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CONTROL SYNTHESIS FOR ELECTROHYDRAULIC SERVOS WITH PARAMETRIC UNCERTAINTY

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Abstract: This paper continues recent research of authors, considering the control synthesis in the presence of a parametric uncertainty, with application to electrohydraulic servos actuating primary flight controls. The uncertain parameter is adjusted during the control process, using in synthesis the methods of Control Lyapunov Functions and backstepping. The obtained control law, containing a dynamic updating of uncertain parameter, renders the closed loop system stable and guarantees asymptotic tracking of position references. Numerical simulations are reported from viewpoint of servo time constant performance.

Key words: nonlinear control synthesis, backstepping, uncertain parameters, Control Lyapunov Function, Barbalat's Lemma, electrohydraulic servo.

1. INTRODUCTION

The present paper continues the theme of recent researches of the authors [1-4], by detailing procedure of adaptive backstepping control synthesis for the Electrohydraulic Servos (EHSs). This procedure of adaptive backstepping is introduced *avant la lettre* by Kanellakopoulos *et al* [5], where a characterization of the class of nonlinear systems to which the new adaptive scheme is applicable is achieved. In our paper, an alternative scheme to that described in Kanellakopoulos *et al.* [5] is proposed for an electrohydraulic servo with unknown or uncertain parameters. So, instead of Kanellakopoulos's adaptive scheme, herein a dynamic update for uncertain parameter is performed during the controlled proces – in other words, on line – in the framework of a recurrent control law based on the Control Lyapunov Functions (CLFs) and backstepping synthesis. The estimation error is proved to be asymptotically stable. The obtained control law renders the closed loop stable and ensures the regulation of the desired output.

Remembering [5], [1], the key idea of backstepping is simple. At every step of backstepping, a new Control Lyapunov Function is constructed by augmentation of the CLF from the previous step by a term which penalizes the error between a state variable and its desired value. A major advantage of backstepping is the construction of a Lyapunov function whose derivative can be made negative definite by a variety of control laws rather than a specific control law. It is obviously that the easy of incorporating uncertainties and unknown parameters with each backstepping contributed to its instant popularity and rapid acceptance.

2. BACKSTEPPING ADAPTIVE CONTROL SYNTHESIS FOR AN ELECTROHYDRAULIC SERVO

In a previous work [4], by using standard backstepping technique, position and force control laws were synthesised for a five-dimensional mathematical model of EHS (see modelling aspects in [6], [7]):

$$\dot{x}_1 = x_2, \; \dot{x}_2 = \frac{1}{m}\left[-kx_1 - fx_2 + S(x_3 - x_4)\right]$$
$$\dot{x}_3 = \frac{B}{V + Sx_1}\left[cx_5\sqrt{p_a - x_3} - Sx_2 - k_\ell(x_3 - x_4)\right], \; \dot{x}_4 = \frac{B}{V - Sx_1}\left[-cx_5\sqrt{x_4} + Sx_2 + k_\ell(x_3 - x_4)\right] \quad (1)$$
$$\dot{x}_5 = \frac{-x_5 + k_v u}{\tau}, \; c := c_d w\sqrt{\frac{2}{\rho}}$$

The state variables are denoted by: x_1 [cm] – EHS load displacement; x_2 [cm/s] – EHS load velocity; x_3 and x_4 are the pressures p_1, p_2 in the cylinder chambers, and x_5 stands for the valve position. The differential equations governing the dynamics of the EHS are those given in [6] and are reported having as a reference point the hydromechanical servomechanism SMHR included in the aileron control chain of Romanian military jet IAR 99. The nominal values of the parameters appearing in equations (1) are: $m = 0.033$ daNs^2/cm – equivalent inertial load of primary control surface reduced at the EHS's rod; $S = 10$ cm^2 – EHS's piston area; $f = 3$ daNs/cm – equivalent viscous friction force coefficient; $k = 100$ daN/cm – equivalent aerodynamic elastic force coefficient; $w = 0.05$ cm – valve-port width; $p_a = 210$ daN/cm^2 – supply pressure; $k_\ell = 5/210$ cm^5/(daN×s) – internal leakage cylinder's coefficient; $\rho = 85/(981\times10^5)$ daNs^2/cm^4 – volumetric density of oil; $c_d = 0.63$ – volumetric flow coefficient of the valve port; $k_c = 30/12\,000$ cm^5/daN $(:= V/(2B))$ – coefficient involving the bulk modulus B of the oil used and the EHS's cylinder semivolume V; $k_v = 0.0085/(0.05\times10)$ cm/V – valve displacement/voltage coefficient; τ = 1/573 s – time constant of the (servo)valve. The valve dynamics is evaded in the mathematical model (1); a proportionality coefficient k_v between the control (input voltage to servovalve) and valve displacement was considered.

The following result [2], [4] describes the structure of the backstepping position control law:

Proposition 1. *Let* $k_1 > 0, \; k_2 > 0$ *be tuning parameters. Under the* (*rather physical*) *assumptions* $0 < x_3 < p_a, \; 0 < x_4 < p_a, \; |x_1| < V/S$, *the control u given by*

$$u = \frac{1}{k_v}\left[x_{5d} + \tau\left(\dot{x}_{5d} - k_2 g_2 e_p\right)\right] \quad (2)$$

$$x_{5d} = -\frac{1}{G_2}\left(k_1 e_p - \dot{p}_d + G_1\right) \quad (3)$$

$$G_1 := G_1(x_1, x_2) := \frac{-2BV(Sx_2 + k_\ell p)}{V^2 - S^2x_1^2}, \; G_2 := G_2(x_1, x_2, x_3) := Bc\left(\frac{\sqrt{p_a - x_3}}{V + Sx_1} + \frac{\sqrt{x_4}}{V - Sx_1}\right) \quad (4)$$

$$e_p := p - p_d, \; p := x_3 - x_4, \;\; p_d(t) := \frac{k}{S}x_{1d}(t) := \frac{k}{S}x_{1s}\left(1 - e^{-t/t_{1r}}\right), \; e_5 := x_5 - x_{5d} \quad (5)$$

when applied to (1), *guarantees asymptotic stability for the position tracking error* $e_1 := x_1 - x_{1d}$; *more precisely,* $\lim_{t\to\infty} e_1(t) = 0$.

The basic backstepping assumes the knowledge of the system parameters. In fact, frequently it may be necessary to *identify* some of these parameters off line or *estimate them using on-line adaptive schemes*. The essence of adaptive control is that, by *learning from the past information* through this parameter adaptation mechanism, the real parametric uncertainty can be evaded. To illustrate the adaptive backstepping machinery, let consider the mathematical model (1). In these equations it is assumed the uncertainty of the coefficient c enclosed in the mixed parameter Bc:

$$\alpha := Bc\,. \tag{6}$$

For the sake of simplicity, one neglects the coefficient k_ℓ of internal leakage. Thus, the pressure equations will be rewritten in the form

$$\dot{x}_3 = \frac{\alpha x_5\sqrt{p_a - x_3}}{V + Sx_1} - \frac{BS}{V + Sx_1}x_2, \quad \dot{x}_4 = \frac{-\alpha x_5\sqrt{x_4}}{V - Sx_1} + \frac{BS}{V - Sx_1}x_2 \tag{1'}$$

and will be added to the other equations of the system (1)

$$\dot{x}_1 = x_2,\ \dot{x}_2 = \frac{1}{m}\left[-kx_1 - fx_2 + S(x_3 - x_4)\right],\ \dot{x}_5 = \frac{-x_5 + k_v u}{\tau} \tag{1''}$$

to define a system whose equations belong to a general class of nonlinear systems treated in [5]

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_0(\boldsymbol{x}) + \sum_{i=1}^{p}\theta_i \boldsymbol{f}_i(\boldsymbol{x}) + \left(\boldsymbol{g}_0(\boldsymbol{x}) + \sum_{i=1}^{p}\theta_i \boldsymbol{g}_i(\boldsymbol{x})\right)u \tag{7}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control vector, $\boldsymbol{f}_0, \boldsymbol{g}_0, \boldsymbol{f}_i, \boldsymbol{g}_i$ are smooth vector fields of appropriate dimensions and $[\theta_1, \dots, \theta_p]^T$ is the vector of unknown (uncertain) constant parameters. Two geometric conditions, so called: a) feedback linearization condition and b) parametric-pure-feedback condition must be fulfilled by the vector fields $\boldsymbol{f}_0, \boldsymbol{g}_0, \boldsymbol{f}_i, \boldsymbol{g}_i$. The geometric approach therein developed, not very familiar to most control engineers, is avoided in the present paper, instead using a simple, intuitive scheme of adaptive backstepping having as object the system (1'), (1"), (6), which represents the electrohydraulic servo as a tracking system. Therefore, for this system the aim of control synthesis is to have a good tracking by the state variable x_1 of the specified x_{1d} desired position references. The closed loop performance of the system can be measured by the *actual* (*realised*) *servo time constant* τ_s. Thus, a good tracking system is characterised by fast (little) time constant τ_s. Both servo time constant and position reference signal are in connection with the response of a first order system to step inputs x_{is}

$$x_{1d} = x_{1s}\left(1 - e^{-t/t_{1r}}\right) \tag{8}$$

x_{1s} stands for stationary value of the state x_1, and t_{1r} stands for associated desired time constants.

The main result of this work is given by the following

Proposition 2. *Consider the uncertain parameter* α *for the EHS mathematical model* (1'), (1"), (6). *Let* $k_1 > 0$, $k_2 > 0$, $k_3 > 0$, $\rho_\alpha > 0$ *be tuning parameters and* $\hat{\alpha} \neq 0$ *the notation for the estimate of the uncertain parameter* α. *Define the* ***learning error*** $\tilde{\alpha} := \alpha - \hat{\alpha}$. *Under the rather physical assumptions* $|x_1| < V/S$, $0 < x_3 < p_a$, $0 < x_4 < p_a$, *the control u given by*

$$u = \frac{1}{k_v}\left[x_{5d} + \tau\left(\dot{x}_{5d} - \hat{\alpha}k_2 g_2' e_p\right)\right] \tag{9}$$

$$x_{5d} = -\frac{1}{\hat{\alpha}g_2'}\left(k_1 e_p - \dot{p}_d + g_1\right) \tag{10}$$

$$\dot{\hat{\alpha}} = \frac{1}{\rho_\alpha}\left(g_2' x_5 e_p + k_3\tilde{\alpha}\right), \dot{\tilde{\alpha}} = -\frac{1}{\rho_\alpha}\left(g_2' x_5 e_p + k_3\tilde{\alpha}\right) \tag{11}$$

$$e_p := p - p_d \,,\; p := x_3 - x_4 \,,\;\; p_d(t) := \frac{k}{S} x_{1d}(t) := \frac{k}{S} x_{1s}\left(1 - e^{-t/t_{1r}}\right),\; e_5 := x_5 - x_{5d} \tag{12}$$

$$g_1(x_1, x_2) := \frac{-2BSVx_2}{V^2 - S^2 x_1^2}, \quad g_2(x_1, x_3, x_4) := \alpha\left(\frac{\sqrt{p_a - x_3}}{V + Sx_1} + \frac{\sqrt{x_4}}{V - Sx_1}\right)$$
$$g_2'(x_1, x_3, x_4) := \frac{g_2(x_1, x_3, x_4)}{\alpha}. \tag{13}$$

when applied to (1'), (1''), (6), *guarantees asymptotic stability for the learning error* $\tilde{\alpha}$ *and the position tracking error* $e_1 := x_1 - x_{1d}$; *more precisely,* $\lim_{t\to\infty} \tilde{\alpha}(t) = 0$, $\lim_{t\to\infty} e_1(t) = 0$.

Proof: By inspecting the system (1'), (1''), (6), it follows that the internal states x_1 and x_2 are stable; indeed, the roots of the characteristic equation

$$m\lambda^2 + f\lambda + k = 0 \tag{14}$$

are stable roots – negative real, or complex with negative real parts – due to the viscous friction force in hydraulic cylinder. Therefore, a special care to stabilise the states x_1, x_2 is not necessary. Thus, evading the equations for the states x_1, x_2 in the backstepping procedure, this technique will be applied only with regard to the variables $x_3 - x_4$ and x_5. Consider now the Lyapunov like function

$$V_1 = \frac{1}{2} e_p^2 + \frac{1}{2k_2} e_5^2 + \frac{1}{2} \rho_\alpha \tilde{\alpha}^2 . \tag{15}$$

Then, its derivative along the system (1'), (1'') is

$$\dot{V}_1 = e_p\left(g_2 x_5 + g_1 - \dot{p}_d\right) + \frac{e_5}{k_2}\left(\frac{-x_5 + k_v u}{\tau} - \dot{x}_{5d}\right) - \rho_\alpha \tilde{\alpha}\dot{\hat{\alpha}}.$$

Now, by using (12), (13), (10), (11), we have

$$e_p\left(g_2 x_5 + g_1 - \dot{p}_d\right) = e_p\left[g_2\left(e_5 + x_{5d}\right) + g_1 - \dot{p}_d\right] = e_p\left(\alpha g_2' e_5 + \alpha g_2' x_{5d} + g_1 - \dot{p}_d\right) =$$
$$e_p\left[\alpha g_2' e_5 + g_1 - \dot{p}_d + \frac{\alpha}{\hat{\alpha}}\left(-g_1 + \dot{p}_d - k_1 e_p\right)\right] = e_p\left[\alpha g_2' e_5 + g_1 - \dot{p}_d + \left(1 + \frac{\tilde{\alpha}}{\hat{\alpha}}\right)\left(-g_1 + \dot{p}_d - k_1 e_p\right)\right] =$$
$$e_p\left[\alpha g_2' e_5 - k_1 e_p + \frac{\tilde{\alpha}}{\hat{\alpha}}\left(-g_1 + \dot{p}_d - k_1 e_p\right)\right] = -k_1 e_p^2 + \alpha g_2' e_p e_5 + \tilde{\alpha}\left[\frac{e_p}{\hat{\alpha}}\left(-g_1 + \dot{p}_d - k_1 e_p\right)\right].$$

Thus

$$\dot{V}_1 = -k_1 e_p^2 + \alpha g_2' e_p e_5 + \tilde{\alpha}\left[\frac{e_p}{\hat{\alpha}}\left(-g_1 + \dot{p}_d - k_1 e_p\right)\right] + \frac{e_5}{k_2}\left(\frac{-x_5 + k_v u}{\tau} - \dot{x}_{5d}\right) - \rho_\alpha \tilde{\alpha}\dot{\hat{\alpha}}.$$

By substituting (11)–(13), one gets

$$\dot{V}_1 = -k_1 e_p^2 - \frac{1}{k_2 \tau} e_5^2 + \tilde{\alpha}\left[\frac{e_p}{\hat{\alpha}}\left(-g_1 + \dot{p}_d - k_1 e_p\right) + g_2' e_p e_5 - \rho_\alpha \dot{\hat{\alpha}}\right] = -k_1 e_p^2 - \frac{1}{k_2 \tau} e_5^2 - k_3 \tilde{\alpha}^2 . \tag{16}$$

The equations for the errors e_p, e_5 can be written as

$$\dot{e}_p = -k_1 e_p + g_2 e_5 + \frac{\tilde{\alpha}}{\hat{\alpha}}\left(-g_1 + \dot{p}_d - k_1 e_p\right), \quad \dot{e}_5 = -\frac{e_5}{\tau} - \hat{\alpha} k_2 g_2' e_p, \quad \dot{\tilde{\alpha}} = -\frac{1}{\rho_\alpha}\left(k_3 \tilde{\alpha} + g_2' x_5 e_p\right). \tag{17}$$

A tedious way to continue the proof is that of checking the asymptotic stability of the errors $e_p, e_5, \tilde{\alpha}$ by using the second method of Lyapunov for systems with variable coefficients (see [8], Theorem 1). As in [4], an alternative and very efficient procedure will be further used: that of Barbalat's Lemma [9]. The reasoning is as follows. Making use of the definitions (11)–(13) for e_p, e_5 and $\tilde{\alpha}$, we have $V_1(0) > 0$ when $t \to 0$ (see $x_{5d} \neq 0$). Since $\dot{V}_1 \leq 0$, it is obvious that $0 \leq V_1(t) \leq V_1(0)$, $(\forall) t > 0$, hence the positive function $V_1(t)$ is bounded and consequently e_p, e_5 and $\tilde{\alpha}$ are bounded; so, $p = p(t)$ is also bounded in the interval $\mathbb{R}_+ = [0, \infty)$. Now, taking the derivative of (16) yields

$$\ddot{V}_1 = 2\left(k_1^2 e_p^2 + \frac{1}{k_2 \tau^2} e_5^2\right) - 2k_1 g_2 e_p e_5 + \frac{2}{\tau}\hat{\alpha} g_2' e_p e_5 + \frac{2k_1 \tilde{\alpha}}{\hat{\alpha}} e_p\left(k_1 e_p - \dot{p}_d + g_1\right) + \frac{2k_3}{\rho_\alpha}\tilde{\alpha}\left(g_2' x_5 e_p + k_3 \tilde{\alpha}\right).$$

Furthermore, $\ddot{V}_1$ is bounded, provided that $g_1, g_2, \hat{\alpha}$ remain bounded during the dynamical process; this condition holds, having in view the assumptions involving the variables x_1, x_3, x_4. So, $\dot{V}_1$ is uniformly continuous (as having a bounded derivative). Let us now consider Barbalat's Lemma:

If the function f(t) is differentiable and has a finite limit $\lim_{t\to\infty} f(t)$, *and if* $\dot{f}$ *is uniformly continuous, then* $\lim_{t\to\infty} \dot{f}(t) = 0$.

Thus, Barbalat's Lemma will be applied to show that the errors e_p and e_5 tend to zero as time tends to infinity. Indeed, applying Barbalat's Lemma, $\dot{V}_1 \to 0$. Hence, e_p and e_5 tend to zero. Now, let's look at the second equation in (1"), which can be rewritten as follows

$$\ddot{x}_1 + 2h\dot{x}_1 + r^2 x_1 = p_1, \quad h := \frac{f}{2m}, \; r := \sqrt{\frac{k}{m}}, \; p_1 = \frac{S}{m} p \tag{18}$$

and p is now seen as a bounded function of t, $p := p(t) = x_3(t) - x_4(t)$. The most usual case is that of complex roots with negative real parts, considered in the precedent works [1-4]. But, the occurrence of negative real roots is not excluded. With initial conditions $x_1(0) = \dot{x}_1(0) = 0$, the solution of (18) in this aperiodic case is

$$x_1(t) = \frac{1}{2q} e^{(q-h)t} \int_0^t e^{(h-q)u} p_1(u) \mathrm{d}u - \frac{1}{2q} e^{-(q+h)t} \int_0^t e^{(h+q)u} p_1(u) \mathrm{d}u \tag{19}$$

with $q^2 = r^2 - h^2 > 0, q, h, r, h - q$ positive. This variant is inherent to hydraulic servo systems owing to small viscous friction in cylinder. Define

$$p_{1d}(t) := \frac{S}{m} p_d(t) \tag{20}$$

and let us also consider

$$\tilde{x}_{1d}(t) := \frac{1}{2q} e^{(q-h)t} \int_0^t e^{(h-q)u} p_{1d}(u) du - \frac{1}{2q} e^{-(q+h)t} \int_0^t e^{(h+q)u} p_{1d}(u) du. \tag{21}$$

Since $e_p \to 0$ when $t \to \infty$, it is clear that $p_1(t) \to p_{1d}(t)$, as $t \to \infty$; this means: $(\forall) \varepsilon > 0$, $(\exists)\, \delta(\varepsilon)$ such that for $t > \delta(\varepsilon)$ we have $\left|p_1(t) - p_{1d}(t)\right| < \varepsilon$. Then, if

$$\left|x_1(t)-\tilde{x}_{1d}(t)\right| \le \frac{1}{2q} e^{(q-h)t} \int_0^t e^{(h-q)u} \left|p_1(t)-p_{1d}(t)\right| \mathrm{d}u + \frac{1}{2q} e^{-(q+h)t} \int_0^t e^{(h+q)u} \left|p_1(t)-p_{1d}(t)\right| \mathrm{d}u \le$$

$$\frac{\varepsilon}{2q}\left(e^{(q-h)t} \int_0^t e^{(h-q)u} \mathrm{d}u + e^{-(q+h)t} \int_0^t e^{(h+q)u} \mathrm{d}u\right) = \frac{\varepsilon}{2q}\left(\frac{1-e^{(q-h)t}}{h-q} + \frac{1-e^{-(q+h)t}}{h+q}\right),$$

$t > \delta(\varepsilon)$

Therefore

$$x_1(t) \to \tilde{x}_{1d}(t) \text{ as } t \to \infty . \tag{22}$$

Then:

$$\tilde{x}_{1d} = \frac{k x_{1s}}{2qm}\left[e^{(q-h)t} \int_0^t e^{(h-q)u}\left(1-e^{-u/t_{1r}}\right) \mathrm{d}u - e^{-(q+h)t} \int_0^t e^{(h+q)u}\left(1-e^{-u/t_{1r}}\right) \mathrm{d}u\right]$$

and, based on definition relations

$$q^2 = h^2 - r^2, \quad h = f/2m, \quad r^2 = k/m$$

simple, successive calculations finally give

$$\tilde{x}_{1d}(t) \to x_{1s} \text{ as } t \to \infty . \tag{23}$$

Thus, from (22) and (23), a standard proceeding gives

$$x_1(t) \to x_{1s} \text{ as } t \to \infty \tag{24}$$

and so ends the proof.

Let notice that if $\tilde{\alpha} = 0$, the control (9) is identical with the “nonadaptive” control given in [2], [4].

3. SIMULATION RESULTS AND CONCLUDING REMARKS

It is was mentioned in Section 2 that the goal of the control is to ensure a good tracking of the specified x_{1d} desired position references, for instance as given in (8). Simulations are used to illustrate the theoretical findings. Thus, fourth order Runge-Kutta integrations of the tandem **controlled system** (1'), (1''), (6) – **compensator** (9)-(13) were performed considering a succession of time intervals of length T = 0.002 s and “zero order hold control” (which means that the control variable is kept constant during the time period T). Numerical data are those already given in Section 2. For the simulations, two sets of values of the tuning parameters were selected as suitable (these values are naturally related to the system of units chosen in Section 2) : 1) $k_1 = 25$, $k_2 = 0.00004$, $k_3 = 0.00000038$, $\rho_\alpha = 0.025 \times k_3$, with reference signal $x_{1s} = 0.255$ cm and $t_{1r} = 0.018$ s (the case of Fig. 1), and 2) $k_1 = 24.5$, $k_2 = 0.00005$, $k_3 = 0.000044$, $\rho_\alpha = 0.023 \times k_3$, with reference signal $x_{1s} = 1$ cm and $t_{1r} = 0.015$ s (the case of Fig. 2). Number of time intervals T in Figures is 150, respectively, 166. Thus, the presented plots show a good working of the proposed adaptive backstepping controller. Certainly, an increased x_{1s} requires a decreased claim on actual servo time constant τ_s. The nominal (i.e., corresponding to $x_{1s} = 0.255$ cm ; see [4]) actual servo time constant $\tau_s = 0.038$ s is very close to the actual servo time constant of the ideal “nonadaptive” system (i.e., for total parameter knowledge), $\tau_s = 0.037$ s [2], [4]. The Figures also show that the estimation error converges to zero. Performing various numerical simulations demonstrated that the initial parameter estimate $\hat{\alpha}(0)$ doesn’t affect the estimation process. Moreover, the servo time constant $\tau_s = 0.038$ s – the main performance criterium of the system – is more or less maintained even when the uncertain parameter B was changed from the decreased value 1000 daN/cm^2 up to the increased value 7300 daN/cm^2.

Taking into account state and control limitations, the global asymptotic stability of tracking errors in Proposition 2 cannot be stipulated. However, suitable input signal x_{1d} and tuning parameters $k_1 > 0$, $k_2 > 0$, $k_3 > 0$, $\rho_\alpha > 0$ can be chosen to preserve these constraints. Specifically, the physical constraint of control saturation $0 < u < 10$ V was considered during the system integration.

This paper is naturally connected with and – and ends – the series of works [1]-[4], wherein the backstepping machinery, Control Lyapunov Functions and Barbalat's lemma – powerfull tools of the nonlinear control [11] – has been applied or adapted for some mathematical models of a servoactuator largely used in most industries and especially in aerospace applications. Now the adaptive backstepping is used in providing control law for an EHS five-dimensional mathematical model with uncertain or unknown parameter. A complicated geometrical approach described in [5] is so avoided, instead promoting a relatively simple, intuitive scheme of adaptive backstepping. Worthy noting, the tracking of input references and the error estimation were proven to be asymptotical. The full state information was considered available.

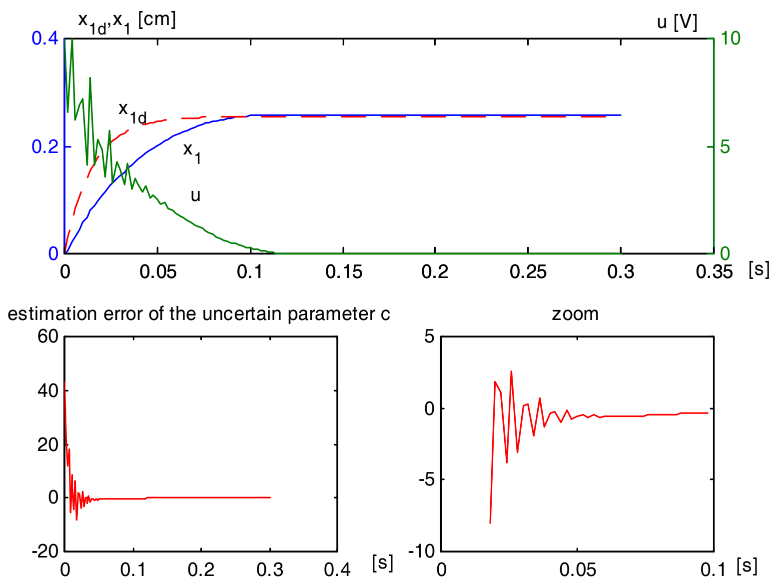

Fig. 1

Response to step reference signal x_{1d} defined by $x_{1s} = 0.255$ cm, $t_{1r} = 0.015$ s. Initial estimate of uncertain parameter $\hat{\alpha}(0) = 0.1Bc$ (initial estimation error of the parameter c: 43.07). Plots of variables x_{1d}, x_1, u and $\tilde{\alpha}/B$. Actual servo performance: $\tau_s \cong 0.038$s.

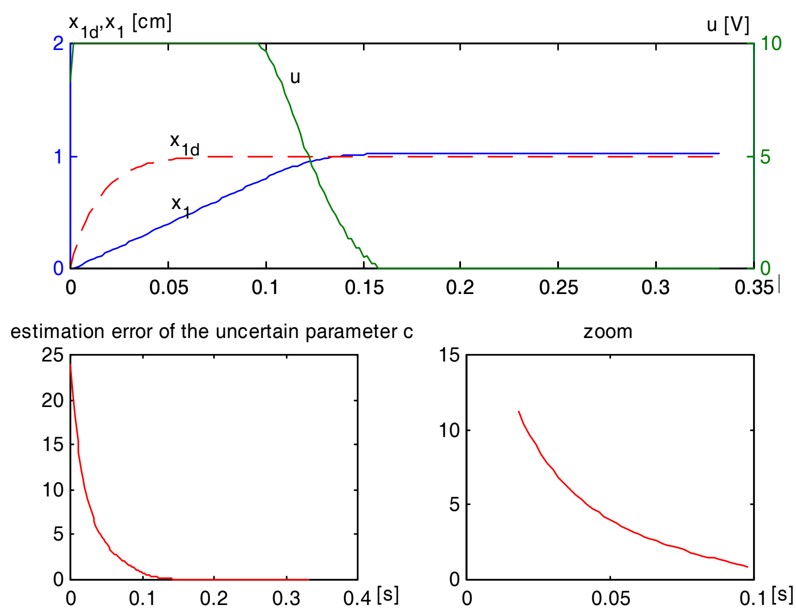

Fig. 2

Response to step reference signal x_{1d} defined by $x_{1s} = 1$ cm, $t_{1r} = 0.084$ s. Initial estimate of uncertain parameter $\hat{\alpha}(0) = 0.5Bc$ (initial estimation error of the parameter c: 23.93). Plots of variables x_1, u and $\tilde{\alpha}/B$. Actual servo performance $\tau s \cong 0.084$ s.

Table 1 Tandem system-controller of Fig. 1: robustness of performance to parameter variations

parameter – system of units	nominal value	maximal admissible value	maximal admissible variation (%)	minimal admissible value	minimal admissible variation (%)
$c - \mathrm{cm}^3\mathrm{s}^{-1}\mathrm{daN}^{-1/2}$	47.85	50.25	*5*	26.32	*-45*
$m - \mathrm{daNs}^2\mathrm{cm}^{-1}$	0.033	0.038	*15*	0.001	*-96.97*
$f - \mathrm{daNscm}^{-1}$	3	4	*33*	2.25	*25*
$k - \mathrm{daNcm}^{-1}$	100	500	*400*	50	*-50*
$V - \mathrm{cm}^3$	30	75	*150*	27	*-10*
$S - \mathrm{cm}^2$	10	18	*80*	9.3	*-7*
$B - \mathrm{daNcm}^{-2}$	6000	7400	*23.33*	1000	*-83.33*
$p_a - \mathrm{daNcm}^{-2}$	210	220.5	*5*	84	*-60*
$k_v - \mathrm{cmV}^{-1}$	0.017	0.0192865	*13.45*	0.0051	*-70*
$\tau - \mathrm{s}$	1/573	0.0020506	*17.5*	3.490401×10^{-4}	*-80*

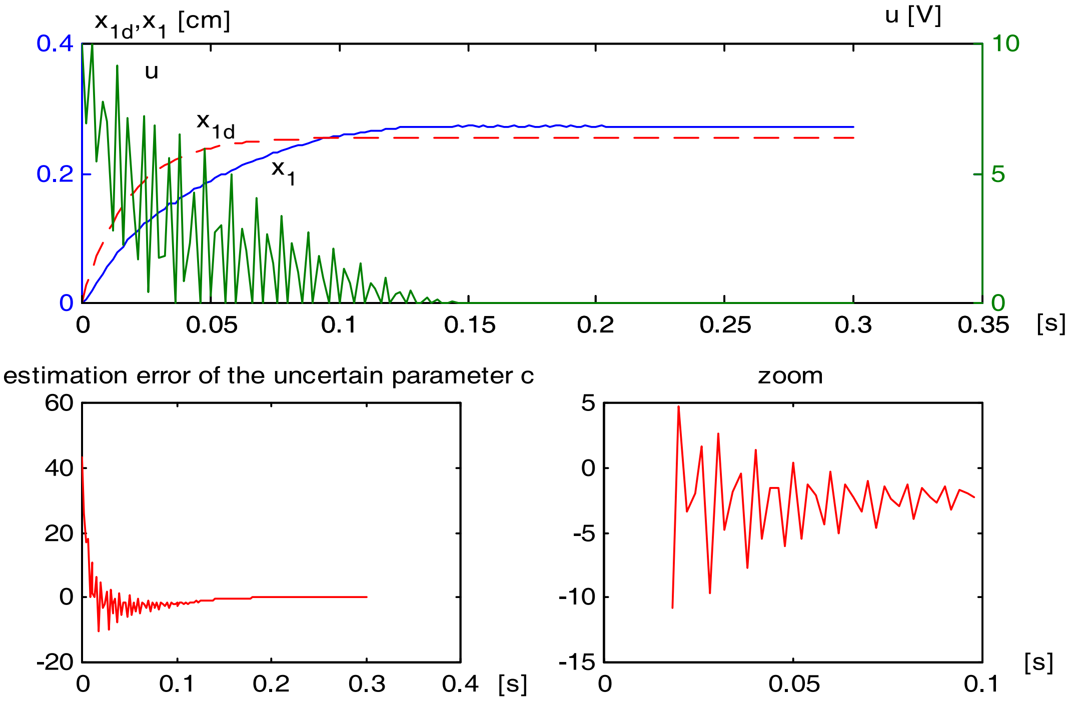

Fig. 3 – Robustness of system in Fig. 1 versus τ maximal admissible value: performance worsened somewhat, $\tau_s \cong 0.042$ s.

The simulation studies validate the theoretical results. Furthermore, a close correspondence between these new findings and our recent results [2-4] has been found. But, in the final, we note a neglected feature in the literature of the field: the fitted key parameters of bakstepping, $k_1 > 0$, $k_2 > 0$, $k_3 > 0$, $\rho_\alpha > 0$, do not ensure a large parameter robustness of the performance, see Table 1. The „admissible values" in the Table refer to that parameters limits which do not irremediable compromise the actual servo time constant and, in fact, the stable response of the system to the specified step input; see Fig. 3 for the case of considered maximal admissible value of τ : 0.0020506 . The sensitivity to some system parameters – c, S, p_a – is to be underlined. Thus, future works need to pay attention to these robustness aspects of the backstepping controllers.

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