THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 7, Number 3/2006, pp. 000-000

RADIATION SPECTRUM OF TWO ELECTRONS MOVINGIN A SPIRAL IN VACUUM

Aurel V. KONSTANTINOVICH, and Ivan A. KONSTANTINOVICH

Chernivtsi National University, 2, Kotsyubynsky St., Chernivtsi, 58012, Ukraine
e-mail: aconst@hotbox.ru
Corresponding author: Aurel V. KONSTANTINOVICH

In this study a special attention is given to the research of the magnitude of radiation power of two electrons moving one by one in vacuum in non-relativistic case in dependence on their location in a spiral. Synchrotron radiation spectra for a single and two electrons in relativistic case are obtained and analyzed.

Key words: synchrotron radiation spectrum of two electrons in vacuum, coherence factor

1. INTRODUCTION

Studies of the radiation spectrum of electrons moving in magnetic fields in vacuum are important from the point of view of their applications in electronics, astrophysics, plasma physics, etc. [1–4].

A question requiring further investigations is the coherence of synchrotron radiation [1, 5–12]. At moving an electron beam through a spiral undulator a laser radiation takes place [13]. The phenomena take the place in free-electron lasers were studied in paper [14].

Using the exact integral relationships for the spectral distribution of radiation power of two electrons moving one by one along a spiral in vacuum the structure of the synchrotron radiation spectrum was investigated by means of analytical and numerical methods. Special attention is given to the research of the dependence of radiation power magnitude of two electrons moving one by one in dependence on their location in a spiral in non-relativistic case. Synchrotron radiation spectrum for a single and two electrons in relativistic case is obtained and analyzed.

2. SPECTRAL DISTRIBUTION OF THE RADIATION POWER OF TWO ELECTRONS MOVING ALONG A SPIRAL IN VACUUM

The law of motion and the velocity of the l^{th} electron in magnetic field are given by the expressions

$$\vec{r}_l(t)=r_0\cos\{\omega_0(t+\Delta t_l)\}\vec{i}+r_0\sin\{\omega_0(t+\Delta t_l)\}\vec{j}+V_\parallel(t+\Delta t_l)\vec{k} \quad , \quad \vec{V}_l(t)=\frac{d\vec{r}_l(t)}{dt} \tag{1}$$

Here $r_0=V_\perp\omega_0^{-1}$, $\omega_0=ec^2B^{ext}\tilde{E}^{-1}$, $\tilde{E}=c\sqrt{p^2+m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\parallel 0Z$, $V_\perp$ and $V_\parallel$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass, respectively.

The time-averaged radiation power of two electrons moving one by one in vacuum is presented in [7]:

$$\overline{P}^{rad}=\int_0^\infty W(\omega)d\omega, \tag{2}$$

Recommended by Horia SCUTARU, member of the Romanian Academy

$$W(\omega)=\frac{2e^2}{\pi}\int_0^\infty dx\,\omega\frac{\mu_0}{4\pi}S_2(\omega)\frac{\sin\left\{\frac{1}{c}\omega\eta(x)\right\}}{\eta(x)}\cos\omega x\left[V_\perp^2\cos(\omega_0 x)+V_\parallel^2-c^2\right], \tag{3}$$

where $\eta(x)=\sqrt{V_\parallel^2x^2+4\frac{V_\perp^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

The coherence factor $S_2(\omega)$ of two electrons is defined as

$$S_2(\omega)=2+2\cos(\omega\Delta t_{12}) \tag{4}$$

Here $\Delta t_{12}=\Delta t_2-\Delta t_1$ is the time shift of the electrons moving along a spiral. The analogous expression for the coherence factor was investigated by Bolotovskii [15].

From relationships (2) and (3) after some transformations the contributions of separate harmonics to the averaged radiation power can be written as [7]:

$$\begin{aligned}\bar{P}^{rad}=&\frac{e^2}{c}\sum_{m=1}^\infty\int_0^\infty \mathrm{d}\omega\omega^2\frac{\mu_0}{4\pi}\int_0^\pi\sin\theta\,\mathrm{d}\theta\; S_2(\omega)\delta\left\{\omega\left(1-\frac{1}{c}V_\parallel\cos\theta\right)-m\omega_0\right\}\times\\&\times\left\{V_\perp^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m'^2(q)\right]+\left(V_\parallel^2-c^2\right)J_m^2(q)\right\}\end{aligned} \tag{5}$$

where $q=\frac{V_\perp}{c}\frac{\omega}{\omega_0}\sin\theta$, $J_m(q)$ and $J_m'(q)$ are the Bessel function with integer index and its derivative, respectively.

Each harmonic is a set of the frequencies, which are the solution of the equation

$$\omega\left(1-\frac{1}{c}V_\parallel\cos\theta\right)-m\omega_0=0 \tag{6}$$

The total radiation power emitted by a single electron moving in a spiral in vacuum is determined, according to [16], as

$$P_{vac}^{tot}=\frac{2}{3}\frac{e^2}{c}\frac{\mu_0}{4\pi}\omega_0^2V_\perp^2\left(1-\frac{V^2}{c^2}\right)^{-2} \tag{7}$$

where $\omega_0=\frac{eB^{ext}}{m_0}\sqrt{1-\frac{V^2}{c^2}}$.

Our high accuracy numerical calculations of the radiation power spectral distribution were performed at $B^{ext}=10^{-4}$ T and $c=0.2997925\cdot10^9$ m/s.

3. SYNCHROTRON RADIATION SPECTRUM OF TWO ELECTRONS MOVING IN A SPIRAL IN NON-RELATIVISTIC CASE

For the velocities components $V_{\perp vac}=0.2\cdot10^8$ m/s and $V_{\parallel vac}=0.2\cdot10^8$ m/s the radiation power spectral distributions of two electrons moving one by one in vacuum depending on their location along a spiral are shown in Figs. 1–5 (curves 2–7).

It is interesting to compare the radiation power spectral distribution for two electrons with the radiation power spectral distribution of a single electron (curve 1 in Fig.1). The radiation power of the single electron in vacuum $P_{vac1}^{tot}=0.713\cdot10^{-24}$ W calculated according to relationship (7) is in good agreement to the power

$P^{\text{int}}_{vac1} = 0.722 \cdot 10^{-24}$ W determined after integration according to relationships (2) and (3). For a single electron we have the coherence factor $S_1 = 1$.

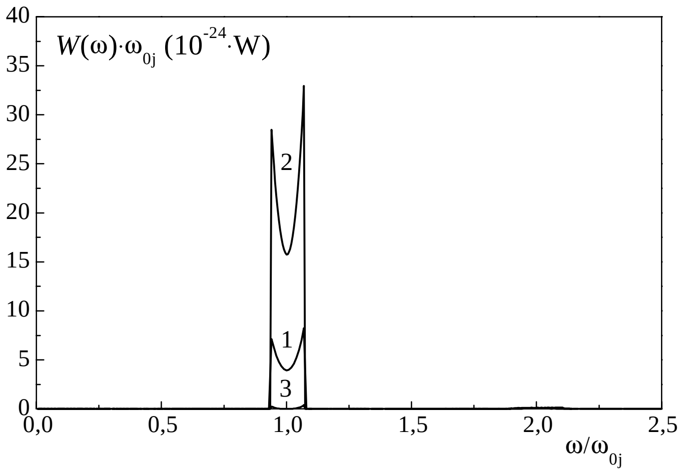

Fig. 1. Spectral distribution of radiation power for the electrons moving in a spiral in non-relativistic case at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.2 \cdot 10^8$ m/s, $V_{\parallel vac} = 0.2 \cdot 10^8$ m/s, $r_{0j} = 1.142$ m, $\omega_{0j} = 0.1751 \cdot 10^8$ rad/s, j=1,2,…,10.

Curve 1 – the radiation spectrum of a single electron at $P^{tot}_{vac1} = 0.713 \cdot 10^{-24}$ W, $P^{\text{int}}_{vac1} = 0.722 \cdot 10^{-24}$ W,

Curve 2 – two electrons at $\Delta t^2_{12} = 0.0001\pi / \omega_{02}$ and $P^{\text{int}}_{vac2} = 0.2888 \cdot 10^{-23}$ W,

Curve 3 – two electrons at $\Delta t^3_{12} = \pi / \omega_{03}$ and $P^{\text{int}}_{vac3} = 0.502 \cdot 10^{-25}$ W.

For the time shift $\Delta t^2_{12} = 0.0001\pi / \omega_{02}$ (curve 2 in Fig. 1) the coherence factor $S_2(\omega) \cong 4$ and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four more than a single electron.

For the time shift $\Delta t^3_{12} = \pi / \omega_{03}$ (curve 3 in Fig. 1) the power of radiation of two electrons is by an order of magnitude lower than that of a single electron. It corresponds to a half of the period of rotation.

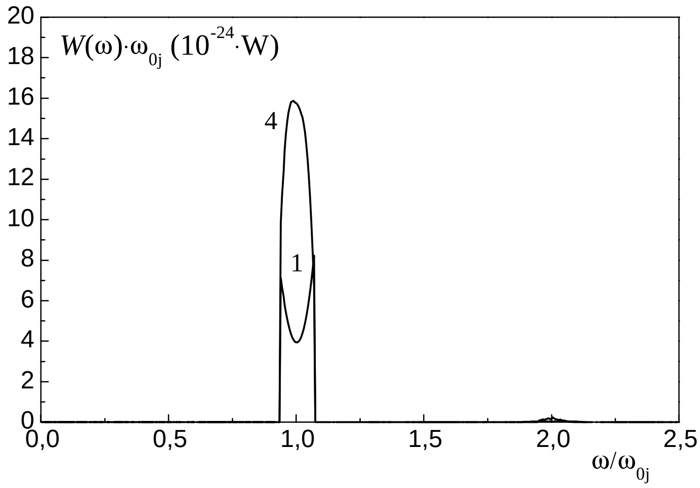

Fig. 2. Spectral distribution of radiation power for the electrons moving in a spiral in non-relativistic case. Curve 1 – the radiation spectrum of a single electron. Curve 4 – two electrons at $\Delta t^4_{12} = 10\pi / \omega_{04}$ and $P^{\text{int}}_{vac4} = 0.1848 \cdot 10^{-23}$ W.

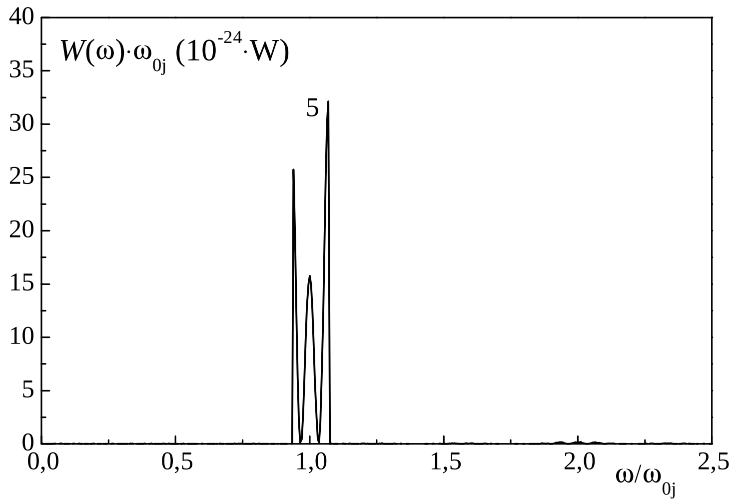

Fig. 3. Spectral distribution of radiation power for two electrons moving in a spiral in non-relativistic case. Curve 5 – $\Delta t_{12}^{5} = 30\pi/\omega_{05}$ and $P_{vac5}^{\text{int}} = 0.1525\cdot 10^{-23}$ W.

At the basic frequency ω_{0j} for the time shift $2i\pi/\omega_{0j}$ (i=4, 5,…) the radiation power spectral distribution of two electrons takes the maximum value (see curves 4 and 5 in Figs 2 and 3).

At the basic frequency ω_{0j} the function of the radiation power spectral distribution of two electrons is equal to zero if the time shift between them in a spiral is equal to $(2i+1)\pi/\omega_{0j}$ (i=0, 1, 2,…) (see Figs 1, 4 and 5).

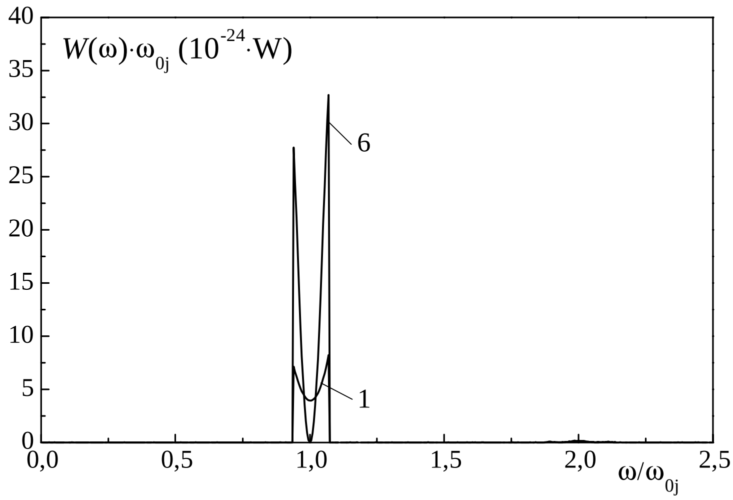

Fig. 4. Spectral distribution of radiation power for two electrons moving in a spiral in non-relativistic case. Curve 6 – $\Delta t_{6} = 15\pi/\omega_{06}$ and $P_{vac6}^{\text{int}} = 0.1684\cdot 10^{-23}$ W.

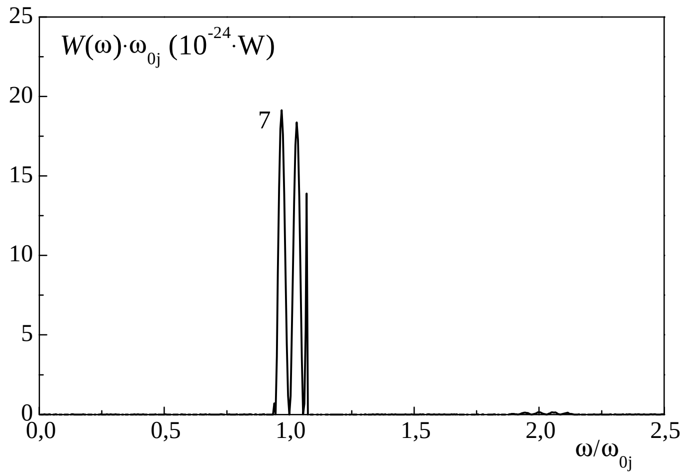

Fig. 5. Spectral distribution of radiation power for two electrons moving in a spiral in non-relativistic case. Curve 7 – $\Delta t_7 = 35\pi/\omega_{07}$ and $P^{\rm int}_{vac\,7} = 0.1205\cdot 10^{-23}$ W.

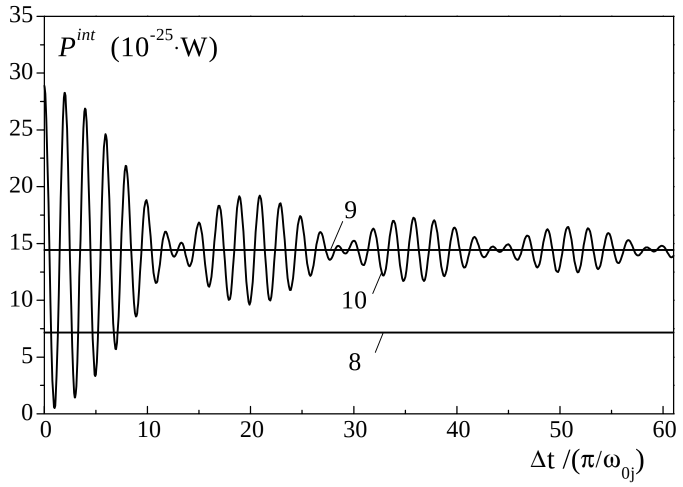

Fig. 6. Radiation power in dependence on time shift between two electrons moving in a spiral in non-relativistic case at $V_{\perp vac} = 0.2\cdot 10^8$ m/s and $V_{\| vac} = 0.2\cdot 10^8$ m/s. Radiation power of a single electron (curve 8). Radiation power of two separate electrons (curve 9). Radiation power of two electrons moves one by one depending on their location in a spiral (curve 10).

The magnitude of the radiation power of two electrons moving one by one in dependence on their location in a spiral in non-relativistic case is presented in Fig. 6. With increasing time shift Δt_{12} the radiation power of two electrons non-monotonously tends to the double radiation power of a single electron.

4. SYNCHROTRON RADIATION SPECTRUM OF TWO ELECTRONS MOVING IN A SPIRAL IN RELATIVISTIC CASE

The influence of the Doppler effect determines the band's boundaries of separate harmonics in the radiation spectrum of two electrons moving one by one along a spiral in a vacuum. In relativistic case for the velocities components $V_{\perp vac} = 0.24\cdot 10^9$ m/s and $V_{\| vac} = 0.15\cdot 10^8$ m/s the radiation power spectral distributions of two electrons in vacuum depending on their location along a spiral are shown in Figs 7–14 (curves 12 and 13, 15 to 18).

The radiation power of the single electron in relativistic case in vacuum $P^{tot}_{vac11} = 0.2852 \cdot 10^{-21}$ W calculated according to relationship (7) is in good agreement to the power $P^{\text{int}}_{vac11} = 0.2092 \cdot 10^{-21}$ W determined after integration according to relationships (2) and (3).

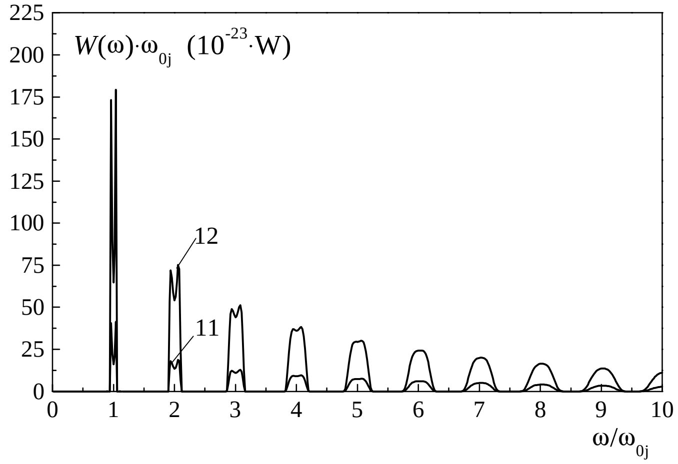

Fig. 7. Spectral distribution of synchrotron radiation power of the electrons moving in a spiral in relativistic case at low harmonics at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.24 \cdot 10^{9}$ m/s and $V_{\| vac} = 0.15 \cdot 10^{8}$ m/s, $\omega_{0j} = 0.105 \cdot 10^{8}$ rad/s $r_{0j} = 22.85$ m, j=11,12,…,18. Curve 11 – a single electron with $P^{\text{int}}_{vac11} = 0.2092 \cdot 10^{-21}$ W, curve 12 – two electrons moving one by one at $\Delta t^{12}_{12} = 0.001\pi / \omega_{012}$ and $P^{\text{int}}_{vac12} = 0.8369 \cdot 10^{-21}$ W.

For the time shifts $\Delta t^{13}_{12} = 0.001\pi / \omega_{013}$ (curve 12 in Fig. 7) the coherence factor $S_2(\omega) \cong 4$ and two electrons in relativistic case radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four more than a single electron ($P^{\text{int}}_{vac12} \cong 4 \cdot P^{\text{int}}_{vav11}$).

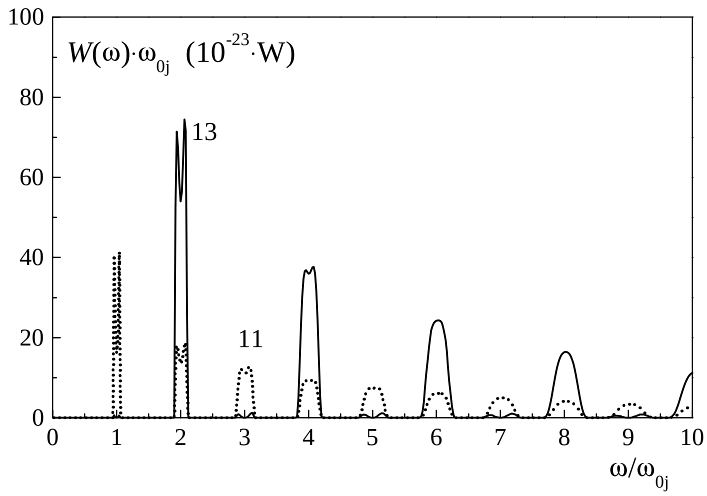

Fig. 8. Spectral distribution of synchrotron radiation power of the electrons moving in a spiral in relativistic case at low harmonics at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.24 \cdot 10^{9}$ m/s, $V_{\| vac} = 0.15 \cdot 10^{8}$ m/s. Curve 11 – a single electron with $P^{\text{int}}_{vac11} = 0.2092 \cdot 10^{-21}$ W, curve 13 – two electrons moving one by one in a spiral at $\Delta t^{13}_{12} = \pi / \omega_{013}$ and $P^{\text{int}}_{vac13} = 0.3928 \cdot 10^{-21}$ W.

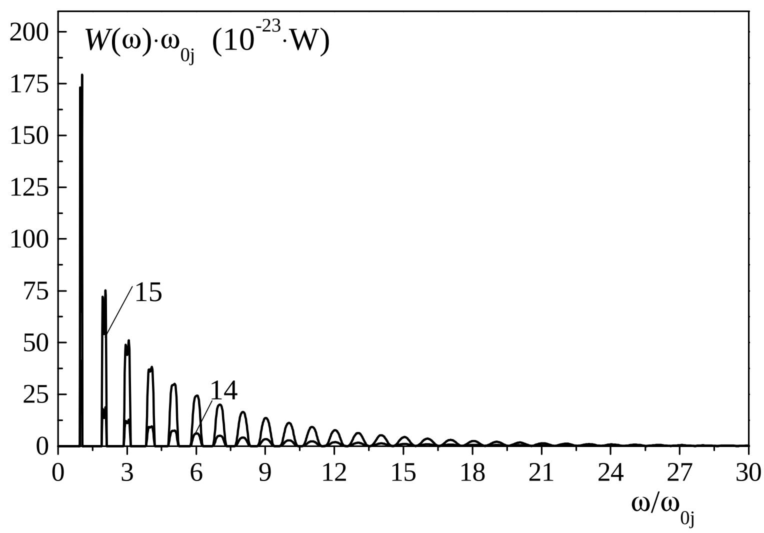

Fig. 9. Spectral distribution of synchrotron radiation power of the electrons moving in a spiral in relativistic case at higher harmonics at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.24 \cdot 10^9$ m/s, $V_{\parallel vac} = 0.15 \cdot 10^8$ m/s. Curve 14. – a single electron with $P^{\text{int}}_{vac14} = 0.2817 \cdot 10^{-21}$ W, curve 15 – two electrons moving one by one at $\Delta t^{15}_{12} = 0.001\pi / \omega_{015}$ and $P^{\text{int}}_{vac15} = 0.1127 \cdot 10^{-20}$ W.

In the case of uniform location of two electrons along a spiral at the time shift $\Delta t^{16}_{12} = \pi / \cdot\, \omega_{016}$ (curve 16 in Fig. 10) we have found that any radiation at the frequencies $(2i+1)\omega_{016}$ (i=0,1,2,…,7,…) is absent.

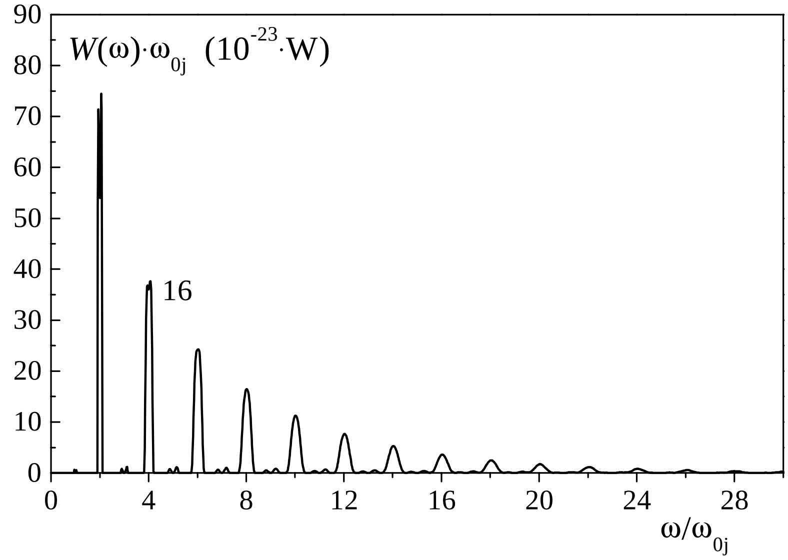

Fig. 10. Spectral distribution of synchrotron radiation power of two electrons moving in a spiral in relativistic case at higher harmonics at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.24 \cdot 10^9$ m/s and $V_{\parallel vac} = 0.15 \cdot 10^8$ m/s. curve 16 – the time shift $\Delta t^{16}_{12} = \pi / \omega_{016}$ and $P^{\text{int}}_{vac16} = 0.5415 \cdot 10^{-21}$ W.

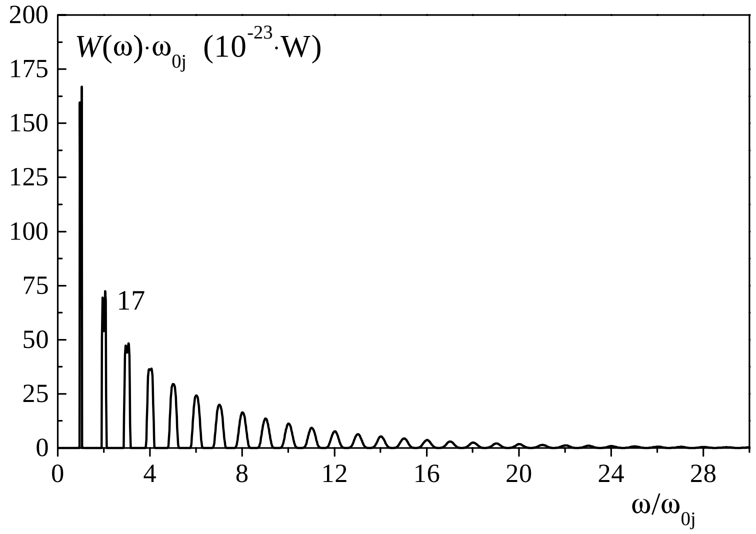

Fig. 11. Spectral distribution of synchrotron radiation power of two electrons moving in a spiral in relativistic case at higher harmonics at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.24 \cdot 10^{9}$ m/s and $V_{\parallel vac} = 0.15 \cdot 10^{8}$ m/s. curve 17 – the time shift $\Delta t_{12}^{17} = 2\pi / \omega_{017}$ and $P_{vac17}^{\text{int}} = 0.9876 \cdot 10^{-21}$ W.

For the time shift $\Delta t_{12}^{17} = 2\pi / \omega_{017}$ (see Figs 11 and 12) the coherence factor is equal to zero at frequencies $(2i+1)\cdot \omega_{018} / 2$ ($i = 0,1,2,\ldots$) and at these frequencies the radiation is absent.

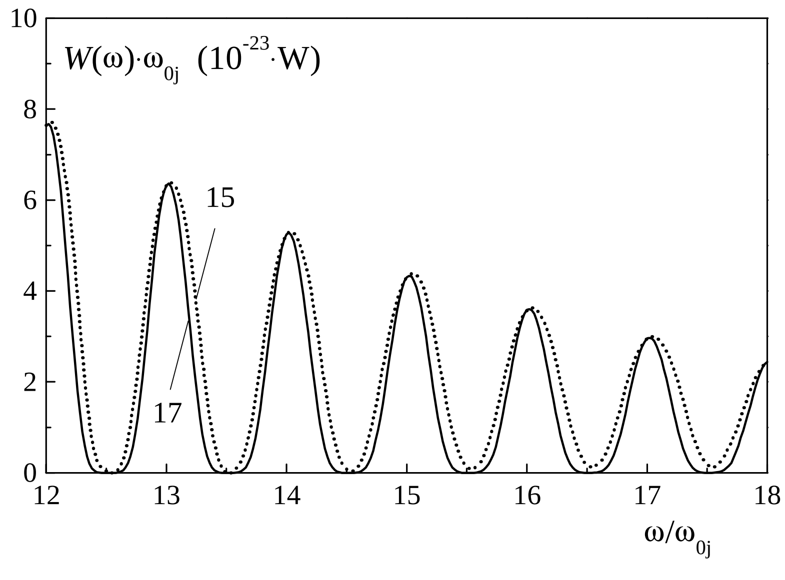

Fig. 12. Spectral distribution of synchrotron radiation power of two electrons moving one by one in a spiral at higher harmonics.

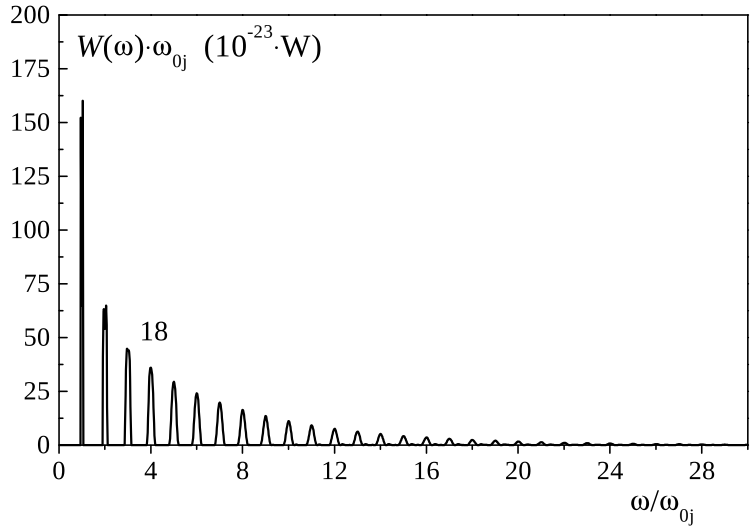

Fig. 13. Spectral distribution of synchrotron radiation power of two electrons moving in a spiral in relativistic case at higher harmonics at $B^{ext} = 10^{-4}$ T, $V_{\perp vac} = 0.24 \cdot 10^{9}$ m/s and $V_{\parallel vac} = 0.15 \cdot 10^{8}$ m/s, curve 18 – the time shift $\Delta t_{12}^{18} = 4\pi / \omega_{018}$ and $P_{vac\,18}^{\text{int}} = 0.7834 \cdot 10^{-21}$ W.

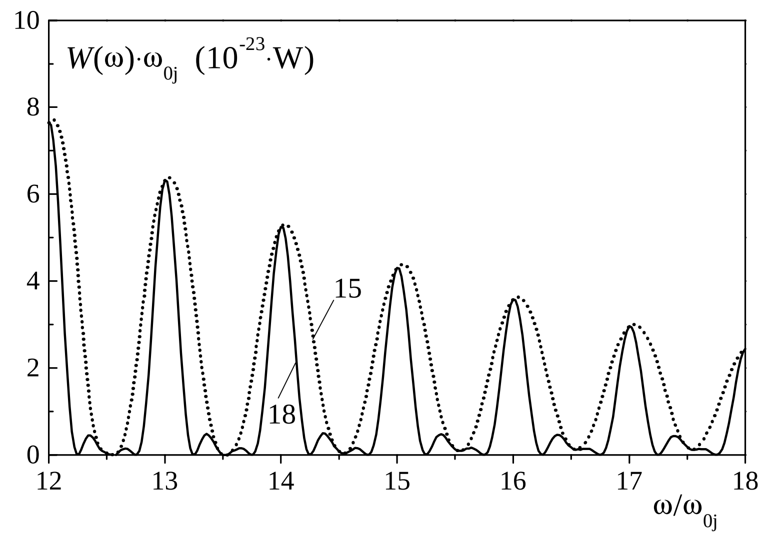

Fig. 14. Spectral distribution of synchrotron radiation power of two electrons moving in a spiral in relativistic case at higher harmonics

For the time shift $\Delta t_{12}^{18} = 4\pi / \omega_{018}$ (see Figs 13 and 14) the coherence factor is equal to zero at frequencies $(2i+1)\cdot \omega_{018} / 4$ ($i = 0,1,2,...$) and at these frequencies the radiation is absent.

The Doppler effect determines the band's boundaries of separate harmonics in the radiation spectra of two electrons moving one by one along a spiral in a vacuum.

5. CONCLUSIONS

The Doppler effect determines the band's boundaries of separate harmonics in the radiation spectra of two electrons moving one by one along a spiral in a vacuum.

For small time shifts the coherence factor in non-relativistic and in relativistic cases is $S_2(\omega) \cong 4$.

In the case of uniform location of two electrons along a spiral at the time shift $\Delta t_{12}^{16} = \pi / \cdot \omega_{016}$ any radiation at the frequencies $(2i+1)\omega_{016}$ (i=0,1,2,…,7,…) is absent.

With increasing time shift Δt_{12} the radiation power of two electrons moving along a spiral in non-relativistic case non-monotonously tends to the double radiation power of a single charge.

The coherence factor leads to essential changes in the radiation power spectral distribution of two electrons in dependence on their position in a spiral.

REFERENCES

1. TERNOV, I.M., *Synchrotron radiation*, Usp. Fiz. Nauk, **165**, 4, pp. 429–456, 1995 (in Russian).
2. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., RARENKO, I.M., KONSTANTINOVICH, I.A., ZHARKOI, V.P., *Radiation spectrum of the system of charged particles moving in non-absorbable isotropic medium*, J. Physical Studies, **4**, 1,, pp. 48–56, 2000 (in Ukrainian).
3. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A., *Classical radiation theory of charged particles. II. Medium influence on the radiation spectrum of charged particles moving in electromagnetic fields*, Bulletin of Chernivtsi National University. Physics and Electronics. No 132, pp. 24-39, 2002 (Ukrainian).
4. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A., *Fine structure of radiation spectrum of charged particles moving in magnetic fields in non-absorbable isotropic media and in vacuum,* Romanian Journal of Physics, **48,** 5–6, pp. 717–725, 2003.
5. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A, *Radiation spectra of electrons moving in a spiral in magnetic fields,* Proceedings of CAOL'2003, 1st International Conference on Advanced Optoelectronics and Lasers, V. 1, Alushta, Crimea, Ukraine, 16–20 September 2003, pp. 161–164.
6. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A., *Radiation power spectral distribution of electrons moving in a spiral in magnetic fields,* Journal of Optoelectronics and Advanced Materials,. **5**, 5, pp. 1423–1431, 2003.
7. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A., *Radiation spectra of charged particles moving in a spiral in magnetic fields,* Proceedings of the Romanian Academy, A, **4**, 3, pp. 175–182, 2003.
8. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH,II.A., *Peculiarities of the radiation spectra of electrons moving in a spiral in magnetic fields,* Proceedings of LFNM 2004, 6th International Conference on Laser and Fiber–Optical Networks Modeling, Kharkov, Ukraine, 6–9 September 2004, pp. 266–268.
9. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A., RARENKO,, I.M., *Radiation power spectral distribution of electrons moving in a spiral in magnetic field,* Proceedings of the Ivth International Hutsulian Workshop on Mathematical Theories and Their Applications in Physics & Technology, Chernivtsi, Ukraine, 28 October – 02 November 2002, Kyiv: TIMPANI. 2004, pp. 321–.336.
10. KONSTANTINOVICH, A.V., KONSTANTINOVICH, I.A., *Radiation spectra of charged particles moving in magnetic fields in vacuum and in medium,* Romanian Journal of Optoelectronics, **12**, 3, pp. 13–25, 2004.
11. KONSTANTINOVICH, A.V., MELNYCHUK, S.V., KONSTANTINOVICH, I.A., *Radiation spectra of charged particles moving in magnetic field,* Romanian Journal of Physics, **50**, 3–4, pp. 347–356, 2005.
12. KONSTANTINOVICH, A.V., KONSTANTINOVICH, I.A *The radiation spectra of electrons moving in a spiral in magnetic fields in media,* Proceedings of LFNM 2005, 7th International Conference on Laser and Fiber- Optical Networks Modeling. Yalta, Crimea, Ukraine, 15–17 September 2005, pp. 68–71.
13. DEACON, D.A., ELIAS, L.R., MADEY, J.M.J., RAIMAN, G.J., SCHWETTMAN, H.A., SMITH, T.I., *First operation of a free-electron laser,* Phys. Rev. Lett., **38**, 16, pp. 892–894, 1977.
14. DATTOLI, G., MEZA, L., MIGLIORATI, M., PALUMBO, L., *Storage ging free-electron laser and microwave type instabilities,* Nuovo Cimento, **115B**, pp. 639–644, 2000.
15. BOLOTOVSKII, B.M., *The theory of the Vavilov-Cherenkov effect,* Usp. Fiz. Nauk, **62**, P. 201-246, 1957 (in Russian).
16. SOKOLOV, A.A., ZHUKOVSKII, V.Ch., KOLESNIKOVA, M.M., NIKITINA, N.S., SHISHANIN, O.E., *On the synchrotron radiation of the electron moving in a spiral,* Izv. Vuzov. Fizika, 2, pp.108–116, 1969 (in Russian).

Received August 16, 2006