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SPHERICITY ERROR EVALUATION BASED ON 3D OPTICAL MEASUREMENT

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Sphericity is an important geometrical feature in modern industry. Most of the former sphericity evaluation is studied based on 2D measurement, because 3D measurement is complex and with high cost. But it is reported that 2D sphericity evaluation method is unreasonable. In order to evaluate sphericity error accurately and effectively, this paper proposes novel 3D sphericity error evaluation method based on SFS technique. If a sphere is placed under SFS vision system, its 3D shape will be recovered from 0° to 180° only by one sphere image. Each pixel in the image represents a 3D data in the sphere. All these 3D data can be used in the sphericity evaluation procedure. Least-square method (LSM), minimum circumscribed sphere, maximum inscribed sphere, and minimum zone solution, which commonly used in sphericity evaluation, are all studied in this paper. Compared with the conventional sphericity evaluation based on 2D measurement, the proposed 3D evaluation has the merits of non-contact, non-destructive, low cost, high speed, and high reliable. 3D shape recovery of a sphere based on SFS and its 3D sphericity evaluation procedure will be analyzed in this paper.

Key words: SFS, LSM, minimum circumscribed sphere, maximum inscribed sphere, minimum zone

1. INTRODUCTION

Because of the rapid improvement of mechanical process, machining of precision spherical surface is in great demand. Moreover, the spherical surface used as measurement sensors, such as the standard sphere on the Coordinate Measuring Machine (CMM), should be more precise. In industry, spherical error (sphericity) has a considerable effect on rotating motion in many machines. Any defects such as surface roughness, waviness and form error on a spherical surface may result in the generation of a large amount of heat and in turn lead to wear and life reduction. Therefore, it is very important to evaluate sphericity error accurately and effectively.

In the ISO (3290) [1], sphericity deviation is measured in two or three equatorial planes at 90º to each other. The value of the sphericity is calculated using the minimum circumscribing circle method applied to the two or three profiles. All these profiles are mostly plotted by a contact circularity instrument, which is shown in Fig. 1. It is reported that a considerable difference exists between 2D and 3D evaluations of surface roughness [2]. This means that the determination of sphericity using 2D measurements is unreasonable. However, the former 3D measurement technology is complex and with high cost, so the ISO (1101) [3] does not deal with sphericity tolerance explicitly. Based on the concepts of 2D measurement of circularity and cylindricity tolerances dealt with in ISO (1101), the 2D sphericity evaluation methods based on LSM, minimum circumscribed sphere, maximum inscribed sphere, and minimum zone solution, are formulated. Some efforts have been devoted to evaluating the sphericity error using different minimum algorithm.

(1) Using discrete Chebyshev approximations, Danish [4] calculated the minimum zone solution for sphericity error;
(2) Kanada [5] computed the minimum zone sphericity using iterative least squares and the downhill simplex search methods;
(3) Fan and Lee [6] proposed an approach with minimum potential energy analogy to the minimum zone solution of spherical form error. And the problem of finding the minimum zone sphericity error is transformed into that of finding the minimum elastic potential energy of the corresponding mechanical system;

(4) Chen [7] constructed three mathematical models to evaluate the minimum circumscribed sphere, the maximum inscribed sphere and the minimum zone sphere by directly resolving the simultaneous linear algebraic equations first. Then, the minimum zone solutions can be obtained by comparing the solutions between the 4-1 models, the 1-4 models, the 3-2 models and the 2-3 models;
(5) Samuel [8] established the minimum circumscribed limacoid, maximum inscribed limacoid and minimum zone limacoid based on the computational geometry to evaluate sphericity error.
(6) With the emergence of computational intelligence, intelligence-oriented algorithms such as genetic algorithms (GAs) have been employed to evaluate form errors such as flatness, straightness, cylindricity, sphericity, [9] etc. Genetic algorithms based on the principles of population evolution have been theoretically and empirically proven to be robust for searching for solutions in complex spaces, and by using GAs the form errors will be evolved toward the optimal solution.
(7) XiuLan Wen [10] using immune algorithm (IA) for sphericity error evaluation. It is proven that IA is a favorable alternative for solving minimum zone sphericity error.

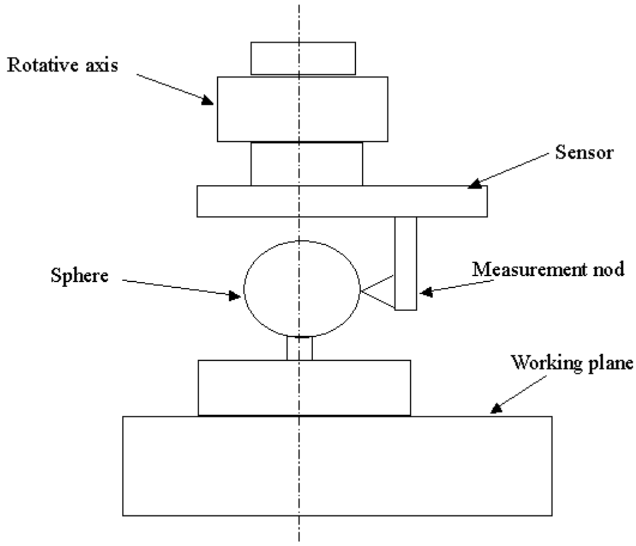

Fig. 1. Sketch of “circularity instrument”.

All the above sphericity evaluation method is based on 2D measurement data. It is unreasonable because sphere is a 3D object. So sphericity evaluation based on 3D measurement is in great demand. In the doctor thesis -“Research on 3D recovery based on only one online measuring image” [11], novel non-contact 3D measurement method-SFS is introduced. SFS is a visual inspection method, but different compared with other vision techniques because it can visualize the 3D-data of an object by measuring only one image. This image is captured by a CCD or other type of camera. The merits of SFS are easy operating, non-contact, non-destructive, low cost, high speed, and high precision. If a sphere is placed under a CCD that is in good measurement status, then the 3D data of the sphere can be recovered by SFS.

It is the purpose of this paper to propose novel 3D sphericity evaluation method, including commonly analyzing method such as LSM, minimum circumscribed sphere, maximum inscribed sphere, and minimum zone solution. These proposed evaluation method is based on SFS technique. Compared with 2D sphericity evaluation method, the proposed evaluation method isn’t using only two or three equatorial planes of a sphere to judge sphericity error, but using all the 3D data of the recovered sphere. The paper is organized as follows: 3D shape recovery of a sphere based on SFS; then 3D sphericity error evaluation method is proposed; the last is conclusion.

2. 3D SHAPE RECOVERY OF A SPHERE BASED ON SFS

In computer vision, the techniques to recover shape are called shape-from-X techniques, where X can be shading, stereo, motion, and texture. SFS deals with the recovery of shape from a gradual variation of shading in the image. To solve SFS problem, it is important to study how the images are formed. A simple model of image formation is the Lambertian model, in which the gray level at a pixel in the image depends on the light source direction and the surface normal. In SFS, given a gray level image, the aim is to recover the light source and the surface shape at each pixel in the image. However, real images do not always follow the Lambertian model, Even if we assume Lambertian reflectance. Therefore, it is difficult to find a unique solution to SFS; it requires additional constraints. In the doctor thesis [11], a good solution for SFS is introduced. This proposed SFS in [11] is to find the slant Φ and tilt θ of each pixel in the image. According to the relationship between slant-tilt and surface normal, the true depth of each pixel can be calculated.

Assume that the energy of the incident light is $I(x, y)$, the vector of the light source direction is (Φ_s, θ_s), the gray of each pixel in the image is E_i, the maximum gray in the image is E_{max}. Then the slant and tilt of each pixel can be got by formula (1) and (2).

$$\phi_i = \arccos \frac{E_i}{E_{\max}} \tag{1}$$

$$\theta_i = \arctan \frac{I_y \cos\theta_s - I_x \sin\theta_s}{I_x \cos\theta_s \cos\phi_s + I_y \cos\phi_s \sin\theta_s} \tag{2}$$

Assume that the normal of each point in the object is (n_{ix}, n_{iy}, n_{iz}), and then the normal can be got from Φ_i and θ_i by formula (3).

$$\begin{aligned} n_x &= \sin\phi_i \cos\theta_i \\ n_y &= \sin\phi_i \sin\theta_i \\ n_z &= \cos\phi_i \end{aligned} \tag{3}$$

When the entire surface normal is got, the shape of the recovered object can be plotted. In our experiment, one sphere is placed in the CCD-SFS system. Its single image is shown in Fig. 2(a), and the 3D recovery is shown in Fig. 2(b). Using these 3D sphere data, 3D sphericity evaluation can be exploited.

3. SPHERICITY ERROR EVALUATION

The sphericity evaluation based on 2D measurement is unreasonable because it only select part of data in the sphere that distributed on two or three circles. The selected data can't represent the overall sphere character. If the max deviation is not placed on the selected part of the sphere, then the sphericity evaluation can't reflect the true character of the sphere. In order to make sphericity evaluation more accurately, the evaluation based on 3D measurement is studied. Four commonly used evaluation method, such as LSM, minimum circumscribed sphere, maximum inscribed sphere, minimum zone, will be introduced in next part.

3.1 LSM sphere based on 3D measurement

For a given set of data points, $P_i\ (x_i, y_i, z_i)$ (i =1,2…n), if the square sum of the distance between P_i and a sphere H_l is least, then the sphere H_l is called LSM sphere. In 2D measurement, the set of data points $P_i\ (x_i, y_i, z_i)$ usually selected from two circles on the sphere, one is the equatorial circle, the other is vertical circle, which is shown in Fig. 3(a). The measurement of P_i is performed by "circularity instrument", which is shown in Fig. 1. The "circularity instrument" can plot any selected circle in the sphere on a paper, which can be shown in Fig. 3(b).

Assume that the image has M rows and N columns, and then the image size is $M \times N$. Because each pixel in the image represents a point in the object, then it will recover $M \times N$ 3D data of the object. In 3D sphericity evaluation, the selection of evaluation data P_i is not only located on two circle described in Fig. 3(a). Furthermore, all the 3D data in the sphere will be selected in the set of $P_i\ (x_i, y_i, z_i)$ (i =1,2… $M \times N$).

If the center of the LSM sphere is (a_l, b_l, c_l), and radius is R_l, then the function of LSM sphere can be described as formula (4).

$$f(a_1,b_1,c_1,R_1)=\min\sum_{i=1}^{M\times N}[\sqrt{(x_i-a_1)^2+(y_i-b_1)^2+(z_i-c_1)^2}-R_1] \tag{4}$$

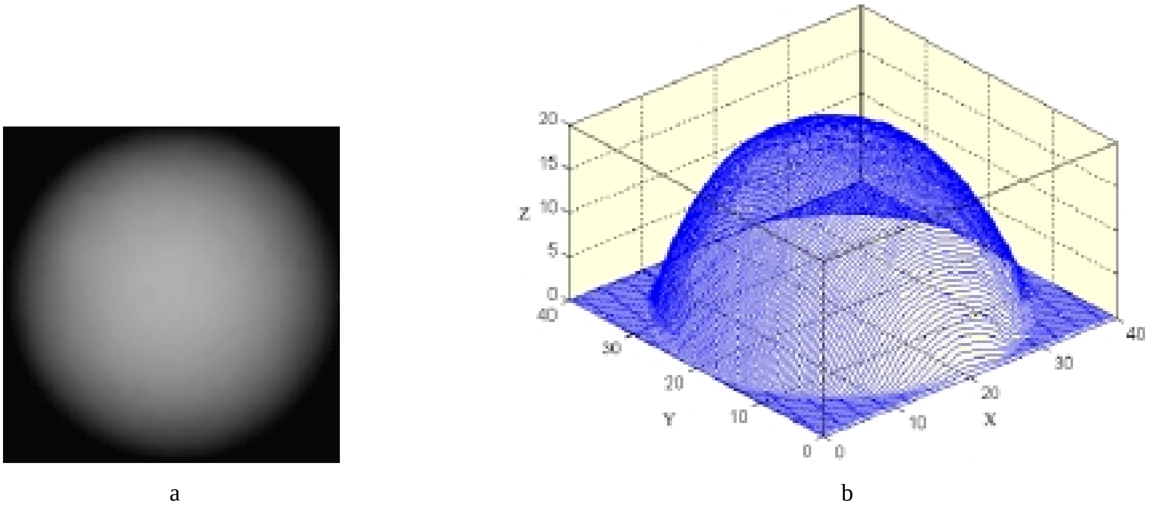

Fig. 2. 3 D recovery of a sphere based on one image; a) The single image of the sphere; b) The 3D recovery shape of the sphere.

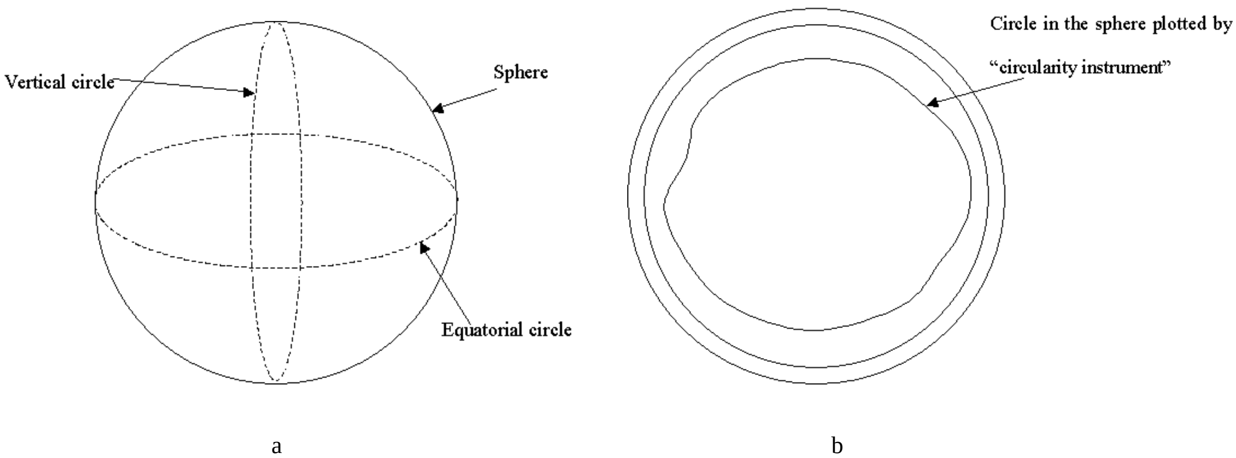

Fig. 3. Selection of two circles in the sphere based on 2D measurement; a) The selection position of the two circle; b)The selected circle plotted by "circularity instrument".

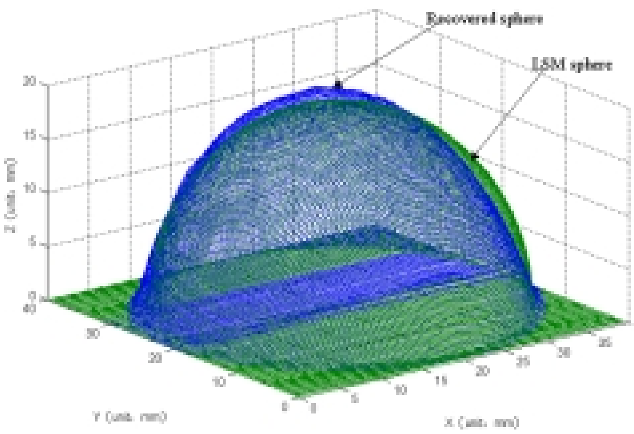

Fig. 4. The recovered sphere and the LSM evaluation sphere in the same coordinate.

In 3D space, X and Y coordinate is in scale with the dimension ($Row\times Col$) in the image. Because the 3D shape of a sphere is recovered from 0° to 180° from the single image, then Z coordinate value of the center is 0. So the searching of center (a_1, b_1, c_1, R_1) is replaced by the searching of $(Row, Col, 0, R_1)$. Plot the recovered sphere and the LSM evaluation sphere in the same coordinate can be shown in Fig. 4.

3.2 Minimum circumscribed sphere based on 3D measurement

For a given set of data points, P_i (x_i, y_i, z_i) (i =1,2…n), if all the data points P_i are inside region of a sphere H_2, and the radius of the sphere H_2 is minimum, then the sphere H_2 is called minimum circumscribed sphere.

Based on 3D measurement, all the 3D data in the sphere will be selected in the set of P_i (x_i, y_i, z_i) (i =1,2… $M \times N$). If the center of the minimum circumscribed sphere is (a_2, b_2, c_2), and radius is R_2, then the function of minimum circumscribed sphere can be described as formula (5).

$$f(a_2,b_2,c_2) = \underset{i=1}{\overset{M\times N}{MAX}} \sqrt{(x_i-a_2)^2+(y_i-b_2)^2+(z_i-c_2)^2} \qquad (5)$$

The plot of the recovered sphere and the minimum circumscribed sphere in the same coordinate is shown in Fig. 5.

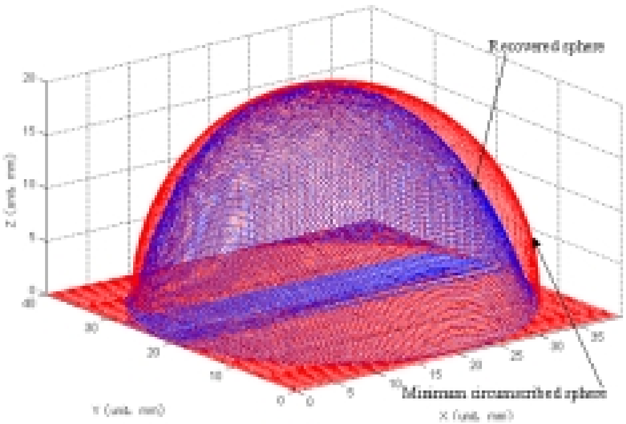

Fig. 5. The recovered sphere and the minimum circumscribed sphere in the same coordinate.

3.3 Maximum inscribed sphere based on 3D measurement

For a given set of data points, P_i (x_i, y_i, z_i) (i =1,2…n), if all the data points P_i are outside region of a sphere H_3, and the radius of the sphere H_3 is maximum, then the sphere H_3 is called maximum inscribed sphere.

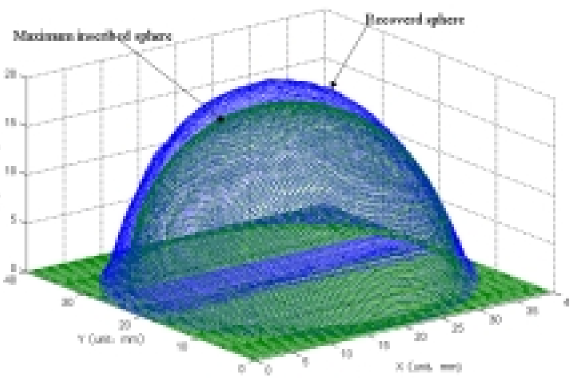

Fig. 6. The recovered sphere and the maximum inscribed sphere in the same coordinate.

Based on 3D measurement, all the 3D data in the sphere will be selected in the set of P_i (x_i, y_i, z_i) (i =1,2… $M \times N$). If the center of the maximum inscribed sphere is (a_3, b_3, c_3), and radius is R_3, then the function of maximum inscribed sphere can be described as formula (6).

$$f(a_3,b_3,c_3)=\underset{i=1}{\overset{M\times N}{MIN}}\sqrt{(x_i-a_3)^2+(y_i-b_3)^2+(z_i-c_3)^2} \tag{6}$$

The plot of the recovered sphere and the maximum inscribed sphere in the same coordinate are shown in Fig. 6.

3.4 Minimum zone solution based on 3D measurement

For a given set of data points, P_i (x_i, y_i, z_i) (i =1,2…n), if all the data points P_i are on or between two concentric spheres, the minimum radius separation between the two concentric spheres is called the minimum zone solution. It is reported that the minimum zone criterion yields more accurate fitting results than other criteria such as the least-square method (LSM) [10].

Based on 3D measurement, all the 3D data in the sphere will be selected in the set of P_i (x_i, y_i, z_i) (i =1,2… $M\times N$). If the center of the two concentric spheres is (a_4, b_4, c_4), and their radius are R_4 and R_5, then the function of maximum inscribed sphere can be described as formula (7).

$$\begin{aligned}
f(a_4,b_4,c_4,R_4)&=\underset{i=1}{\overset{M\times N}{MIN}}\sqrt{(x_i-a_4)^2+(y_i-b_4)^2+(z_i-c_4)^2}\\
f(a_4,b_4,c_4,R_5)&=\underset{i=1}{\overset{M\times N}{MAX}}\sqrt{(x_i-a_4)^2+(y_i-b_4)^2+(z_i-c_4)^2}\\
f(a_4,b_4,c_4,R_5)&=\min(R_4-R_5)
\end{aligned} \tag{7}$$

The plot of the recovered sphere and the minimum zone concentric sphere in the same coordinate are shown in Fig. 7.

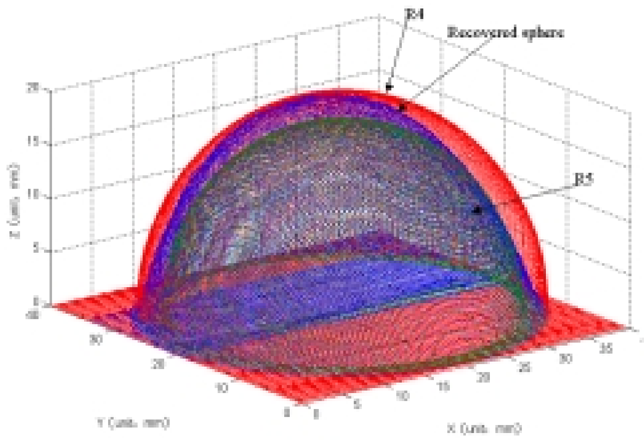

Fig. 7. The recovered sphere and the minimum zone concentric sphere in the same coordinate.

3.5 Experiment result of four sphericity evaluation method

In our experiment, the radius of the sphere being measured is 20 mm. Four 3D sphericity evaluation methods mentioned above are all adopted to evaluate the geometrical character. The sphericity error by four methods is shown in Table 1.

Table 1.
Sphericity error of four 3D sphericity evaluation method (unit: mm).

Methods	LSM	Minimum circumscribed sphere	Maxmum inscribed sphere	Minimum zone
Sphericity error	1.2	1.9	1.5	3.4

4. CONCLUSIONS

In this paper, four sphericity evaluation method based on 3D measurement are presented. LSM, minimum circumscribed sphere, maximum inscribed sphere, and minimum zone are all adopted to evaluate sphericity error. The acquirement of the 3D data is performed by SFS, which is a non-contact, non-destructive 3D shape recovery method. Compared with conventional sphericity evaluation based on 2D contact measurement, the proposed 3D sphericity evaluation method is more reliable, because all the 3D data in the sphere is used in the procedure of the sphericity evaluation. The merits of the 3D sphericity evaluation is as follows:

(1) It is a non-destructive evaluation method. The sensor of the 3D measurement is CCD, which just like our eyes, without any touching and destruction of the sphere.

(2) It is a non-contact evaluation method. Compared with “circularity instrument” mostly used in the former sphericity evaluation based on 2D measurement, the proposed method will not bring any impact on the sphere.

(3) It is a 3D sphericity evaluation method. It is more reliable than 2D evaluation method because it can use all the 3D data in the evaluation procedure.

(4) It is easy operated. When the sphere is located under CCD, the software will do all the evaluation work. No further operation by people is needed.

(5) It can be spread to other evaluation field, such as the 3D evaluation of linearity, cylindricity, etc.

(6) The evaluation result can be visualized. Because the evaluation work is based on 3D measurement, all the shape of the sphere and the concentric evaluation sphere can be plotted on the screen in time.

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