$(\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial f}{\partial q_i})\delta q_i = (p_i - \frac{\partial f}{\partial q_i})\delta q_i. \tag{2.12}$$

Case 3. If there exists a function $g(q_i, \dot{q}_i)$ such that

$$\delta L = \frac{\mathrm{d}}{\mathrm{d}t} g(q_i, \dot{q}_i), \tag{2.13}$$

the integral of motion becomes

$$\frac{\partial L}{\partial \dot{q}_i}\delta q_i - g(q_i, \dot{q}_i) = p_i \delta q_i - g(q_i, \dot{q}_i). \tag{2.14}$$

We see that (2.11) and (2.12) are particular cases of the third case.

THEOREM 2.2. (Noether). *To every infinitesimal transformation of the form* $q_k' = q_k + \delta q_k$, $k = 1,2,...,n$, *due to a variation of the Lagrangian given by* (2.13), *it corresponds to a conserved quantity defined by* (2.14).

In conclusion, from the motion equation of Lagrange (1.14), and the Noether's theorems, we can define a mechanical model for a hibrid system as a quadruple $\Sigma = \{\Lambda_n, g, F, q\}$, where Λ_n is the configuration space (Lagrange's space), g the metric tensor, F the vector of input forces related with by generalized forces by $Q_k = \sum_{i=1}^{n} F_i \cdot \frac{\partial r_i}{\partial q_k}$, $\tilde{Q}_m = Q_k g^{km}$, $k = 1,2,...,n$, and q the generalized coordinate vector. We are interested in a class of hybrid systems that interact with the surrounding environment via holonomic or nonholonomic constraints. Clamping a sliding body to a surface is an example of a holonomic constraint. Rolling without sliding is an example of a nonholonomic constraint. The characterizing feature of this model is that the constraint can become active at any configuration so that the constraint distribution is defined at least over an open subset of Λ_n. The advantage of this model is that both holonomic and nonholonomic constraints can be represented in the same way. As an example, we consider the woodpecker model first considered by Glocher and Pfeiffer [12].

4. THE MODEL OF GLOCHER AND PFEIFFER

The woodpecker model is described by the set $\mathbf{q} = (y, \phi, \phi_1)^T$, where y is the vertical displacement of the sleeve, ϕ is the absolute angle of rotation of the woodpecker, and ϕ_1 is the absolute angle of rotation of the sleeve. The mass of woodpecker is m, the moment of inertia of woodpecker with respect to the mass center O, is J. Horisontal deviations are negligigle. The mass of sleeve is m_1, and the moment of inertia of the sleeve with respect to the mass center O_1, is J_1. The contact without friction occurs when the beak of the woodpecker hits the pole (constraint 1). The diameter of the hole in the sleeve is a little larger than the diameter of the pole, and then the lower or upper edge of the sleeve may come into contact with the pole without friction (constraints 2 and 3). We suppose that the displacement are small. This is a planar mechanism moving in a vertical plane with gravity. The coordinates (x_C, y_C) define the joint C. The constraint 1 permit transition from separation to separation at the beak impact. The constraint 2 permit transition from separation to sliding at the lower sleeve impact, from sliding to sticking, from sticking to sliding, and from sliding to separation. The constraint 3 permit transition from separation to separation at the upper sleeve impacts.

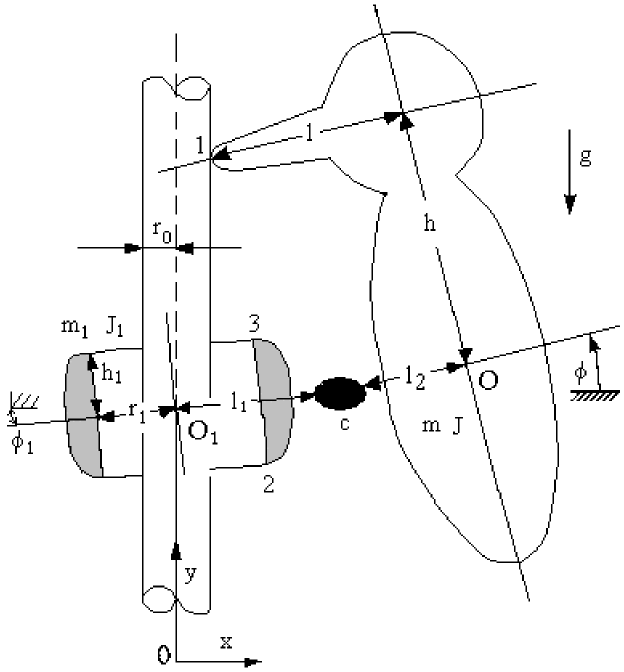

Fig. 3.1. The Glocker and Pfeiffer's model [12]).

The motion equations (1.14) yield to the motion equations derived by Glocker and Pfeiffer ([12], [13]) in the case of constrained motion of a multibody system

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{h} - \sum_{j=1}^{3} (\mathbf{w}_{Nj}\lambda_{Nj} + \mathbf{w}_{Tj}\lambda_{Tj}) = 0 , \tag{4.1}$$

The inertia matrix $\mathbf{M}$ of the woodpecker is given by

$$\mathbf{M} = \begin{pmatrix} m+m_1 & ml_1 & ml_2 \\ ml_1 & J_1 + ml_1^2 & ml_1 l_2 \\ ml_2 & ml_1 l_2 & J + ml_2^2 \end{pmatrix}. \tag{4.2}$$

The vector of the generalized forces $\mathbf{h}$, and the vector of generalized coordinates $\mathbf{q}$, are

$$\mathbf{h} = \begin{pmatrix} -mg - m_1 g \\ -c\phi_1 + c\phi - mgl_1 \\ -c\phi + c\phi_1 - mgl_2 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} y \\ \phi_1 \\ \phi \end{pmatrix}. \tag{4.3}$$

The last three term of the motion equations (4.1) represent the normal and tangential contact forces proportional to the Lagrange multipliers λ_N and λ_T. The constraints vectors $\mathbf{w}_N$ and $\mathbf{w}_T$ in the contact points are

$$\mathbf{w}_{N1} = \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}, \ \mathbf{w}_{N2} = \begin{pmatrix} 0 \\ h_1 \\ 0 \end{pmatrix}, \ \mathbf{w}_{N3} = \begin{pmatrix} 0 \\ -h_1 \\ 0 \end{pmatrix}, \tag{4.4}$$

$$\mathbf{w}_{T1} = \begin{pmatrix} 1 \\ l_1 \\ l_2 - l \end{pmatrix}, \ \mathbf{w}_{T2} = \begin{pmatrix} 1 \\ r_1 \\ 0 \end{pmatrix}, \ \mathbf{w}_{T3} = \begin{pmatrix} 1 \\ r_1 \\ 0 \end{pmatrix}. \tag{4.5}$$

The motion equation (4.1) is subject to the initial conditions on the impact intervals

$$\dot{\mathbf{q}}_A = \dot{\mathbf{q}}(t_A), \ \dot{\mathbf{q}}_C = \dot{\mathbf{q}}(t_C), \ \dot{\mathbf{q}}_E = \dot{\mathbf{q}}(t_E),$$

$$\dot{g}_{iA} = \dot{g}_i(t_A), \ \dot{g}_{iC} = \dot{g}_i(t_C), \ \dot{g}_{iE} = \dot{g}_i(t_E), \ i = N \text{ or } i = T, \tag{4.6}$$

where t_A, t_C are t_E represent time instances at the beginning of the impact, end of compression and end of expansion. Here, $\dot{\mathbf{g}}$ is the relative velocity vector in the normal and tangential direction, which can be stated in terms of the generalized velocity vector $\dot{\mathbf{q}}$

$$\dot{g}_{NCj} = \mathbf{w}_{Nj}^T(\dot{\mathbf{q}}_C - \dot{\mathbf{q}}_A) + \dot{g}_{NAj}, \ \dot{g}_{TCj} = \mathbf{w}_{Tj}^T(\dot{\mathbf{q}}_C - \dot{\mathbf{q}}_A) + \dot{g}_{TAj}, \ j = 1,2,3. \tag{4.7}$$

The computations are carried out for the same data considered in [12]: $m = 0.0003$ kg, $J = 5\times10^{-9}\,\text{kgm}^2$, $m_1 = 0.0045$ kg, $J_1 = 7\times10^{-7}\,\text{kgm}^2$, $c = 0.0056$ Nm, $g = 9.81\text{m/s}^2$, $r_0 = 0.0025$ m, $r_1 = 0.0031$ m, $h_1 = 0.0058$ m, $l_1 = 0.01$ m, $l_2 = 0.015$ m, $h = 0.02$ m, $l = 0.0201$ m. The contact is characterized by the following coefficients of restitution $\varepsilon_{N1} = 0.5$, $\varepsilon_{N2} = \varepsilon_{N3} = 0$ and $\varepsilon_{T1} = \varepsilon_{T2} = \varepsilon_{T3} = 0$, defined as ration of the relative velocities after and before the impact (Newton law) (Stănescu, Munteanu, Chiroiu and Pandrea [14]).

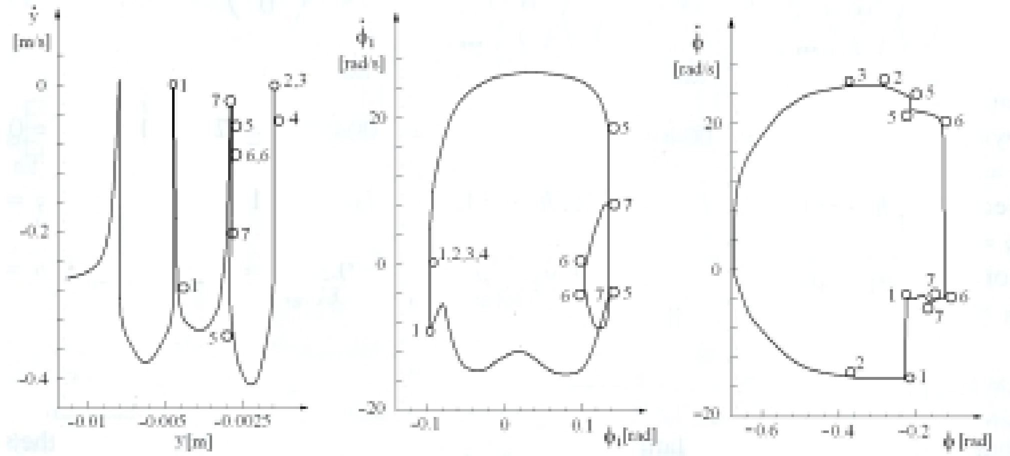

Fig. 4.1. Phase space portraits.

The phase portraits are shown in fig. 4.1. At point 1 the lower edge of the sleeve hits the pole. At point 2 the sleeve has a sliding and then a sticking. When the woodpecker reaches point 3, the tangential constraint is passive and the sleeve slides to point 7 where the contact is lost. At point 5 the upper edge of the sleeve hits the pole. Point 6 is an elastic impact of the beak to the pole. At point 7 the upper edge of the sleeve hits the pole and has a separation. The results are similar to those of Glocker and Pfeiffer [12], with the difference that the impact is without friction.

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