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# ANALYSIS OF THE RADIAL MAGNETIC FIELD IN AN ELECTRIC MACHINE SLOT

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The present paper analyse the magnetic field in an open slot of the armature of synchronous generator by using the conformal mapping method. In the paper have been developed analytical relations that have general character allowing the determination of the magnetic field radial component and the additional losses in the winding bars. Such practical relations are useful in windings optimisation of high power generators

*Key words:* magnetic field, Schwarz – Christoffel formula, electrical machines.

## 3. INTRODUCTION

The solution to a field problem may be obtained in analytical or numerical manner.

The problem-solving method is chosen using the hypotheses of the problem and the shape of the calculation domain. In electric machines, the solution of such problem is not easy at all, due to the calculation domain, which, most of the times, is non-linear and holds high number of inhomogenities and boundaries with complex forms. Notwithstanding, there have been developed several computer programs that provide the magnetic field in an electric machine by using exclusively numerical methods, especially the finite element method [1], [2], [3].

Such programs are of high use in the analysis of the magnetic field in the machine, but they have certain limitations as well: they lead to an approximate solution; results are expressed as a series of numerical values, demanding high manoeuvrability; the solution obtained represents just a particular case.

The analytical calculation methods of fields hold reduced area of applicability. These are used in model problems, holding homogenous and linear domains and simple forms of boundaries. But they hold an outstanding advantage: allow the obtainment of the exact solution of the field problem as finite terms, with known elementary or special functions. The analytical solution allows qualitative interpretation of results and it has to be always preferred, if it can be obtained [4], [5], [6].

In calculating electric machines, especially when designing them, it is of high importance to know the magnetic field in several areas of the magnetic circuit. The magnetic field influences the value of machine parameters [7], leads to losses in the conducting media [8] and other particular effects that have to be taken into account in calculations.

This paper shows an example of calculating the magnetic field from an open slot of the armature of a synchronous generator.

The radial component of such field develops internal circular currents through the shorted elementary conductors of generator winding and, as a result, additional losses in the winding bars. The minimization of such additional losses represents an optimization criterion of the structure of Roebel bars [9], [10].

Using the method of conformal mapping, in this paper it is found the magnetic field in the slot. Instead of the real calculation domain, there have been analyzed two simpler configurations of boundaries, which allow the analytical solving of the field problem.

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## 2. FIRST CONFIGURATION

The exciting magnetic field of a synchronous generator passes also trough the gap in the armature slots. To analyze the field in the slot, figure 1.a suggests first option of the calculation domain. This configuration of boundaries allows the obtaining of the solution using the superposition method.

By using the Schwarz – Christoffel formula, it is done the conformal mapping of the Z plane (figure 1.a) over the W upper half plane (figure 1.b). Using the notations in figure 1, it is obtained:

$$z = C_1 \int \sqrt{\frac{w-1}{w-a}} \frac{\mathrm{d}w}{w} + C_2 \tag{1}$$

To calculate the integral, it is made the change of variable $q^2 = \dfrac{w-1}{w-a}$, and it results $w = \dfrac{1-aq^2}{1-q^2}$ and $\mathrm{d}w = \dfrac{2(1-a)q}{(1-q^2)^2}\mathrm{d}q$ .

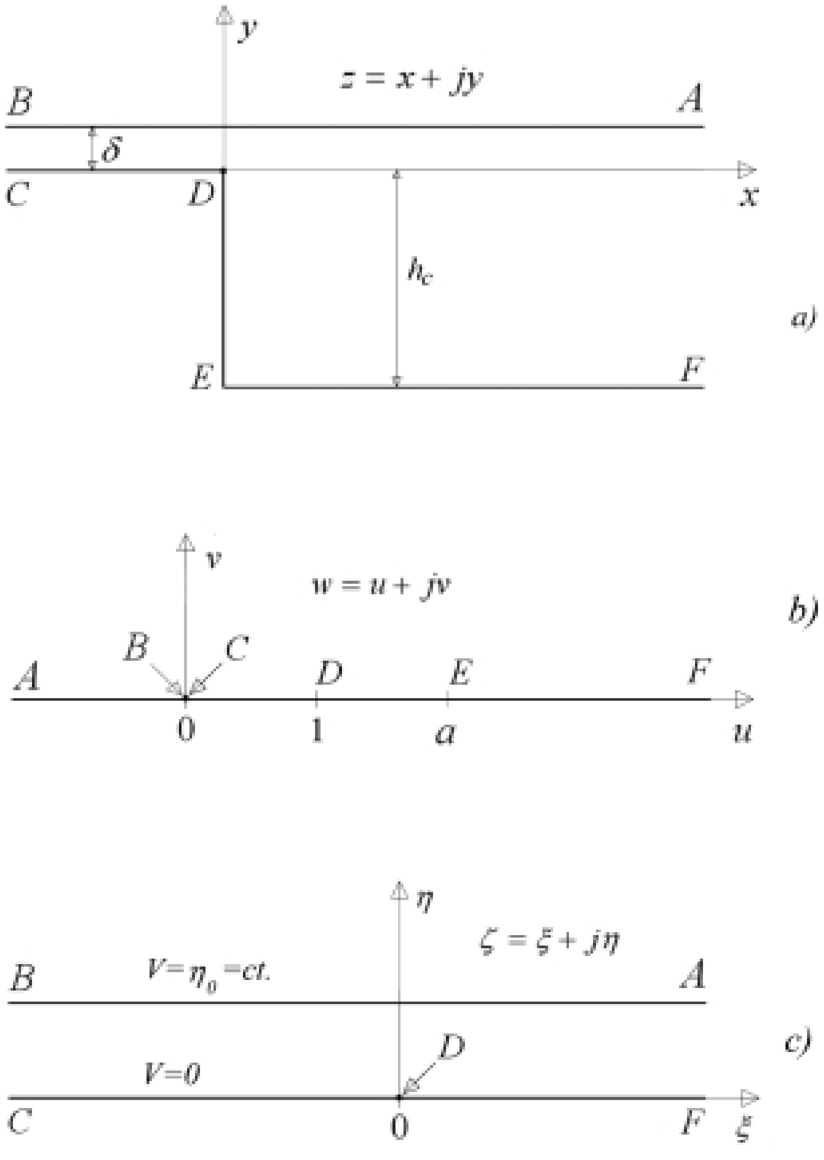


Fig.1 The conformal mapping of the complex planes:
a) Z-plane; b) W-plane; c) ζ-plane.

In this way, the integral becomes:

$$z = C_1 \int \sqrt{\frac{2(1-a)q^2}{(1-aq^2)(1-q^2)}} \mathrm{d}q + C_2 \tag{2}$$

By solving the integral, we obtain in the end:

$$z = C_1 \left( \frac{1}{\sqrt{a}} \ln \frac{\sqrt{w-a} - \sqrt{a(w-1)}}{\sqrt{w-a} + \sqrt{a(w-1)}} + \ln \frac{\sqrt{w-a} + \sqrt{w-1}}{\sqrt{w-a} - \sqrt{w-1}} \right) + C_2 \tag{3}$$

**Determination of constants**

By following the correspondence between certain points of the W and Z planes, some constants can be found. As example, for $w$=1, it has to result $z$=0; consequently, from (3) it is obtained $C_2$=0.

To determine constant $C_1$, it is calculated the distance $\overline{\mathrm{BC}}$ in the Z plane, integrating in the W plane across very small semicircle radius.

Using $w = r \cdot \mathrm{e}^{\mathrm{j}\varphi}$ and $\mathrm{d}w = r \cdot \mathrm{e}^{\mathrm{j}\varphi} \cdot \mathrm{j} \cdot \mathrm{d}\varphi$ it is obtained:

$$\mathrm{BC} = \text{-j}\delta = \lim_{r \to 0} C_1 \int_{\pi}^{0} \sqrt{\frac{r \cdot \mathrm{e}^{\mathrm{j}\varphi} - 1}{r \cdot \mathrm{e}^{\mathrm{j}\varphi} - a}} \cdot \mathrm{j} \cdot \mathrm{d}\varphi = C_1 \frac{1}{\sqrt{a}} \mathrm{j}(-\pi) \tag{4}$$

resulting $C_1 = \dfrac{\delta\sqrt{a}}{\pi}$.

In the end, if $w$=$a$, then $z$=-$ih_c$; as a result, from (3) it is obtained: $C_1 \left( \dfrac{\pi}{\sqrt{a}} + \pi \right) i = -ih_c$, resulting: $a = \left( \dfrac{h_c + \delta}{\delta} \right)^2$.

Relation (3) assures the conformal mapping of the Z plane in the W upper half plane.

By using the Schwarz – Christoffel formula, it is further obtained the relation of transformation of the W upper half plane in the ABCD strip of the ζ plane (figure 1.c) as:

$$\zeta = C_3 \ln w + C_4 \tag{5}$$

Seeing the correspondence of points $w$=1 and $w$=-∞ from the W plane in the ζ plane, it is obtained: $C_4 = 0$ and $C_3 = \dfrac{\eta_0}{\pi}$.

By using the two relations of conformal mapping, it is calculated the flux density in a point of the Z plane as:

$$\overline{B} = \mu_0 H_x - \mathrm{j}\mu_0 H_y = -\mu_0 \left( \frac{\mathrm{d}\zeta}{\mathrm{d}w} \cdot \frac{\mathrm{d}w}{\mathrm{d}z} \right) = -\mu_0 \frac{\eta_0}{\delta\sqrt{a}} \sqrt{\frac{w-a}{w-1}} \tag{6}$$

From figure 1.a it is noticed that the maximum value of the induction in the air gap $(B_{\delta\max})$ corresponds to point $z \to -\infty + \mathrm{j} \cdot 0$, i.e. $w = 0$; it results $B_{\delta\max} = -\mu_0 \dfrac{\eta_0}{\delta}$.

In the end, it is obtained in reported values:

$$\left( \frac{\overline{B}}{\overline{B}_{\delta\max}} \right) = \frac{1}{\sqrt{a}} \sqrt{\frac{w-a}{w-1}} \tag{7}$$

By using relation (7), for $\overline{B}_{\delta\max} = 1\,\mathrm{T}$, it has been found the variation of the modulus of the flux density over the AB boundary from the domain of figure 1.a.

The solution is showed in figure 2, where it is also suggested the way in which has been applied the superposition method in order to get the variation of the induction across the direction of the slot width; it has been considered an open slot, with width $b_c = 30$ mm and height $h_c = 100$ mm and an air gap $\delta = 5$ mm.

In figure 3 can be seen the influence of height $h_c$ upon the solution of the field problem obtained by superposition, for $b_c = 30$ mm and $\delta = 5$ mm. It is found that for a certain value of $h_c$, the depth of the slot does no longer influence significantly the field distribution in the air gap. For example, if the depth of the slot raises from $h_c = 100$ mm to $h_c = 1000$ mm, the radial field in the slot axe decreases with about 1.6%. In case of high power synchronous generators, the height of the slot is, usually $h_c = (4...7)b_c$. Due to this reason, in order to calculate the field in the air gap it is preferable to adopt an ideal slot with infinite depth; in this way it is taken into account the reciprocal influence of slot walls, and it is avoided the superposition method.

The finite depth of the usual slots has irrelevant influence, as it comes out from the above analysis.

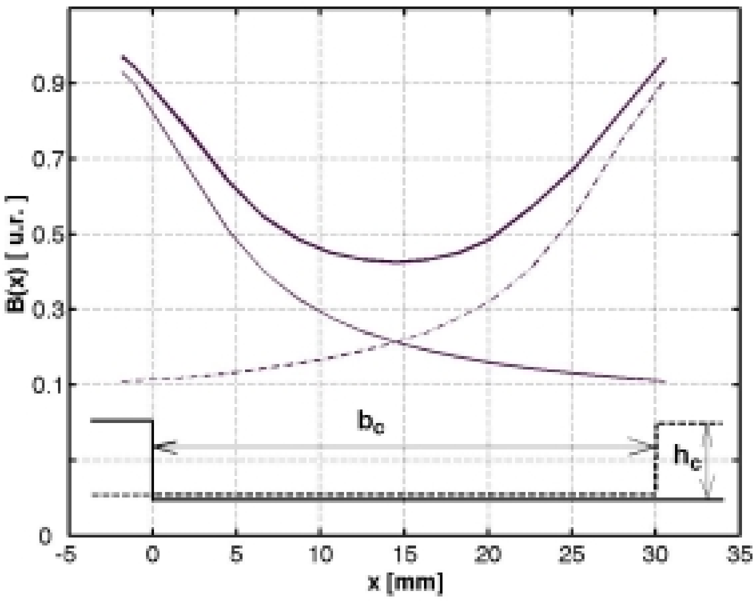


Fig. 2 The superposition method: flux density distribution on the AB boundary, for δ=5 mm, $h_c$=100 mm and 0<$x$<30 mm.

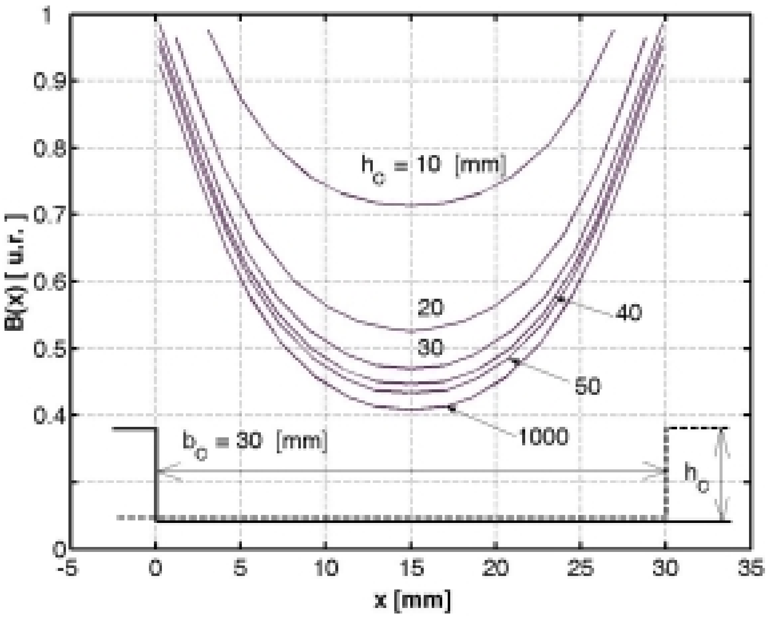


Fig. 3. The field solution for $b_c$=30 mm and different slot hights: $h_c$=10 mm, 20 mm, 30 mm, 40 mm, 50 mm, 1000 mm.

## 3. SECOND CONFIGURATION

It is considered an open slot, idealized (figure 4.a), with infinite depth; the curvature of the armature surfaces and the influence of other slots are not taken into account.

The Schwarz-Cristoffel relation, which makes the conformal mapping of the domain from the Z plane in the W plane (figure 4) is:

$$z = C_1 \int \frac{\sqrt{(w-a)(w-b)}}{w(w-1)} \mathrm{d}w + C_2 \tag{8}$$

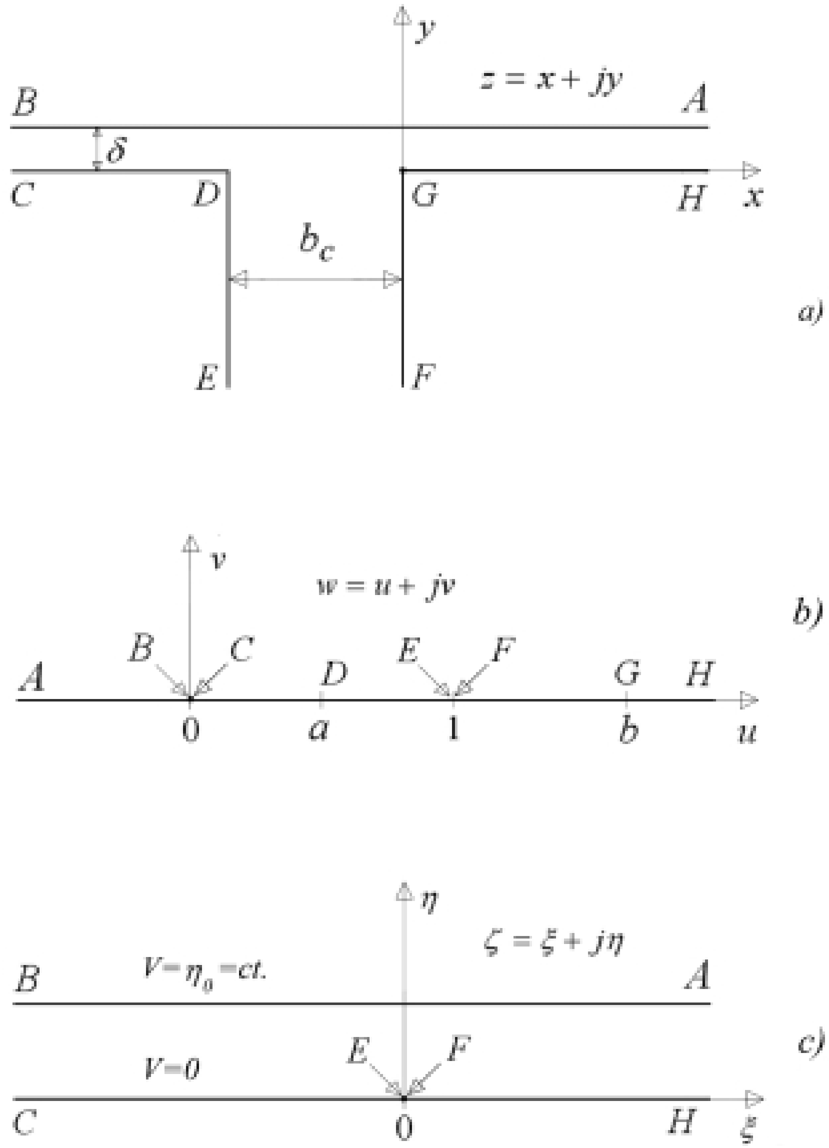


Fig. 4 The conformal mapping of the complex planes: Z, W, ζ

Using the change of variable $p = \frac{\sqrt{w-b}}{\sqrt{w-a}}$, it is obtained in the end [11, 12]:

$$z=\frac{\delta}{\pi}\left(\ln\frac{\sqrt{w-a}+\sqrt{w-b}}{\sqrt{w-a}-\sqrt{w-b}}-\ln\frac{b\sqrt{w-a}+\sqrt{w-b}}{b\sqrt{w-a}-\sqrt{w-b}}-\frac{2(b-1)}{\sqrt{b}}\cdot\operatorname{arctg}\frac{1}{\sqrt{b}}\frac{\sqrt{w-b}}{\sqrt{w-a}}\right), \tag{9}$$

where:

$$b=\frac{\left(\frac{b_c}{\delta}\right)^2+2+\sqrt{\left(\left(\frac{b_c}{\delta}\right)^2+2\right)^2-4}}{2}\ ; \quad a=\frac{1}{b}. \tag{10}$$

The conformal mapping of the W upper half plane in the ABCH strip of the $\zeta$ plane is made using relation (5) from the previous example:

$$\zeta=\frac{\eta_0}{\pi}\ln w \tag{11}$$

Using these, it is obtained following relation to calculate the magnetic induction in a point of the Z plane, in relative values:

$$\left(\frac{\overline{B}}{\overline{B}_{\delta\max}}\right)=\frac{w-1}{\sqrt{(w-a)(w-b)}}. \tag{12}$$

Relation (12) allows the calculation of the values of the magnetic flux density in any point of the calculation domain: the space of the air gap and the slot, respectively.

With $B_{\delta\max}=1\,\text{T}$, there have been done calculations for $\delta=5$ mm, $b_c=24$ mm. It has been aimed to find the radial component of the magnetic induction within the slot area.

In figure 5 and figure 6 has been drawn the distribution of the radial component of the magnetic induction across the direction of the slot width, for particular depths measured from the air gap ($y$=0) towards the base of the slot. In figure 5 can be seen that the distribution shape changes significantly for some depth, especially near the air gap. For larger depths (figure 6), the distribution highly resembles the sinusoidal one.

Several calculations have been done for particular values of the air gap ($\delta$) and of the slot width ($b_c$). It has been found that the distribution of the radial component remains the same should $b_c/\delta$ remains constant. In other words, curves in figure 5 and figure 6 correspond to any case in which $\frac{b_c}{\delta}=4.8$. It has been also found that for depth $|y|\geq 2\delta$, the distribution of the radial component $(B_r)$ of the induction in the slot may be approximated by following term:

$$B_r(x)=B_{rA}\sin\left(\frac{x}{b_c}\pi\right) \tag{13}$$

where, $B_{rA}$ represents the value of the radial component of the magnetic induction in the slot axe.

For $|y|\approx 1.5\,\delta$ it can be used relation $B_r(x)=B_{rA}\left(\sin\left(\frac{x}{b_c}\pi\right)\right)^{0,8}$, and if $|y|\approx\delta$ it may be considered $B_r(x)=B_{rA}\left(\sin\left(\frac{x}{b_c}\pi\right)\right)^{0,5}$. For $|y|<\delta$, the distribution of the radial component can no longer be approximated by a „sine” function; the variation forms are the ones in figure 5, for $|y|<5$ mm.

In order to establish as general as possible analytical relations to calculate the magnetic field in the slot, it has been analyzed the variation of the radial component of the induction in the slot axe. In figure 7 has been drawn the variation of such component $(B_{rA})$ according to the position of the current point across the direction of the slot depth; in the figure have been drawn the variation curves for four slots with following widths: $b_c = \delta$; $b_c = 2\delta$; $b_c = 3\delta$; $b_c = 5\delta$.

By using the entire family of curves obtained as a result of calculation, as well as the MATLAB software, it has been found a formula to approximate the values of the induction in the slot axe, as follows:

$$\frac{B_{rA}}{B_{\delta\max}} = \mathrm{e}^{\alpha} \tag{14}$$

where:

$$\alpha = -m\left|\frac{y}{\delta}\right| - n \tag{15}$$

and:

$$m = \frac{38{,}39\left(\frac{b_c}{\delta}\right)^3 - 665{,}1\left(\frac{b_c}{\delta}\right)^2 + 6507\left(\frac{b_c}{\delta}\right)^1 + 1061}{\left(\frac{b_c}{\delta}\right)^3 + 909{,}4\left(\frac{b_c}{\delta}\right)^2 + 2654\left(\frac{b_c}{\delta}\right)^1 - 1246}; \qquad n = 0{,}0857\left(\frac{b_c}{\delta}\right) + 0{,}3786\,. \tag{16}$$

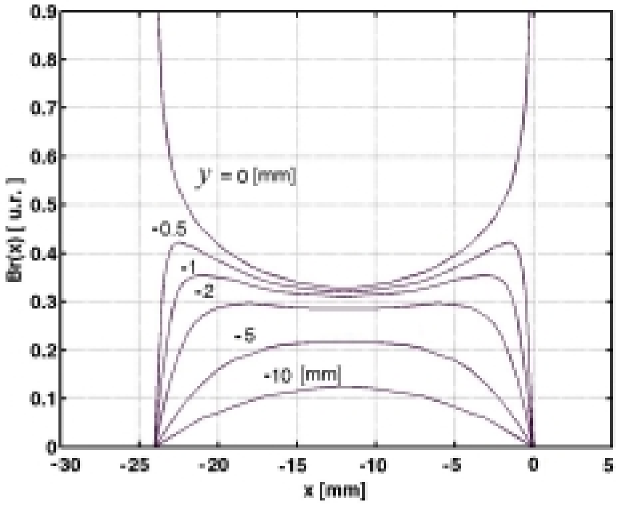


Fig. 5 The distribution of the flux density radial component across the direction of the slot width ($b_c$=24 mm), near the air gap ($y$=0…-10 mm); δ=5 mm

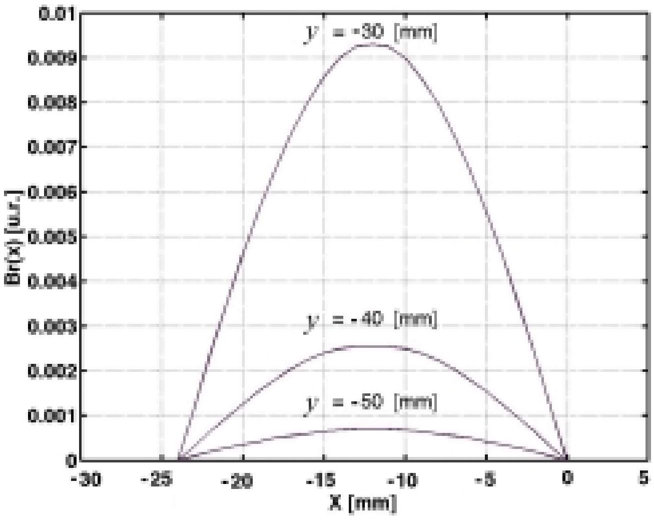


Figure 6. The distribution of the flux density radial component for larger depths: $y$=-30 mm; $y$=-40 mm; $y$=-50 mm.

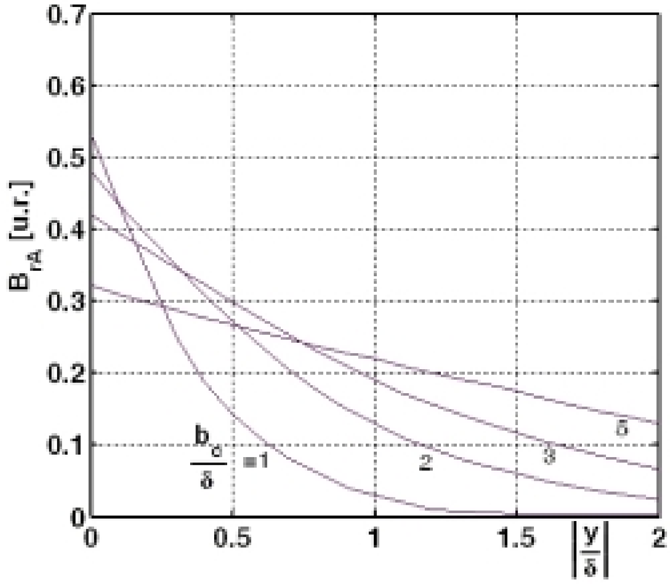


Fig. 7 The variation of the flux density radial component in the slot axe (near the air gap)

By using the approximate relation (14), it can be found in any point the magnetic induction in the slot axe avoiding the use of the exact relation (12). Errors introduced in calculations by relation (14) are lower than 1%, if $\left|\frac{y}{\delta}\right| > 0{,}3$.

In case of synchronous generators found in practice, the active part of the slot is at depth $|y| > 0{,}3\,\delta$, so that relation (14) can be used and it gives negligible errors in design calculations.

## 4. CONCLUSION

This paper shows an example of calculating the magnetic field in an open slot of the armature of a synchronous generator by using the method of conformal mapping. There have been found exact relations to calculate the magnetic field in the slot from two geometrical configurations that approximate the real calculation domain; by using such configurations, there have been developed simple analytical relations that approximate the field.

Numerical calculations showed that the values of the magnetic field in the slot are significant in the upper part of the slot, near the air gap, decreasing rapidly to the base of the slot.

The analytical relations that have been developed have general character and allow the determination of the radial component of the magnetic field in any point of the slot area, for any dimensions of the slot, of the air gap and for any values of the induction in the air gap.

Such relations introduce acceptable errors in the calculation of electric machines and are useful in designing and optimizing the windings of high power synchronous generators.

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