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A CLASS OF EXACT SOLUTIONS OF THE SYSTEM OF ISENTROPIC TWO-DIMENSIONAL GAS DYNAMICS

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A class of exact solutions of the system of isentropic two-dimensional gas dynamics is presented exhaustively. The elements of this class are characterized to be one-dimensional or multidimensional simple waves solutions or regular interactions of simple waves solutions.

1. INTRODUCTION

We consider in this paper the homogeneous quasilinear system

$$\sum_{j=1}^{n}\sum_{k=0}^{m} a_{ijk}(u)\frac{\partial u_j}{\partial x_k}=0,\quad 1\le i\le n \tag{1.1}$$

together with its concrete two-dimensional gasdynamic version

$$\begin{cases} \dfrac{\partial c}{\partial t}+v_x\dfrac{\partial c}{\partial x}+v_y\dfrac{\partial c}{\partial y}+\dfrac{\gamma-1}{2}c\left(\dfrac{\partial v_x}{\partial x}+\dfrac{\partial v_y}{\partial y}\right)=0 \\ \dfrac{\partial v_x}{\partial t}+v_x\dfrac{\partial v_x}{\partial x}+v_y\dfrac{\partial v_x}{\partial y}+\dfrac{2}{\gamma-1}c\dfrac{\partial c}{\partial x}=0 \\ \dfrac{\partial v_y}{\partial t}+v_x\dfrac{\partial v_y}{\partial x}+v_y\dfrac{\partial v_y}{\partial y}+\dfrac{2}{\gamma-1}c\dfrac{\partial c}{\partial y}=0 \end{cases} \tag{1.2}$$

corresponding to an isentropic description [in usual notations: c is the sound velocity, v_x, v_y are fluid velocities].

Teminology 1.1 [M.Burnat]. For the system (1.1) we say that at a point u^* of the hodograph space, a real vector $\overline{\kappa}$ is a *hodograph dual* of a real exceptional vector $\overline{\beta}$ defined at the point if this vector satisfies at u^* the duality condition:

$$\sum_{j=1}^{n}\sum_{k=0}^{m} a_{ijk}(u^*)\beta_k\kappa_j=0,\ 1\le i\le n \tag{1.3}$$

We also say that a dual direction $\overline{\kappa}$ is a *hodograph characteristic* direction. In certain cases, for defining a dual vector $\overline{\kappa}$ we could ignore, in a first step, the duality relation which is implicit in the terminology above. Such a case corresponds to $n=m+1$(see (1.2)]. It is easy to be seen, cf.(1.3), that a dual direction $\overline{\kappa}$ at a point u^* of the hodograph space satisfies in this case the condition:

$$\det\left[\sum_{j=1}^{n} a_{ijk}(u^*)\kappa_j\right]=0 \quad ; \quad i,k=1,\dots,n \tag{1.4}$$

which is formally independent of (1.3). In case of the system (1.2) the restriction (1.4) takes the form

$$c^2\kappa_1\left[\left(\frac{2}{\gamma-1}\right)^2\kappa_1^2-\left(\kappa_2^2+\kappa_3^2\right)\right]=0 \tag{1.5}$$

We notice that in case of the system (1.2) each dual pair associates at the mentioned point u^* to a vector $\bar{\kappa}$ a *single* dual vector $\bar{\beta}$.

<u>*Definition 1.2*</u> [M. Burnat, Z. Peradzynski]. A smooth curve in the hodograph space is said to be a *hodograph characteristic* if it is tangent at each point of it to a characteristic direction $\bar{\kappa}$.

- A nonconstant continuous solution of the system (1.1) whose hodograph is a *genuinely nonlinear* ([4], [5]) arc of characteristic curve is said to be a simple waves solution.
- A nonconstant continuous solution of the system (1.1) with the hodograph on a hypersurface with a system of genuinely nonlinear characteristic coordinates is said to be a *regular interaction of simple waves solutions*. Given a hypersurface in a hodograph space of (1.2) we could eventualy construct such system of characteristics coordinates by intersecting this hypersurface with a cone (1.5) (see exemples in [5]).

2. A CLASS OF EXACT SOLUTIONS OF THE ISENTROPIC GAS DYNAMICS

In order to obtain (local) solutions of the system (1.2) of the isentropic two-dimensional gas dynamics we put around the point (x_0, y_0, t_0) of the physical space

$$\xi=\frac{x-x_0}{t-t_0}\ ,\ \eta=\frac{y-y_0}{t-t_0} \tag{2.1}$$

and present the mentioned system in the form

$$\begin{cases}(v_x-\xi)\dfrac{\partial c^2}{\partial\xi}+(v_y-\eta)\dfrac{\partial c^2}{\partial\eta}+(\gamma-1)c^2\left(\dfrac{\partial v_x}{\partial\xi}+\dfrac{\partial v_y}{\partial\eta}\right)=0\\ \dfrac{\partial c^2}{\partial\xi}+(\gamma-1)(v_x-\xi)\dfrac{\partial v_x}{\partial\xi}+(\gamma-1)(v_y-\eta)\dfrac{\partial v_x}{\partial\eta}=0\\ \dfrac{\partial c^2}{\partial\eta}+(\gamma-1)(v_x-\xi)\dfrac{\partial v_y}{\partial\xi}+(\gamma-1)(v_y-\eta)\dfrac{\partial v_y}{\partial\eta}=0.\end{cases} \tag{2.2}$$

We consider for the system (2.2) local solutions for which

$$v_x=\Phi\xi+\Psi\eta+\Xi\ ,\ v_y=\hat{\Phi}\xi+\hat{\Psi}\eta+\hat{\Xi}\ ,\quad \text{real constant } \Phi,\hat{\Phi},\Psi,\hat{\Psi},\Xi,\hat{\Xi} \tag{2.3}$$

3. AN EXHAUSTIVE LIST OF SOLUTIONS IN THE CLASS CONSIDERED ABOVE

In paragraph 5 we get the folowing exhaustive list of the mentioned solutions to (2.2):

[cf. (5.12)] $$v_x\equiv\Xi,\ v_y\equiv\hat{\Xi}\ ,\ c^2\equiv K \quad ; \quad \text{arbitrary } K \tag{3.1}$$

[cf. (5.13)] $$v_x \equiv \Xi,\ v_y \equiv \eta,\ c^2 \equiv 0 \tag{3.2}$$

[cf. (5.14)] $$v_x \equiv \Xi,\ v_y \equiv \frac{2}{\gamma+1}\eta + \hat{\Xi},\ c^2 \equiv \left(\frac{\gamma-1}{\gamma+1}\eta - \hat{\Xi}\right)^2 \tag{3.3}$$

[cf. (5.15)] $$v_x \equiv \xi,\ v_y \equiv \hat{\Xi},\ c^2 \equiv 0 \tag{3.4}$$

[cf. (5.16)] $$v_x \equiv \xi,\ v_y \equiv \eta,\ c^2 \equiv 0 \tag{3.5}$$

[cf. (5.17)] $$v_x \equiv \xi,\ v_y \equiv \frac{3-\gamma}{\gamma+1}\eta + \hat{\Xi},\ c^2 \equiv \frac{3-\gamma}{4}\left(2\frac{\gamma-1}{\gamma+1}\eta - \hat{\Xi}\right)^2 \tag{3.6}$$

[cf. (5.18)] $$v_x \equiv \frac{2}{\gamma+1}\xi + \Xi,\ v_y \equiv \hat{\Xi},\ c^2 \equiv \left(\frac{\gamma-1}{\gamma+1}\xi - \Xi\right)^2 \tag{3.7}$$

[cf. (5.19)] $$v_x \equiv \frac{3-\gamma}{\gamma+1}\xi + \Xi,\ v_y \equiv \eta,\ c^2 \equiv \frac{3-\gamma}{4}\left(2\frac{\gamma-1}{\gamma+1}\xi - \Xi\right)^2 \tag{3.8}$$

[cf. (5.20)] $$\begin{gathered} v_x = \frac{1}{\gamma}\xi + \Xi,\ v_y = \frac{1}{\gamma}\eta + \hat{\Xi}, \\ c^2 = \frac{1}{2}\left[\left(\frac{\gamma-1}{\gamma}\xi - \Xi\right)^2 + \left(\frac{\gamma-1}{\gamma}\eta - \hat{\Xi}\right)^2\right] \end{gathered} \tag{3.9}$$

[cf. (5.30)] $$\begin{gathered} v_x = \Phi\xi \pm \eta\sqrt{\Phi(1-\Phi)} + K\sqrt{1-\Phi},\ K = \frac{\Xi}{\sqrt{1-\Phi}} = \mp\frac{\hat{\Xi}}{\sqrt{\Phi}}, \\ v_y = \pm\xi\sqrt{\Phi(1-\Phi)} + \eta(1-\Phi) \mp K\sqrt{\Phi}, \qquad 0 < \Phi < 1 \\ c^2 \equiv 0, \end{gathered} \tag{3.10}$$

[cf. (5.31)] $$\begin{gathered} v_x = \sqrt{\Phi}\left(\xi\sqrt{\Phi} \pm \eta\sqrt{\frac{2}{\gamma+1} - \Phi}\right) + \Xi \\ v_y = \pm\sqrt{\frac{2}{\gamma+1} - \Phi}\left(\xi\sqrt{\Phi} \pm \eta\sqrt{\frac{2}{\gamma+1} - \Phi}\right) + \hat{\Xi} \qquad 0 < \Phi < \frac{2}{\gamma+1} \\ c^2 = \frac{\gamma+1}{2}\left[\frac{\gamma-1}{\gamma+1}\left(\xi\sqrt{\Phi} \pm \eta\sqrt{\frac{2}{\gamma+1} - \Phi}\right) - \left(\Xi\sqrt{\Phi} \pm \hat{\Xi}\sqrt{\frac{2}{\gamma+1} - \Phi}\right)\right]^2 \end{gathered} \tag{3.11}$$

$$v_x = \Phi\xi \pm \eta\sqrt{(1-\Phi)\left(\Phi - \frac{3-\gamma}{\gamma+1}\right)} + K\sqrt{1-\Phi},\ K = \frac{\Xi}{\sqrt{1-\Phi}} = \mp\frac{\hat{\Xi}}{\sqrt{\Phi - \frac{3-\gamma}{\gamma+1}}},$$

[cf. (5.32)]
$$v_y = \pm\xi\sqrt{(1-\Phi)\left(\Phi - \frac{3-\gamma}{\gamma+1}\right)} + \eta\left(\frac{4}{\gamma+1} - \Phi\right) \mp K\sqrt{\Phi - \frac{3-\gamma}{\gamma+1}},\qquad \frac{3-\gamma}{\gamma+1} < \Phi < 1, \quad (3.12)$$

$$c^2 = \frac{(3-\gamma)(\gamma-1)}{2(\gamma+1)}\left(\xi\sqrt{1-\Phi} \mp \eta\sqrt{\Phi - \frac{3-\gamma}{\gamma+1}} - K\right)^2,$$

[cf. (5.36)]
$$v_x \equiv \Xi,\ v_y \equiv \hat{\Phi}\xi + \eta + \hat{\Xi},\ c^2 \equiv 0 \quad (3.13)$$

[cf. (5.37)]
$$v_x \equiv \xi,\ v_y \equiv \hat{\Phi}\xi + \hat{\Xi},\ c^2 \equiv 0 \quad (3.14)$$

[cf. (5.38)]
$$v_x \equiv \Phi\xi + \Psi\eta + \Xi,\ v_y \equiv \frac{\Phi(1-\Phi)}{\Psi}\xi + \eta(1-\Phi) - \frac{\Phi}{\Psi}\Xi,\ c^2 \equiv 0. \quad (3.15)$$

4. NATURE OF SOLUTIONS ON THE EXHAUSTIVE LIST

Incidentally, and remarkably, *all* the solutions on the exhaustive list could be characterized according to the facts of paragraph 1.

- Solutions (3.3), (3.7) and (3.11) are one-dimensional simple waves solutions.
- Solution (3.9) is a regular interaction of *multidimensional* simple waves solutions. This solution is considered in every detail in [5]. Its conical hodograph is endowed with three characteristic genuinely nonlinear coordinate fields [two conical helicoidal fields and a family of horizontal circles].
- Solutions (3.6), (3.8) and (3.12) are regular interactions of *one-dimensional* simple waves solutions.
- Solution (3.12) is taken into account in every detail in [5] too. A *linearly degenerate* coordinate field is present in this case requiring a *criterion of admiossibility* guaranteeing the (genuinely nonlinear) *nondegeneracy*.
- Solutions (3.2), (3.4), (3.5), (3.10), (3.13), (3.14), (3.15) are constitutively inadmissible because of the requirement $c^2 \equiv 0$.
- In [6] some *nondegenerate* solutions are still presented which are not regular interactions.

5. DETAILS CONCERNING THE CLASS MENTIONED ABOVE

From $(2.2)_{2,3}$ we obtain cf. (2.3)

$$\frac{\partial v_x}{\partial \xi} + \frac{\partial v_y}{\partial \eta} = \Phi + \hat{\Psi} \quad (5.1)$$

$$\begin{cases} -\dfrac{\partial c^2}{\partial \xi} = (\gamma-1)\Phi[(\Phi-1)\xi + \Psi\eta + \Xi] + (\gamma-1)\Psi[\hat{\Phi}\xi + (\hat{\Psi}-1)\eta + \hat{\Xi}] \\ -\dfrac{\partial c^2}{\partial \eta} = (\gamma-1)\hat{\Phi}[(\Phi-1)\xi + \Psi\eta + \Xi] + (\gamma-1)\hat{\Psi}[\hat{\Phi}\xi + (\hat{\Psi}-1)\eta + \hat{\Xi}] \end{cases} \quad (5.2)$$

The requirement $\frac{\partial c^2}{\partial\xi\partial\eta}=\frac{\partial c^2}{\partial\eta\partial\xi}$ takes, cf. (5.2), the form

$$(\Psi-\hat{\Phi})(\Phi+\hat{\Psi}-1)=0. \tag{5.3}$$

Now, the expression of c^2 could be calculated in two ways. On one hand, an expression of c^2 results from (5.2) and (5.3). On the other hand, an expression of c^2 results from $(2.2)_1$, (5.1) and (5.2). Since the two expressions obtained for c^2 are identical we get, by identifying the coefficients of $\xi^2,\xi\eta,\eta^2,\xi,\eta$ respectively:

$$\frac{1}{2}[\Phi(\Phi-1)+\Psi\hat{\Phi}][(\gamma+1)\Phi+(\gamma-1)\hat{\Psi}-2]+\hat{\Phi}^2(\Phi+\hat{\Psi}-1)=0 \tag{5.4}$$

$$2\Phi\Psi(\Phi-1)+(\Psi+\hat{\Phi})(\Phi-1)(\hat{\Psi}-1)+\Psi\hat{\Phi}(\Psi+\hat{\Phi})+2\hat{\Phi}\hat{\Psi}(\hat{\Psi}-1)+(\gamma-1)\Psi(\Phi+\hat{\Psi})(\Phi+\hat{\Psi}-1)=0 \tag{5.5}$$

$$\frac{1}{2}[\hat{\Psi}(\hat{\Psi}-1)+\Psi\hat{\Phi}][(\gamma-1)\Phi+(\gamma+1)\hat{\Psi}-2]+\Psi^2(\Phi+\hat{\Psi}-1)=0 \tag{5.6}$$

$$2\Phi\Xi(\Phi-1)+\Psi\hat{\Xi}(\Phi-1)+\Psi\Xi\hat{\Phi}+\Xi\hat{\Phi}^2+\hat{\Phi}\hat{\Xi}(\Phi-1)+2\hat{\Phi}\hat{\Psi}\hat{\Xi} \tag{5.7}$$

$$2\hat{\Psi}\hat{\Xi}(\hat{\Psi}-1)+\hat{\Phi}\hat{\Xi}(\hat{\Psi}-1)+\Psi\hat{\Phi}\hat{\Xi}+\hat{\Xi}\Psi^2+\Psi\Xi(\hat{\Psi}-1)+2\Phi\Psi\Xi+(\gamma-1)(\Phi+\hat{\Psi})(\hat{\Phi}\Xi+\hat{\Psi}\hat{\Xi})=0. \tag{5.8}$$

Therefore we have a nonlinear algebraic system (5.3)-(5.8) with six equations for six coefficients $\Phi,\hat{\Phi},\Psi,\hat{\Psi},\Xi,\hat{\Xi}$ in (2.3). We begin by presenting an exhaustive list of solutions for the system (5.3)-(5.8).

The requirements (5.3) suggests the importance of two cases.

Case 1. This case takes into account the circumstance

$$\Psi-\hat{\Phi}=0 \tag{5.9}$$

in (5.3). From (5.4)-(5.6) and (5.9) we obtain the following system for $\Phi,\Psi,\hat{\Psi}$:

$$\begin{cases}\Psi^2\{2[2(\Phi-1)+\hat{\Psi}]+(\gamma-1)(\Phi+\hat{\Psi})\}+\Phi(\Phi-1)[2(\Phi-1)+(\gamma-1)(\Phi+\hat{\Psi})]=0\\ \Psi\{2\Psi^2+[2\Phi(\Phi-1)+2(\Phi-1)(\hat{\Psi}-1)+2\hat{\Psi}(\hat{\Psi}-1)+(\gamma-1)(\Phi+\hat{\Psi})(\Phi+\hat{\Psi}-1)]\}=0\\ \Psi^2\{2[2(\hat{\Psi}-1)+\Phi]+(\gamma-1)(\Phi+\hat{\Psi})\}+2\hat{\Psi}(\hat{\Psi}-1)[2(\hat{\Psi}-1)+(\gamma-1)(\Phi+\hat{\Psi})]=0.\end{cases} \tag{5.10}$$

Next, we have to distinguish, cf. $(5.10)_2$, between the possibilities $\Psi=0$ or $\Psi\neq 0$.
We begin our analysis with the subcase $\Psi=0$. In this subcase, from $(5.10)_{1,3}$ we obtain for $\Phi,\hat{\Psi}$ the system

$$\begin{cases}\Phi(\Phi-1)[(\gamma+1)\Phi+(\gamma-1)\hat{\Psi}-2]=0\\ \hat{\Psi}(\hat{\Psi}-1)[(\gamma-1)\Phi+(\gamma+1)\hat{\Psi}-2]=0.\end{cases} \tag{5.11}$$

Therefore we get the following exhaustive list of solutions of (5.10) corresponding to the mentioned subcase [we complete this list with the information concerning $\hat{\Phi},\Xi,\hat{\Xi}$; cf. (5.7), (5.8), (5.9)]

$$\Phi=0,\qquad \Psi=0,\qquad \hat{\Psi}=0,\qquad \hat{\Phi}=\Psi,\qquad \text{arbitrary } \Xi,\hat{\Xi} \tag{5.12}$$

$$\Phi=0,\qquad \Psi=0,\qquad \hat{\Psi}=1,\qquad \hat{\Phi}=\Psi,\qquad \text{arbitrary } \Xi;\ \hat{\Xi}=0 \tag{5.13}$$

$$\Phi = 0, \qquad \Psi = 0, \qquad \hat{\Psi} = \frac{2}{\gamma+1}, \qquad \hat{\Phi} = \Psi, \qquad \text{arbitrary } \Xi, \hat{\Xi} \tag{5.14}$$

$$\Phi = 1, \qquad \Psi = 0, \qquad \hat{\Psi} = 0, \qquad \hat{\Phi} = \Psi, \qquad \Xi = 0, \text{ arbitrary } \hat{\Xi} \tag{5.15}$$

$$\Phi = 1, \qquad \Psi = 0, \qquad \hat{\Psi} = 1, \qquad \hat{\Phi} = \Psi, \qquad \Xi = 0, \quad \hat{\Xi} = 0 \tag{5.16}$$

$$\Phi = 1, \qquad \Psi = 0, \qquad \hat{\Psi} = \frac{3-\gamma}{\gamma+1}, \qquad \hat{\Phi} = \Psi, \qquad \Xi = 0, \text{ arbitrary } \hat{\Xi} \tag{5.17}$$

$$\Phi = \frac{2}{\gamma+1}, \qquad \Psi = 0, \qquad \hat{\Psi} = 0, \qquad \hat{\Phi} = \Psi, \qquad \text{arbitrary } \Xi, \hat{\Xi} \tag{5.18}$$

$$\Phi = \frac{3-\gamma}{\gamma+1}, \qquad \Psi = 0, \qquad \hat{\Psi} = 1, \qquad \hat{\Phi} = \Psi, \qquad \text{arbitrary } \Xi; \ \hat{\Xi} = 0 \tag{5.19}$$

$$\Phi = \frac{1}{\gamma}, \qquad \Psi = 0, \qquad \hat{\Psi} = \frac{1}{\gamma}, \qquad \hat{\Phi} = \Psi, \qquad \text{arbitrary } \Xi, \hat{\Xi} \tag{5.20}$$

We extend our analysis by considering the subcase $\Psi \neq 0$. In this subcase we use $(5.10)_2$ in order to eliminate Ψ^2 from $(5.10)_{1,3}$. We notice that the equations $(5.10)_{1,3}$ are not distinct in this subcase. In fact, we denote

$$X = \Phi - 1, \quad Y = \hat{\Psi} - 1, \quad Z = X + Y = \Phi + \hat{\Psi} - 2 \tag{5.21}$$

and obtain the following common form of equations $(5.10)_{1,3}$

$$(\gamma+1)^2 Z^3 + (\gamma+1)(5\gamma-1)Z^2 + 2(4\gamma^2 - \gamma - 1)Z + 4\gamma(\gamma-1) = 0 \tag{5.22}$$

with the roots

$$Z_1 = -\frac{2\gamma}{\gamma+1}, \quad Z_2 = -1, \quad Z_3 = -\frac{2(\gamma-1)}{\gamma+1}. \tag{5.23}$$

Finally we put (5.23) in the form

$$\Phi + \hat{\Psi} = \frac{2}{\gamma+1} \quad ; \quad [\text{cf. } (5.23)_1] \tag{5.24}$$

$$\Phi + \hat{\Psi} = 1 \quad ; \quad [\text{cf. } (5.23)_2] \tag{5.25}$$

$$\Phi + \hat{\Psi} = \frac{4}{\gamma+1} \quad ; \quad [\text{cf. } (5.23)_3]. \tag{5.26}$$

For (5.25) we obtain cf. $(5.10)_2$

$$\Psi^2 = \Phi(1-\Phi)$$

and therefore

$$0 \le \Phi \le 1 \quad \text{and} \quad \Psi = \pm\sqrt{\Phi(1-\Phi)}. \tag{5.27}$$

Similarly, we get, cf. $(5.10)_2$,

$$\Psi^2 = \Phi\left(\frac{2}{\gamma+1} - \Phi\right)$$

hence

$$0 \le \Phi \le \frac{2}{\gamma+1} \quad \text{and} \quad \Psi = \pm\sqrt{\Phi\left(\frac{2}{\gamma+1} - \Phi\right)} \tag{5.28}$$

for (5.24), and

$$\Psi^2 = \left(\Phi - \frac{3-\gamma}{\gamma+1}\right)(1-\Phi)$$

or, equivalently,

$$\frac{3-\gamma}{\gamma+1} \le \Phi \le 1 \quad \text{and} \quad \Psi = \pm\sqrt{\left(\Phi - \frac{3-\gamma}{\gamma+1}\right)(1-\Phi)} \tag{5.29}$$

for (5.26).

Consequently, we complete the list (5.12)-(5.20) which corresponds, for $\Psi = 0$, to the case (5.9) with the following circumstances [which take into account (5.7), (5.8) and (5.24)-(5.26)]:

$$0 < \Phi < 1,\ \ \Psi = \pm\sqrt{\Phi(1-\Phi)},\quad \hat{\Phi} = \Psi,\quad \hat{\Psi} = 1-\Phi,\quad \text{arbitrary } \Xi;\ \ \hat{\Xi} = \mp\,\Xi\sqrt{\frac{\Phi}{1-\Phi}} \tag{5.30}$$

$$0 < \Phi < \frac{2}{\gamma+1},\ \ \Psi = \pm\sqrt{\Phi\left(\frac{2}{\gamma+1} - \Phi\right)},\quad \hat{\Phi} = \Psi,\quad \hat{\Psi} = \frac{2}{\gamma+1} - \Phi,\quad \text{arbitrary } \Xi, \hat{\Xi} \tag{5.31}$$

$$\begin{gathered} \frac{3-\gamma}{\gamma+1} < \Phi < 1,\ \ \Psi = \pm\sqrt{\left(\Phi - \frac{3-\gamma}{\gamma+1}\right)(1-\Phi)},\quad \hat{\Phi} = \Psi,\quad \hat{\Psi} = \frac{2}{\gamma+1} - \Phi,\quad \text{arbitrary } \Xi; \\ \hat{\Xi} = \mp\,\Xi\sqrt{\frac{\Phi - \frac{3-\gamma}{\gamma+1}}{1-\Phi}} \end{gathered} \tag{5.32}$$

<u>Case 2.</u> This case considers in (5.3) the circumstance

$$\Phi + \hat{\Psi} - 1 = 0. \tag{5.33}$$

We use (5.33) in order to give to (5.4)-(5.6) the form

$$\begin{cases} [2(\Phi-1) + (\gamma-1)][\Psi\hat{\Phi} + \Phi(\Phi-1)] = 0 \\ (\Psi + \hat{\Phi})[\Psi\hat{\Phi} + \Phi(\Phi-1)] = 0 \\ [2\Phi - (\gamma-1)][\Psi\hat{\Phi} + \Phi(\Phi-1)] = 0 \end{cases} \tag{5.34}$$

of a system for $\Phi, \Psi, \hat{\Phi}$.

A single relation results from (5.34) for $\Phi, \Psi, \hat{\Phi}$:

$$\Psi\hat{\Phi} + \Phi(\Phi - 1) = 0. \tag{5.35}$$

Now, the circumstance (5.33) could be completely described by the following list of possibilities [which also considers the contribution of equations (5.7), (5.8) for $\Xi, \hat{\Xi}$]:

$$\Phi = 0, \quad \Psi = 0, \quad \text{arbitrary } \hat{\Phi}, \quad \hat{\Psi} = 1, \quad \text{arbitrary } \Xi; \ \hat{\Xi} = -\hat{\Phi}\Xi \tag{5.36}$$

$$\Phi = 1, \quad \Psi = 0, \quad \text{arbitrary } \hat{\Phi}, \quad \hat{\Psi} = 0, \quad \Xi = 0, \ \text{arbitrary } \hat{\Xi} \tag{5.37}$$

$$\text{arbitrary } \Phi, \ \text{arbitrary } \Psi \neq 0, \quad \hat{\Phi} = \frac{\Phi(\Phi - 1)}{\Psi}, \quad \hat{\Psi} = 1 - \Phi, \quad \text{arbitrary } \Xi, \ \hat{\Xi} = -\frac{\Phi}{\Psi}\Xi \tag{5.38}$$

We notice that (5.12)-(5.20), (5.30)-(5.32), (5.36)-(5.38) represents an *exhaustive* list of possibilities. An exhaustive list of local solutions of the form (2.3) for the system (2.2) of the isentropic gas dynamics results from the above mentioned list; cf. paragraph 3.

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