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SUBHARMONIC GENERATION OF LOVE WAVES IN A FERRITE-DIELECTRIC PLATE WITH CANTOR-LIKE STRUCTURE

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The purpose of this paper is to analyse the subharmonic generation of magnetoelastic Love waves in a ferrite-dielectric plate with Cantor-like structure. It is shown that the waves generated by interacting mechanical waves and an electromagnetic field, are expressed as a nonlinear superposition of cnoidal waves in both phonon and fracton vibration regime.

Key words: Love waves, ferrite-dielectric plate, Cantor-like structure, vibration modes

1. INTRODUCTION

The electroelastic and magnetoelastic waves are produced by interacting mechanical waves and an electromagnetic field and their behavior are of interest in various acoustic devices design. The regularly laminated structures media such as the piezoelectric-epoxy and ferrite-dielectric media were studied in Zinchuk and Podlipenets [1], Savin [2], Tucker and Rampton [3].

Crăciun *et al.* [4] and Alippi *et al*. [5–7] show the experimental evidence of extremely low thresholds for subharmonic generation of ultrasonic waves in one-dimensional artificial piezoelectric plates with Cantor-like structure, as compared to the corresponding homogeneous and periodical plates. An anharmonic coupling between the extended-vibration (phonon) and the localized-mode (fracton) regimes explained this phenomenon.

Crăciun and Alippi demonstrate that the large enhancement of non-linear interaction results from the more favorable frequency and spatial matching of coupled modes (fractons and phonons) in the Cantor-like structure. Also, Chiroiu C. *et al*. [8], Munteanu and Donescu [9] have analysed the nonlinear behavior of piezoelectric plates with Cantor-like structure and provide theoretically the existence of multiple fracton and multiple phonon-mode regimes in the displacement field. The nonlinear wave motion is a linear superposition of cnoidal waves plus additional terms, which include nonlinear interactions among the waves.

The nonlinear interactions among the cnoidal waves are significant for explaining the draining of the energy away from the input wave towards the low frequency spectrum.

Shul'ga and Ratushnyak [10] have studied the distribution of mechanical and magnetic fields in magnetoelastic Love waves for laminate ferrite-dielectrics media. They have shown that ferrite-dielectric structures exhibit different vibrations modes at different frequencies.

The main purpose of this paper is to study the motion of magnetoelastic Love waves in ferrite-dielectric media with Cantor-like structure. We show that these materials exhibit a special behavior inposed by their Cantor-like structure. The large enhancement of nonlinear interaction results from the more favorable frequency and spatial matching of fractons and phonons modes.

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2. FORMULATION OF THE PROBLEM

We consider a composite plate formed by alternating elements of a ferrite material (Fe) and a dielectric material (D), following a triadic Cantor sequence (fig. 2.1). We consider a triadic Cantor sequence up to the fourth generation (31 elements). A rectangular coordinate system $Ox_1x_2x_3$ (or $Oxyz$) is employed. The origin of the coordinate system $Ox_1x_2x_3$ is located at the left end, in the middle plane of the sample, with the axis Ox_1 (Ox) in-plane and normal to the layers and parralel to the crystallographic axis of the ferrite, and Ox_3 (Oz) out-plane, normal to the plate. The problem is bidimensional, all quantities depending only of x_1 (x) and x_2 (y).

The length of the plate is l, the width of the smallest layer is $l/81$ and the thickness of the plate is h. The width of the plate is d. Let the regions occupied by the plate be $V = V^f \cup V^d$ where V^f and V^d are the regions occupied by ferrite and dielectric layers. Let the unit outward normal of S be n_i the interfaces between constituents be I^{fd}. Wesuppose that the layers are in perfect mechanical and electromagnetic contact.

An index followed by a comma represents partial differentiation with respect to space variables and a superposed dot indicates differentiation with respect to time. Throughout the present paper repeated indices denote summation over the range (1,2,3).

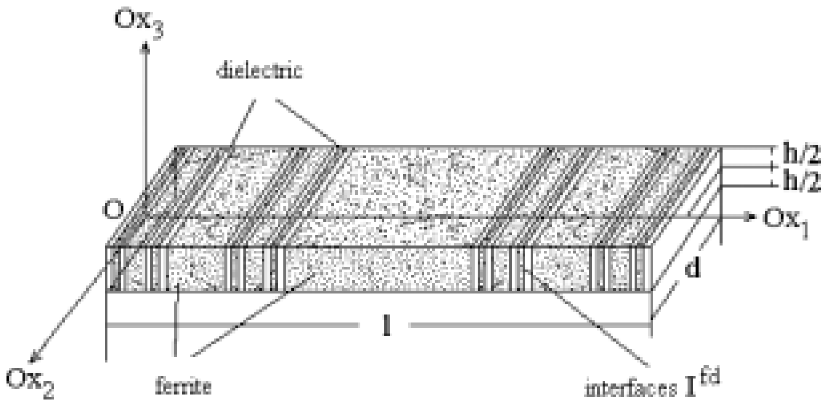

Fig.2.1. The plate with Cantor-like structure. The dashed regions are occupied by ferrite and white regions are occupied by dielectric.

We consider that a magnetizing force $\mathbf{H} = H_0 e_z$ along the z direction periodical electric field is applied to the both surfaces of the plate to excite the Love waves, over a wide frequency range $(10\text{kHz} < \frac{\omega}{2\pi} < 10\text{MHz})$.

The governing equations are linearized about the point of magnetic saturation (M_0, H_0), $\mathbf{M} = M_0 e_z$ (Shul'ga and Ratushnyak [10]) and are composed from :

The motion equation

$$\sigma_{13,1} + \sigma_{23,2} = \rho \ddot{u}_3 \text{ in } V, \qquad (2.1)$$

where u_3 is the displacement in the direction of z axis, σ_{13} and σ_{23} are the components of the stress tensor and ρ is the density.

The flux continuity law

$$b_{1,1} + b_{2,2} = 0\,,\; b_i = h_i + 4\pi m_i\,,\; h_i = -\varphi_{,i}\,,\; i = 1,2\,,\; \text{in } V\,, \tag{2.2}$$

where b_i, $i = 1,2$ are the components of the magnetic flux density vector, h_i, $i = 1,2$, the components of the magnetic field intensity vector, m_i, $i = 1,2$, the components of the magnetization density vector, and $\varphi(x_1, x_2)$ is the potential function.

The magnetization law

$$\dot{m}_1 = -\gamma(H_0 m_2 - M_0 h_2 + b u_{3,2})\,,\; \dot{m}_2 = -\gamma(H_0 m_1 - M_0 h_1 + b u_{3,1}) \;\text{ in } V\,, \tag{2.3}$$

where γ is the gyromagnetic ratio $\gamma = 1.759\times10^{11}\,\mathrm{C/kg}$, M_0 is the saturation magnetization, and H_0 is the saturation of the magnetizing force, and b the magnetoelastic coupling factor.

The constitutive law

$$\sigma_{13} = C_{55} u_{3,1} + \frac{b}{M_0} m_1, \quad \sigma_{23} = C_{55} u_{3,2} + \frac{b}{M_0} m_2 \;\text{ in } V\,, \tag{2.4}$$

where C_{55} is the elastic constant of the ferrite with cubic symmetry.

The equations (2.1) –(2.4) can be reduced to a system of equations for unknowns u_3, σ_{13}, φ and b_1, if the oscillasions are harmonic (Shul'ga and Ratushnyak [10])

$$\delta_1 = \sigma_{13,1} + \mathrm{i} r_{11}\sigma_{3,2} + \mathrm{i} r_{21} b_{1,2} + \mathrm{i}\alpha_{11} u_{3,22} + \rho\omega^2 u_3 + \alpha_{12}\varphi_{,22} = 0\,, \tag{2.5}$$

$$\delta_2 = b_{1,1} + \mathrm{i} r_{21}\sigma_{13,2} + \mathrm{i} r_{22} b_{1,2} + \alpha_{21} u_{3,22} + \alpha_{22}\varphi_{,22} = 0\,, \tag{2.6}$$

$$\delta_3 = u_{3,1} - \frac{\mu}{\Delta_0}\sigma_{13} - \frac{b_{51}}{\Delta_0} b_1 + \mathrm{i} r_{11} u_{3,2} + \frac{r_{21}}{4\pi}\varphi_{,2}\,, \tag{2.7}$$

$$\delta_4 = \varphi_{,1} - \frac{4\pi b_{51}}{\Delta_0}\sigma_{13} + \frac{C'_{55}}{\Delta_0} b_1 + 4\pi \mathrm{i} r_{12} u_{3,2} + r_{22}\varphi_{,2}\,, \tag{2.8}$$

where

$$r_{11} = \frac{4\pi\omega\gamma^2 b^2}{\Delta}\,,\; r_{12} = \frac{\omega\omega_M \gamma b C_{55}}{\Delta}.\; r_{21} = \frac{4\pi\omega\omega_M\gamma b}{\Delta}\,,\; r_{22} = -\frac{\omega\omega_M^2 C_{55}}{\Delta}\,, \tag{2.9}$$

$$\alpha_{11} = C'_{55} + r_{11} C'_{54} + 4\pi r_{12} b_{52}\,,\; \alpha_{21} = 4\pi b_{51} + r_{21} C'_{54} + r_{22} b_{52}\,,\; \alpha_{12} = \frac{1}{4\pi}\alpha_{21}\,, \tag{2.10}$$

$$\alpha_{22} = -\mu + \alpha r_{22} + b_{52} r_{21}\,,\; \Delta_0 = \mu C'_{55} + 4\pi b_{51}^2\,,\; \Delta = C_{55}\omega_M(\omega^2 - \omega_0^2) + 4\pi\omega_M\gamma^2 b^2\,, \tag{2.11}$$

$$\omega_H = \gamma H_0\,,\; \omega_M = 4\pi\gamma M_0\,,\; \omega_0^2 = \omega_H(\omega_H + \omega_M)\,. \tag{2.12}$$

The modified elastic moduli C'_{55} and C'_{54} are given by

$$C'_{55} = C_{55} + \frac{4\pi\omega_H\gamma^2 b^2}{\omega_M(\omega^2 - \omega_H^2)}\,,\quad C'_{54} = \frac{4\pi\omega\gamma^2 b^2}{\omega_M(\omega^2 - \omega_H^2)}\,. \tag{2.13}$$

The components of the Polder tensor μ and α, and the piezomagnetic moduli b_{51} and b_{52} are defined by

$$\mu = 1 - \frac{\omega_H \omega_M}{\omega^2 - \omega_H^2} . \quad \alpha = \frac{\omega \omega_M}{\omega^2 - \omega_H^2} , \tag{2.14}$$

$$b_{51} = \frac{\omega_H \gamma b}{\omega^2 - \omega_H^2} , \quad b_{52} = \frac{\omega \gamma b}{\omega^2 - \omega_H^2} . \tag{2.15}$$

The boundary conditions on the metallized end of the plate $x_1 = 0$

$$\delta_5 = \sigma_{13}(0, x_2, t) = 0 , \ \delta_6 = b_1(0, x_2, t) = 0 , \tag{2.16}$$

The conditions on interfaces I^{fd}

$$\delta_7 = \sigma_{13}(x_1^+, x_2, t) = \sigma_{13}(x_1^-, x_2, t) , \ \delta_8 = b_1(x_1^+, x_2, t) = b_1(x_1^-, x_2, t) , \tag{2.17}$$

$$\delta_9 = u_3(x_1^+, x_2, t) = u_3(x_1^-, x_2, t) , \quad \delta_{10} = \varphi(x_1^+, x_2, t) = \varphi(x_1^-, x_2, t) , \tag{2.18}$$

To solve the problem (2.5) –(2.18) we consider the solution $S \equiv \{u_3, \sigma_{13}, b_1, \varphi\}$ of the form of cnoidal waves

$$S = \sum_{j=1}^{M} c_{ij} \mathrm{cn}^2 (k_{ij} x_1 + l_{ij} x_2 - a_{ij} t) , \ i = 1, 2, 3, 4 , \tag{2.19}$$

where c_{ij} are the amplitude coefficients, k_{ij} and l_{ij} are the wave numbers, and a_{ij} the angular frequencies, $i = 1, 2, 3, 4$, $j = 1, 2, ..., M$. These quantities are determined by an inverse problem.

3. INVERSE PROBLEM

Given a set of measured frequencies $f_n = \omega_n / 2\pi$, $n = 1, 2 ... N$ of the plate we aim at determining the unknown system parameters

$$P = \{ c_{ij} , k_{ij} , l_{ij} , a_{ij} \} , \ i = 1, 2, 3, 4 , \ j = 1, 2, ..., M , \tag{3.1}$$

necessary to analytically construct the solutions of the set of governing equations, i.e. (2.5)–(2.18).

The inverse problem we want to consider is, given the solutions representations (2.19), to find the parameters P given by (3.1) by inversion of the measured natural frequencies of the plate. Because we do not have an experimental set of eigenfrequencies we will use for the inverse problem the set of eigenfrequencies computed from the direct problem.

To extract the parameters P from the "*measured*" natural frequencies data by the nonlinear least-squares optimization technique, an objective function $\Im$ must be chosen that measures the agreement between theoretical and experimental data

$$\Im(P) = \sum_{i=1}^{N} \{[f_i^e - f_i \ (P)]^2\} \ \to \min \tag{3.2}$$

where f_i^e are the measured *i*'th eigenfrequency, f_i the corresponding model prediction, and N the number of measurements of frequencies $N > K$, with K is the number of unknown parameters which are given by P . The primary goal of the optimisation procedure is the minimization of the objective function $\Im(P)$, where P are the design variables which make up the solutions of the problem. The signs "$\to \min$" or "$\to \max$" mean the minimization or maximization of the objective function with some required precision: suppose six decimal places for the variable values is desirable. To measure the accuracy of the identification of P we introduce an error indicator δ to estimate the verification of the governing equations (Chiroiu and Chiroiu [11])

$$\delta(P_{opt}) = \sum_{k=1}^{10} \delta_k \quad , \tag{3.3}$$

with δ_i, $i = 1,2,...,10$ defined by (2.5) –(2.8) and (2.16) –(2.18).

4. RESULTS AND CONCLUSIONS

Numerical simulation was carried out for a single example. We have paid attention on the accurate numerical model to avoid erroneous parameter estimates. The calculus was carried out for l = 67.5 mm and h = 0.3 mm. The material constants are shown in table 4.1. The other constatns are : the coupling factor $b = 6.3\times10^9$ erg/m^3, the gyromagnetic ratio $\gamma = 1.76\times10^7$ Oe^{-1}s^{-1}, the saturation magnetization of ferrite $H_0 = 100Oe$, $M_0 = 1750$Oe.

Table 4.1. The material constants [10]

	dielectric	ferrite
ρ	4.82×10^3 kg/m^3	$.17\times10^3$ kg/m^3
C_{55}	1.504 N/m^2	7.64 N/m^2

The eigenfrequencies $\omega_n / 2\pi$ are determined from the the eigenvalue problem. Table 4.2 shows the computed frequencies and the errors obtained by the eigenvalue-problem.

Table 4.2. Estimation results: computed eigenfrequencies

$\omega_n / 2\pi$								
112.6 ± 0.05	177 ± 0.01	223.3 ± 0.01	262.4 ± 0.1	341 ± 0.05	371.9 ± 0.01	427.4 ± 0.02	511.2 ± 0.04	599.5 ± 0.03
622.3 ± 0.02	677,2 ± 0.02	840.5 ± 0.03	951.2 ± 0.03	1108.5 ± 0.1	1171.3 ± 0.06	1277.1 ± 0.04	1510.3 ± 0.04	1688 ± 0.1
1777.9 ± 0.2	1991.3 ± 0.14	2125.7 ± 0.04	2260.5 ± 0.2	2472.6 ± 0.2	2665.1 ± 0.02	2677 ± 0.02	2980.6 ± 0.2	3355.2 ± 0.4
3547.9 ± 0.04	3579.9 ± 0.03	3697.7 ± 0.02	3784.4 ± 0.14	3975.6 ± 0.04	3995.3 ± 0.25	4247.8 ± 0.08	4264.3 ± 0.05	4299.9 ± 0.03
4362.7 ± 0.07	4528.7 ± 0.2	4666.7 ± 0.1	4699.6 ± 0.08	4776.5 ± 0.2	4799.8 ± 0.04	4866.3 ± 0.02	4875.6 ± 0.07	4888.7 ± 0.05
4899.9 ± 0.01	4921.5 ± 0.04	4944.2 ± 0.1	5013.9 ± 0.2	5021.9 ± 0.17	5125.3 ± 0.08	5156.7 ± 0.18	5244.3 ± 0.2	5259.9 ± 0.3
5298.9 ± 0.2	5318.2 ± 0.02	5319.6 ± 0.08	5322.5 ± 0.3	5356.4 ± 0.15	5377.7 ± 0.41	5421.9 ± 0.25	5433 ± 0.01	5439.9 ± 0.01

The undetermined system parameters P are computed by using a genetic algorithm. The number of the "*measured*" eigenfrequencies is $N = 63$ (table 4.2). The genetic parameters are: number of populations = 45, ratio of reproduction = 1, number of multi-point crossovers = 1, probability of mutation = 0.25 and maximum number of generations = 550.

The genetic algorithm exhibits very good convergence and accuracy. In all computations, the "*measured*" eigenfrequencies are computable from the direct problem. A "*measurement noise*" has been artificially introduced by multiplication of the data values by $1+r$, r being random numbers uniformly distributed in $[-\varepsilon,\varepsilon]$, with $\varepsilon = 10^{-1}, 10^{-2}, 10^{-3}$.

Figs.4.1 and 4.2 show the displacements of the normal modes $\omega/2\pi$ = 427.4 kHz and 951.2 kHz and respectively of the subharmonic modes $\omega/4\pi$ = 213.7 kHz and 475.6 kHz.

Two kind of vibration regimes are found: a localised-mode (fracton) regime represented in fig.4.3 for $\omega/2\pi$ = 2980.6 kHz, 3697.7 kHz and 4247.8 kHz and an extended-vibration(phonon) regime represented in fig. 4.4 for $\omega/2\pi$= 3355.2 kHz. A sketch of the plate geometry is given on the abscissa (dashed, ferrite and white, dielectric).

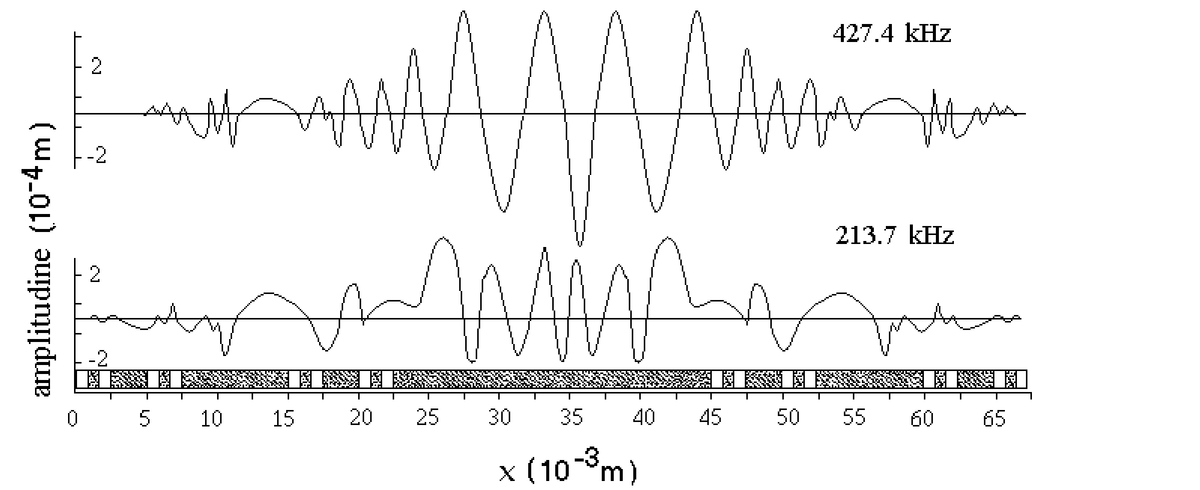

Fig. 4.1. The amplitudes of the Love displacement of the normal mode $\omega/2\pi$ = 427.4 kHz and of the subharmonic mode $\omega/4\pi$ = 213.7 kHz.

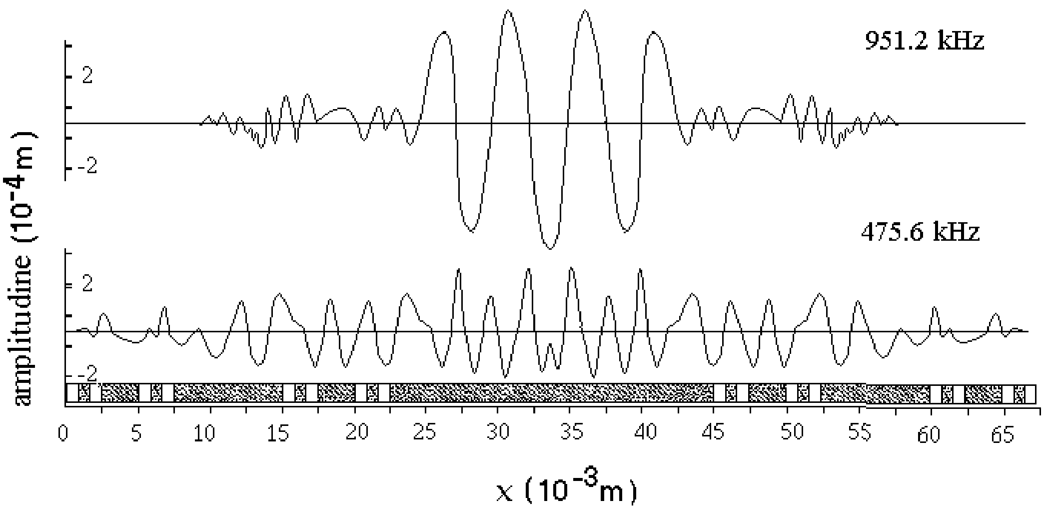

Fig. 4.2. The amplitudes of the Love displacement of the normal mode $\omega/2\pi$ = 951.2 kHz and of the subharmonic mode $\omega/4\pi$ = 475.6 kHz.

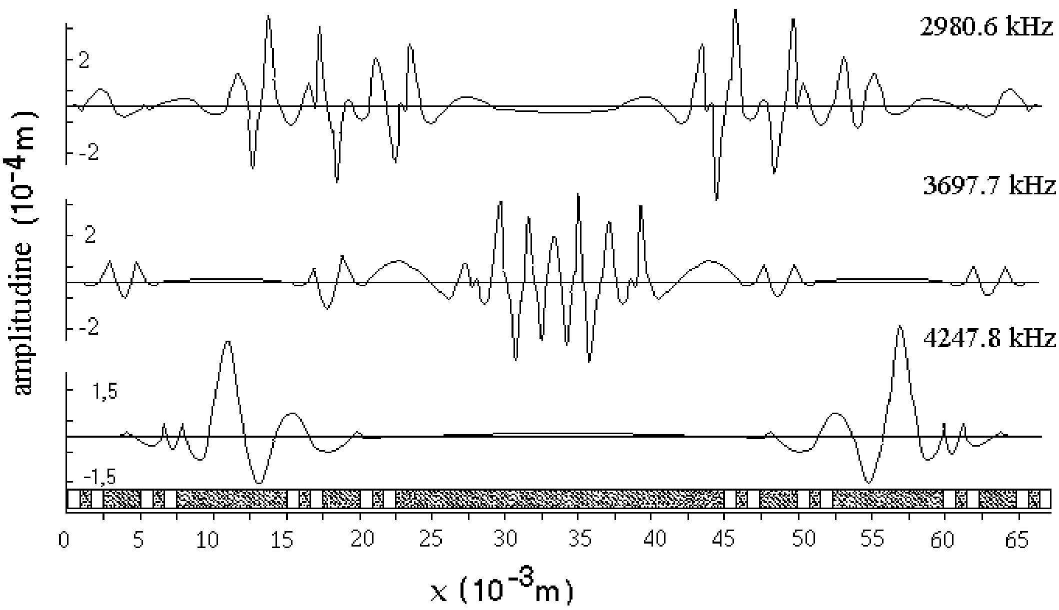

Fig.4.3. The normal amplitudes for three localised vibration modes ($\omega\ /2\pi$ = 2980.5 kHz, $\omega\ /2\pi$ =3697.7 kHz and $\omega\ /2\pi$ =4247.8 kHz).

The fracton vibrations are mostly localised on a few elements, while the phonon vibrations essentially extend to the whole plate. In the case of a periodical plate the dispersion prevents good frequency matching between the fundamental and appropriate subharmonic modes. For the homogeneous plate the mismatch $\omega_n - \omega/2$ is due to the symmetry of fundamental modes with respect to x, but never $\omega/2$ coincides with a plate vibration mode.

For a Cantor plate, the ability of generating the $\omega/2$ subharmonic, is determined by the existence of a normal mode with: (i) small frequency mismatch $\omega_n - \omega/2$, and, (ii) large spatial overlap between the fundamental and subharmonic displacement field

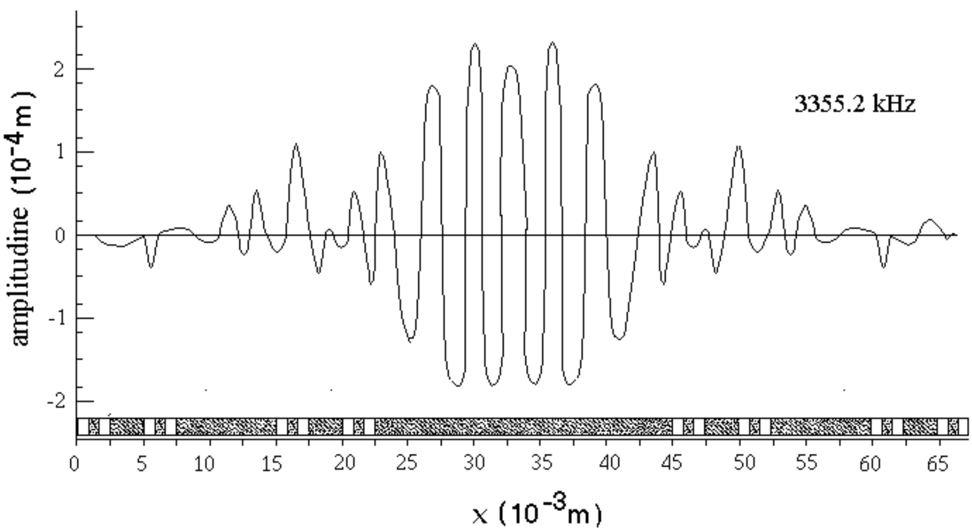

Fig.4.4. The normal amplitudes for two extended vibration modes ($\omega\ /2\pi$ = 3355.2 kHz).

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