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ON THE PROPAGATION OF MAGNETOELASTIC WAVES

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The purpose of this paper is to analyse the motion of the transverse magnetoelastic waves in an incompressible, neo-Hookean solid interacting with a magnetic field. It is shown a decay of the waves amplitudes as $t \to \infty$, and a resonant interaction between the fast and/or the slow waves, for nonzero initial conditions.

Key words: magnetoelastic waves, neo-Hookean solid, resonant interaction, fast and slow waves.

1. INTRODUCTION

The electroelastic and magnetoelastic waves are produced by the interaction between the mechanical waves and the electromagnetic field and their behavior are of interest in various acoustic devices design. With development of nanotechnology and nanoapplications in physics and engineering, there is a significant increase of interest to magnetoelasticity (Zinchuk and Podlipenets [1], Savin [2], Tucker and Rampton [3]).

Besides practical importance there is a theoretical interest in the propagation of magnetoelastic waves in different media, in the understanding of the evolution of waves, their stability and instability, the role of localized waves, etc.

In contrast to the known results in elastodynamics, where no decay of elastic waves is proved, the interaction between elastic and magnetic effects determine a decay of amplitudes of solutions. The energy decay of magnetoelastic waves in bounded conductive media is proved by Menzala and Zuazua [4]. Shul'ga and Ratushnyak [5] have studied the behavior of magnetoelastic Love waves in laminate ferrite-dielectrics media. Chiroiu *et al* [6] have analysed the subharmonic generation of magnetoelastic Love waves in a ferrite-dielectric plate with Cantor-like structure. It is shown that the waves generated by interacting mechanical waves and an electromagnetic field, are expressed as a nonlinear superposition of cnoidal waves in both phonon and fracton vibration regime.

Domanski [7] has applied the asymptotic expansion of high-frequency small amplitude weaves to study the nonlinear equations of magnetoelasticity, and the resonant interaction effects. Rivera and Santos [8], and Andreouand Dassios [9] have shown also that the 3D magnetoelastic waves decays to zero as time goes to infinity, provided the initial data is smooth enough. Di Perna and Majda [10], Joly *et al.* [11] and Majda and Rosales [12] have studies the resonantly interacting of nonlinear hyperbolic waves.

The main purpose of this paper is to study the motion of the transverse magnetoelastic waves in a incompressible, neo-Hookean solid interacting with a magnetic field, and to evidentiate a time decay of amplitude of solutions, and also the resonant interactions between the fast and slow waves.

The model of neo-Hookean solid assumes that the extra stresses due to deformation are proportional to Finger tensor (the left Cauchy-Green deformation tensor)

$$\mathbf{T} = -p\mathbf{I} + G\mathbf{B}, \tag{1.1}$$

Recommended by Radu VOINEA, member of the Romanian Academy where $\mathbf{T}$ is the stress tensor, p the pressure, $\mathbf{I}$ the unity tensor, G the shear modulus, and $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ the Finger tensor, with $\mathbf{F} = \nabla\mathbf{x}$ the deformation gradient tensor. This model is used for modelling materials, for which the deformation is not small.

2. FORMULATION OF THE PROBLEM

Consider an incompressible, conductor of electricity and thermally non-conducting medium, with constant entropy. Consider that the medium is isotropic in its underformed state. The system of equations in Eulerian coordinates are consisted from two Maxwell equations of the infinitely conducting medium, the balance of elastic momentum equation and the mass balance equation [7]

$$\mathrm{div}\mathbf{B} = 0\,, \tag{2.1a}$$

$$\mathbf{B}_{,t} + \mathrm{rot}(\mathbf{B}\times\mathbf{u}) = 0\,, \tag{2.1b}$$

$$\rho(\mathbf{v}_{,t} + \mathbf{v}\cdot\mathrm{grad}\mathbf{v}) - \mathrm{div}\mathbf{T} - \mathrm{div}\mathbf{M} = 0\,, \tag{2.1c}$$

$$\rho_{,t} + \mathrm{div}(\rho\mathbf{v}) = 0\,, \tag{2.1d}$$

where $\mathbf{v}$ the velocity vector, ρ density, $\mathbf{B}$ is the magnetic induction vector, $\mathbf{T}$ elastic stress tensor and $\mathbf{M}$ magnetic stress tensor. The term $\mathrm{div}\mathbf{M}$ represents the action of the magnetic stresses. The comma represents the differentiation with respect to the specified variable, and the dot is the scalar product.

In the Lagrangian coordinates, with considering only the 1D transverse wave motion (with $\mathbf{e}_1$ the unit vector in the coordinate direction x), the system (2.1) yields

$$B_{1,x} = 0\,, \tag{2.2a}$$

$$\mathbf{B}_{,t} - (\mathbf{v}B_1)_{,x} = 0\,, \tag{2.2b}$$

$$\rho\mathbf{v}_{,t} - (\mathbf{e}_1\cdot\mathbf{T})_{,x} - (\mathbf{e}_1\cdot\mathbf{M})_{,x} = 0\,, \tag{2.2c}$$

$$\rho_{,t} = 0\,, \tag{2.2d}$$

where the components of the magnetic stress tensor $\mathbf{M}$ are (μ is the permiability)

$$M_{11} = \frac{1}{2\mu}(B_1^2 - B_2^2 - B_3^2)\,,\ M_{12} = \frac{1}{\mu}B_1 B_2\,,\ M_{13} = \frac{1}{\mu}B_1 B_3\,. \tag{2.3}$$

So, the sytem of equations (2.2) becomes

$$B_{i,t} - (v_i B_1)_{,x} = 0\,,\ i = 1,2,3\,. \tag{2.4a}$$

$$\rho v_{i,t} - (2\phi_{,N} w_i + \frac{1}{\mu}B_1 B_i)_{,x} = 0\,,\ i = 1,2,3\,, \tag{2.4b}$$

$$w_{i,t} - v_{i,x} = 0\,,\ i = 1,2,3\,. \tag{2.4c}$$

In (2.4) $\mathbf{w}$ is the transverse strain vector of components w_i (w_1, w_2 and w_3), $\phi(N)$ is the internal energy function of $N = w_2^2 + w_3^2$. The magnetoelastic coupling in a magnetoelastic material couples the mechanical strain in a material to the magnetic field.

The total energy of a magnetoelastic material in a magnetic field is a sum $\phi = \phi_H + \phi_{anis} + \phi_{me} + \phi_{elas} + \phi_{stress}$, where ϕ_H is the energy due to the interaction between the applied magnetic field and the magnetization of the material, ϕ_{anis} is the energy due to the magnetic anisotropy of the material (the tendency of the magnetization to have a preferred direction), ϕ_{me} is the magnetoelastic coupling energy, ϕ_{elas} is the energy due to the intrinsic stiffness of the material, and ϕ_{stress} is the energy due to an externally applied stress. For 1D neo-Hookean solid, isotropic n its underformed state, the internal energy depends on two independent invariants $I_1 = \frac{1}{2}N$ and $I_2 = \frac{1}{2}N + \frac{1}{4}N^2$, $N = w_2^2 + w_3^2$.

Considering $\rho = 1$, we have $v_1 = 0$, and from (2.1a), $B_1 = \text{const.}$. Also, $w_1 = 0$. The dependent nonzero variables are B_2, B_3, v_2, v_3, w_2 and w_3. The initial conditions are

$$B_2(x,0) = B_2^0(x),\ B_3(x,0) = B_3^0(x),\ v_2(x,0) = v_2^0(x),\ v_3(x,0) = v_3^0(x),$$
$$w_2(x,0) = w_2^0(x),\ w_3(x,0) = w_3^0(x). \tag{2.5}$$

3. SOLUTIONS AND RESULTS

To solve the problem (2.4) –(2.5) we see that these nonlinear hyperbolic equations have the form

$$u_{k,t} + A_{kj}(\mathbf{u})u_{j,x} = 0,\ k = 1,2...,6 \tag{3.1}$$

where $u_k \equiv \{B_2, B_3, v_2, v_3, w_2, w_3\}$ and

$$A_{13} = A_{24} = B_1,\ A_{31} = -A_{42} = -\frac{B_1}{\mu\rho},\ A_{35} = 2\phi_{,N} + 4w_2^2\phi_{,NN},\ A_{45} = 4w_2w_3\phi_{,NN},$$
$$A_{46} = 2\phi_{,N} + 4w_3^2\phi_{,NN},\ A_{53} = A_{64} = 1. \tag{3.2}$$

The characteristic eqyation of (3.1) $\det(\lambda\mathbf{I} - \mathbf{A}) = 0$ has the solutions (Domanski [7]):

- Case.1.

$$\lambda_1 = \lambda_2 = 0,\ \lambda_3 = -\lambda_4 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N} + 4N\phi_{,NN}},\ \lambda_5 = -\lambda_6 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N}}, \tag{3.3a}$$

if $\phi_{,NN} > 0$ and $\phi_{,N} + \frac{1}{2}\frac{B_1^2}{\mu\rho} > 0$, and

- Case 2.

$$\lambda_1 = \lambda_2 = 0,\ \lambda_3 = -\lambda_4 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N}},\ \lambda_5 = -\lambda_6 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N} + 4N\phi_{,NN}}, \tag{3.3b}$$

if $\phi_{,NN} < 0$ and $\phi_{,NN} + \frac{1}{4N}\left(\frac{B_1^2}{\mu\rho} + 2\phi_{,N}\right) > 0$.

The relation of the internal energy $\phi(N)$ to the first Piolla-Kirchhoff stress tensor is obtained from (1.1)

$$T_{11} = 2\phi_{,N} - p\,,\ T_{1k} = 2w_k\phi_{,N}\,,\ k = 2,3\,. \tag{3.4}$$

The calculations are carried out for cnoidal periodical variation of the initial data

$$B_2(x,0) = \mathrm{cn}(b_2 x)\,,\ B_3(x,0) = \mathrm{cn}(b_3 x)\,,\ v_2(x,0) = \mathrm{cn}(v_2^0 x)\,,\ v_3(x,0) = \mathrm{cn}(v_3^0 x)\,,$$
$$w_2(x,0) = \mathrm{cn}(w_2^0 x)\,,\ w_3(x,0) = \mathrm{cn}(w_2^0 x)\,, \tag{3.5}$$

where $b_2, b_3, v_2^0, \ldots$ are known constants.

To solve the system (3.1) and (3.5) we apply the cnoidal method (Munteanu and Donescu [13]) consider the solution $\mathbf{u}$ of the form of cnoidal waves

$$u_i = \sum_{j=1}^{M} c_{ij}(t)\mathrm{cn}^2(k_{ij}x - a_{ij}t)\,,\ i = 1,2,\ldots,6\,, \tag{3.6}$$

where $c_{ij}(t)$ are the amplitude functions, k_{ij} are the wave numbers, and a_{ij} the angular frequencies, $i = 1,2,\ldots,6$, $j = 1,2,\ldots,M$, and

$$c_{ij}(t) = 1 + \left(b + \left(\frac{t}{t_0}\right)^{-p}\right)\exp\left(-\frac{t}{t_0}\right). \tag{3.7}$$

The parameters k_{ij}, a_{ij}, $i = 1,2,\ldots,6$, $j = 1,2,\ldots,M$, and t_0, p and b are determined by a numerically inverse problem based on a genetic algortirhm (Chiroiu et Chiroiu [14]), through the condition to satisfy the equations (3.1) and (3.5). The critical time t_0 marks the moment of time from which the decay of amplitude is acting.

Similar time decay functions ares found in the modeling of liquid crystal in the isotropic phase from very short to very long time (Gottke *et al.* [15]). The calculations show that two kinds of waves exist : a fast wave of velocity c_1, and a slow wave of velocity c_2 (Domanski [7])

$$\text{Case 1}\quad c_1 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N} + 4N\phi_{,NN}}\,,\ c_2 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N}}\,, \tag{3.8a}$$

$$\text{Case 2}\quad c_1 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N}}\,,\ c_1 = \sqrt{\frac{B_1^2}{\mu\rho} + 2\phi_{,N} + 4N\phi_{,NN}}\,. \tag{3.8b}$$

For nonzero initial conditions (3.5), the propagation of waves are characterized by two new effects : a decay in time of the amplitudes and a resonant interactions between the fast and slow waves.

We consider the case of a ferrite characterized by $\rho = 4800\mathrm{kg/m^3}$, $\mu = 10^{-5}$ H/m and modulus of elasticity $1.2\times10^{11}\mathrm{N/m^2}$.

Figs. 3.1 and 3.2 illustrates the spatial variation profiles of the fast and slow strain waves w_2 and respectively, w_3 for $t = t_0$ in the case 1. An interesting feature of the solutions (3.6) and (3.7) consists in a decay of amplitudes for $t \to \infty$, for nonzero initial conditions (3.5). The decay of the waves amplitudes w_2 and respectively, w_3 as $t \to \infty$, for the case 1 are represented in figs. 3.3 and 3.4.

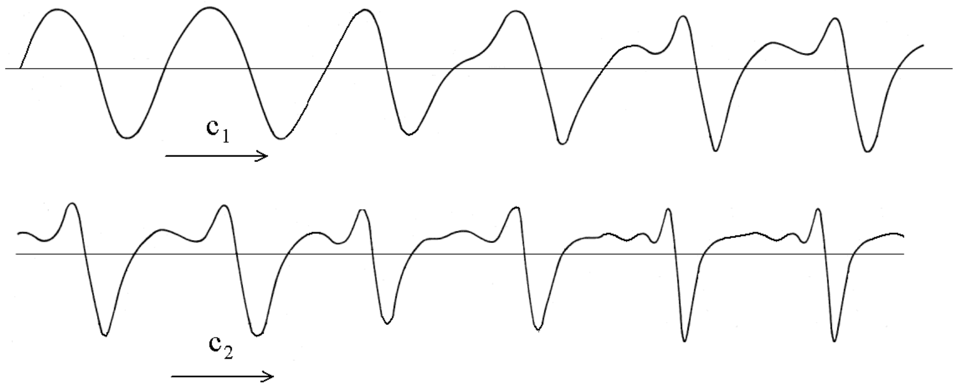

Fig. 3.1. The space variation profiles of fast and slow strain waves w_2 for $t = t_0$ (case 1)

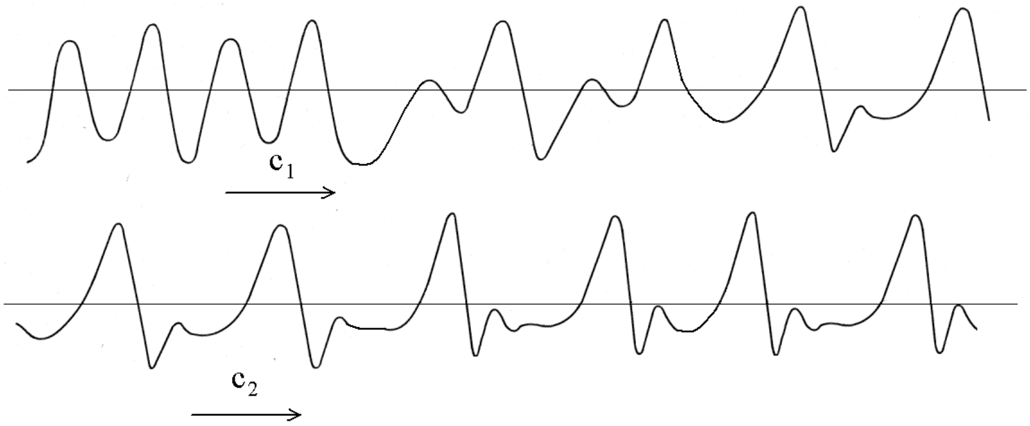

Fig. 3.2. The space variation profiles of fast and slow strain waves w_3 for $t = t_0$ (case 1).

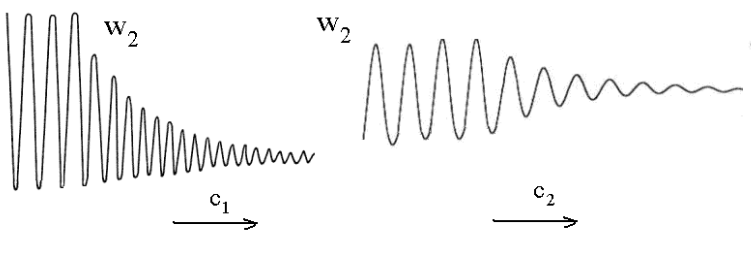

Fig. 3.3. The decay of the wave amplitudes w_2 as $t \to \infty$ (case 1).

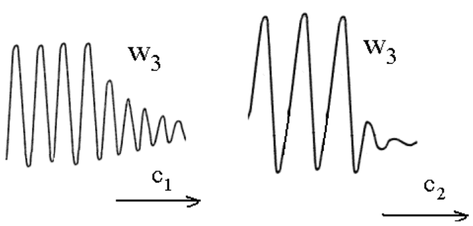

Fig. 3.4. The decay of the wave amplitudes w_3 as $t \to \infty$ (case 1).

By the interaction of two waves, the fast or/and slow waves, the resonance phenomenon can appear. Resonance occurs when two waves share the same vibrational frequency, or a very closed range of frequencies. When one of the wave is propagating, it forces the second wave with the same natural frequency, into vibrational motion. New resonant waves are produced and propagate with the same frequency or with the frequencies being a linear combination of the frequencies of the previous waves. The result of resonance is a large amplitude that may cause damage in the material.

Different patterns of resonance interaction between two waves, e.g. the fast-fast, the slow-slow and the fast-slow strain waves w_2 in the case 1, which share the same vibrational frequency ω_1 are shown in fig. 3.5. A new resonant wave is produced at the colission of the previous two waves. For n waves which interact resonantly with frequencies and wave numbers (ω_i, k_i), $i = 1,2,...,n$, satisfying $\omega_i = \lambda_i k_i$, $i = 1,2,...,n$, a new resonant wave is produced with the frequency and wavenumber $(\bar{\omega}, \bar{k})$ being linear combinations of the previous waves.

The resonant conditions are

$$\omega_1 + \omega_2 + ...\omega_n + \bar{\omega} = 0, \tag{3.9a}$$

$$k_1 + k_2 + ...k_n + \bar{k} = 0, \tag{3.9b}$$

$$b_2 \neq 0,\ b_3 \neq 0, v_2^0 \neq 0,\ v_3^0 \neq 0,\ w_2^0 \neq 0,\ w_3^0 \neq 0. \tag{3.9c}$$

So, for zero initial conditions, the resonant interaction between waves does not exists.

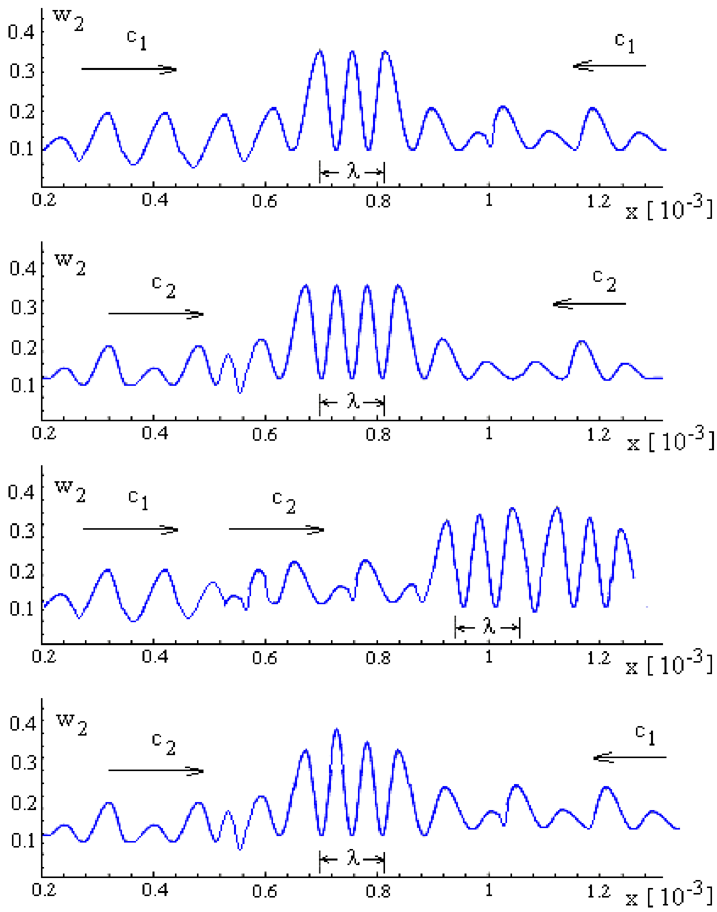

Fig. 3.5. Different patterns of resonance interaction between the fast and/or slow strain waves w_2 (case 1).

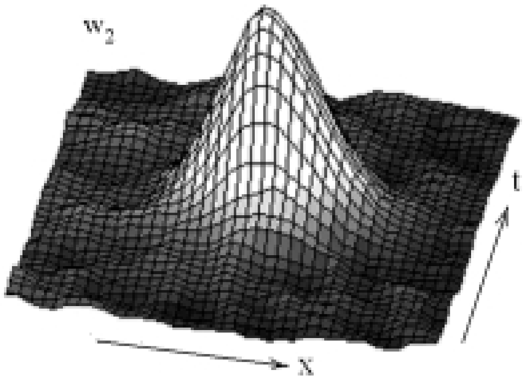

Fig. 3.6. The evolution of a new resonant strain wave w_2 during the interaction between two fast waves (case 1).

The time and spatial evolution of the new resonant strain wave w_2 resulted during the interaction between two fast strain waves w_2 (case 1) with different frequencies and wave numbers (ω_i, k_i), $i = 1, 2$, is shown in fig. 3.6. A larger amplitude is observed in this case then before.

4. CONCLUSIONS

The behavior of magnetoelastic waves are of interest in various acoustic devices design, in which the time decay of wave amplitudes and the resonance intercation between waves are important. In this paper we study the motion of magnetoelastic waves in an incompressible, neo-Hookean solid which interacts with a magnetic field. In contrast to the known results in elastodynamics, where no decay of elastic waves is proved, the interaction between elastic and magnetic effects determine a time decay of wave amplitudes for $t > t_0$, where the critical time t_0 marks the moment of time from which the decay of amplitude is acting.

. Also, a resonant interaction between waves, e..g. between the fast-fast, the slow-slow and the fast-slow waves are observed, for nonzero initial conditions. Resonance occurs when two or more waves waves share the same vibrational frequency, or a very closed range of frequencies. New resonant waves with large amplitudes are formed, which propagate with the same frequency or with the frequencies being a linear combination of the frequencies of the previous waves.

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