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THE MOTION OF A MICROPOLAR FLUID IN INCLINED OPEN CHANNELS

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The turbulent flow of a micropolar fluid downwards on an inclined open channel is studied. The wave profile moves downstream as a linear superposition of solitons at a constant speed and without distortion. The model parameters are determined by using a genetic algorithm.

Key words: turbulent flow, micropolar fluid, solitons, roll-waves, Chezy resisting force, genetic algorithm.

1. INTRODUCTION

In a micropolar fluid the motion is described not only by a deformation but also by a micro-rotation giving six degree of freedom (Eringen, 1966, 1970 [1,2], Brulin, 1982 [3]). The interaction between parts of the fluid is transmitted not only by a force but also by a torque, resulting in asymmetric stresses and couple stresses. The theory of hydrodynamic turbulence is still lacking a fundamental theory from which the physical phenomena can be predicted and understood. The micropolar theory is employed here to obtain solutions which are periodic with respect to distance, describing the phenomenon called "roll-waves" for water flow along a wide inclined channel and to discuss the behavior of the solutions. In this work we study the turbulent flow of a micropolar fluid in a wide channel inclined at an angle $\theta > 0$ below the horizontal.

The principal aim of this paper is to represent the periodic waves as a linear superposition of equally spaced solitons. A similar situation exists for Korteweg–de Vries, various modified Korteweg–de Vries, Boussinesq and Burger equations (Whitham, 1974 [4], Lamb, 1980 [6], Munteanu, Donescu [7]). In the light of inverse scattering theory, this representation may be viewed as a "clean interaction" of solitons, in that they are superimposed but retain their identity and do not destroy each other under the non-linear coupling (Whitham, 1984 [5]).

This representation requires determination of the model parameters: the wave numbers, the frequencies and the phases. For computing these unknown parameters we propose a new method based on a genetic algorithm. Details on the genetic algorithm are available in Goldberg, 1989 [8] and Tanaka, Nakamura, 1994 [9]).

The soliton representation is not so surprisingly in this case because Dressler [10] in 1949 studying the roll-waves motion of the shallow water in inclined open channels has found an equivalent form expressed as a cnoidal wave for a flow subject to the Chezy turbulent resisting force. Our theory generalizes the Dressler theory to the case of a micropolar fluid flow subject to a Chezy resisting force, and demonstrates that the presence of a resistance force, which varies with velocity, is sufficient to permit the construction of periodic solutions.

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2. EQUATIONS OF THE NONLINEAR SHALOW MICROPOLAR FLUID

In the shallow flow the vertical dimensions are small compared to the horizontal dimensions. The motion equations of a micropolar, viscous fluid are given by Eringen [1,2]:

$$\rho\dot{v} + \rho v \text{grad} v = X - \text{grad}\,\pi - (\mu+\alpha)\text{curlcurl}\, v + (2\mu+\lambda)\text{grad div}\, v + 2\alpha \text{curl}\, w, \tag{2.1}$$

$$\rho J\dot{w} + \rho J v \text{grad}\, w = Y - (\gamma+\varepsilon)\text{curlcurl}\, w + (2\gamma+\zeta)\text{grad div}\, w - 4\alpha w + 2\alpha \text{curl}\, v, \tag{2.2}$$

where X is the exterior body forces, Y is the exterior body couples, π is the thermodynamic pressure, ρJ is the inertia tensor density, v is the velocity vector $v = \frac{\partial}{\partial t}u$, u the displacement vector, φ the micro-rotation vector, w the micro-rotation velocity $w = \frac{\partial}{\partial t}\varphi$, ρ the fluid density.

The superposed dot indicates the partial differentiation with respect to time $\dot{a} = \frac{\partial}{\partial t}a$. In (2.1), (2.2) λ and μ are the classical viscosities coefficients of the Navier-Stokes theory. The constants α, ζ, γ and ε are the micropolar coefficients of viscosity. The elastic coefficients must fulfill the condition

$$\mu \ge 0, \quad 2\mu + 3\lambda \ge 0, \quad \alpha \ge 0, \;\; \gamma \ge 0, \quad 2\gamma + 3\zeta \ge 0, \quad \varepsilon \ge 0. \tag{2.3}$$

Equations (2.1) and (2.2) are six equations with unknown vector fields the velocity v and micro-rotation w. These equations must be supplemented by the equation of continuity for an incompressible fluid

$$\text{div}\, v = 0\,. \tag{2.4}$$

In this case, the thermodynamic pressure π must be replaced by an unknown pressure p to be determined through the solution of each problem. The constitutive relations are

$$\sigma_{ij} = (-p + \lambda v_{k,k})\delta_{ij} + (\mu+\alpha)v_{j,i} + (\mu-\alpha)v_{i,j} - 2\alpha\varepsilon_{kij}w_k\,, \tag{2.5}$$

$$\mu_{ij} = \zeta w_{k,k}\delta_{ij} + (\gamma+\varepsilon)w_{j,i} + (\gamma-\varepsilon)w_{i,j}\,, \tag{2.6}$$

where σ_{ij} is the stress tensor and μ_{ij} is the couple-stress tensor.

The field of equations (2.1), (2.2) and (2.4) are subject to certain boundary and initial conditions:

- traction conditions on the surface S of the body B

$$\sigma_{kl}n_k = t_l\,,\;\; \mu_{kl}n_k = \mu_l \;\; \text{on} \;\; S\,, \tag{2.7}$$

where t_l are the surface traction and μ_l the surface couple acting on S.

- velocity conditions of adherence of the fluid to a solid boundary

$$v_k(x,0) = v_k^0(x), \quad w_k(x,0) = w_k^0(x) \quad \text{in} \quad B\,, \tag{2.8}$$

where v_k^0, w_k^0 are the velocity and micro-rotation velocity of the solid boundary. For a rigid stationary boundary we have $v_k^0 = w_k^0 = 0$.

3. TWO-DIMENSIONAL FLOW

Consider a two-dimensional flow of a micropolar, isotropic, incompressible, viscous fluid in a wide channel over a rigid bottom. We have chosen a wide channel to be sure that the motion will bee two-dimensional only. The *x*-axis is horizontally and the bottom is given by $h(x)$. The channel bed is linear and is inclined at an angle $\theta > 0$ below the horizontal $y = -mx$ with $m = \tan\theta > 0$ (fig.1). The vertical distance of the surface above the *x*-axis is denoted by η (x).

The horizontal and vertical components of v and w are, respectively v_1, v_2 and w_1, w_2. We write $X_1 = -r^2\rho v_1 | v_1 | / R = -r^2\rho v_1 | v_1 | / \eta$, $X_2 = -\rho g$, $Y_1 = Y_2 = 0$ in (2.1) and (2.2). Thus, equations (2.1), (2.2) and (2.4) are then

$$\rho\dot{v}_1 + \rho v_1 v_{1,x} + \rho v_2 v_{1,y} = -p_{,x} + (\mu + \alpha)\Delta v_1 - \frac{r^2\rho v_1 | v_1 |}{\eta}, \tag{3.1}$$

$$\rho\dot{v}_2 + \rho v_1 v_{2,x} + \rho v_2 v_{2,y} = -p_{,y} + (\mu + \alpha)\Delta v_2 - \rho g, \tag{3.2}$$

$$\rho J\dot{w}_1 + \rho J v_1 w_{1,x} + \rho J v_2 w_{1,y} = (2\gamma + \zeta) w_{1,xx} + (\gamma + \zeta - \varepsilon) w_{2,xy} + (\gamma + \varepsilon) w_{1,yy} - 4\alpha w_1, \tag{3.3}$$

$$\rho J\dot{w}_2 + \rho J v_1 w_{2,x} + \rho J v_2 w_{2,y} = (2\gamma + \zeta) w_{2,yy} + (\gamma + \zeta - \varepsilon) w_{1,xy} + (\gamma + \varepsilon) w_{2,xx} - 4\alpha w_2, \tag{3.4}$$

$$v_{2,x} - v_{1,y} = 0, \quad w_{2,x} - w_{1,y} = 0, \quad v_{1,x} + v_{2,y} = 0. \tag{3.5}$$

The coma represents the differentiation with respect to the shown variable. In (3.2) g is the acceleration of gravity. In the right side of (3.1) the term $-r^2 v_1 | v_1 | / R$ represents the resisting body force always acting opposite to that of the flow, where r^2 is a constant depending upon the roughness of the channel walls and *R* is the hydraulic radius.

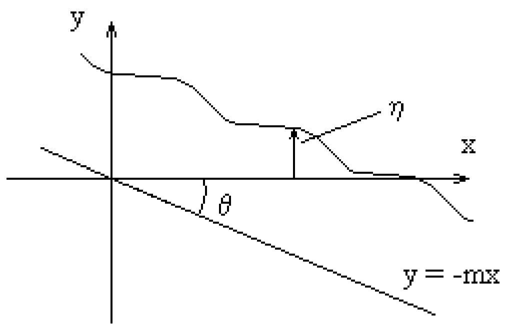

Fig. 1 Geometry of the flow

According to this formula, the turbulent fluctuations exert on the main flow a resistive body force at every point of magnitude $r^2 v_1^2 / R$. Since the most flows in practice are highly turbulent we take account of the resistive force due to the momentum transport of the secondary flow exerted on the average flow at each point. The resistance effects due to the dynamic viscosity of the water are neglected. The above expression of the resisting force was given by Chezy (Dressler [4], 1949). The hydraulic radius is defined as the ratio of the area of a cross section of the water normal to the channel to its "wetted perimeter". That means that part of the perimeter excluding the free surface of the water. The Chezy formula thus expresses the fact that the resistance will be greater in shallow regions where all of the water is closer to the rough boundary. The Chezy formula is valid only for uniform flows, and although it is used for non-uniform flows when the flow vary slowly with respect to *x*, *y* and *t*. In our case $R = \eta$. In equations (3.1)-(3.5) the unknown functions are v_i, w_i, $i = 1,2$, p and η. The boundary conditions are

$$\dot{\eta} + v_1 \eta_{,x} = v_2 \quad \text{at} \quad y = \eta, \tag{3.6}$$

$$p = 0 \quad \text{at} \quad y = \eta, \tag{3.7}$$

$$v_1 m - v_2 = 0 \quad \text{at} \quad y = h, \tag{3.8}$$

The condition (3.6) says that a particle at the surface will remain at the surface and (3.8) - the velocity at the bottom is tangential to the bottom. We add the following condition

$$w_i = 0, \quad i = 1,2 \quad \text{at} \quad y = h. \tag{3.9}$$

Let the constant H be a typical vertical dimension of the resulting flow, and L a typical horizontal dimension. We $H^2/L^2=\delta$ the expansion parameter which we will use for a perturbation procedure. New dimensionless variables are defined by

$$\alpha=\frac{x}{L},\quad \beta=\frac{y}{H},\quad \tau=\frac{\sqrt{gH}}{L}t,\quad \Upsilon=\frac{\eta}{H},\quad P=\frac{p}{\rho gH},\quad V_1=\frac{v_1}{\sqrt{gH}}\ ,$$

$$d=\frac{h}{H}\,,\ V_2=\frac{H\ \ v_2}{L\sqrt{gH}},\quad W_1=\frac{w_1}{\sqrt{g/H}},\quad W_2=\frac{Hw_2}{L\sqrt{g/H}}\,, \tag{3.10}$$

$$\bar{\mu}=\frac{\mu}{\rho L\sqrt{gH}},\quad \bar{\alpha}=\frac{\alpha}{\rho L\sqrt{gH}}\quad ,\ \bar{r}^2=\frac{r^2H}{L},\ \bar{\zeta}=\frac{\zeta}{\rho LJ\sqrt{gH}},\quad \bar{\gamma}=\frac{\gamma}{\rho LJ\sqrt{gH}},\quad \bar{\varepsilon}=\frac{\varepsilon}{\rho LJ\sqrt{gH}}.$$

In terms of the dimensionless variables (3.1)-(3.9) become

$$\delta Y[V_{1,\tau}+V_1V_{1,\alpha}-(\bar{\mu}+\bar{\alpha})V_{1,\alpha\alpha}+P_{,\alpha}]+V_2V_{1,\beta}Y-(\bar{\mu}+\bar{\alpha})V_{1,\beta\beta}Y+\bar{r}^2V_1^2=0\,, \tag{3.11}$$

$$\delta[V_{2,\tau}+V_1V_{2,\alpha}-(\bar{\mu}+\bar{\alpha})V_{2,\alpha\alpha}+P_{,\beta}+1]+V_2V_{2,\beta}-(\bar{\mu}+\bar{\alpha})V_{2,\beta\beta}=0\,, \tag{3.12}$$

$$\delta[W_{1,\tau}+V_1W_{1,\alpha}-(2\bar{\gamma}+\bar{\zeta})W_{1,\alpha\alpha}+4\bar{\alpha}W_1]+V_2W_{1,\beta}-(\bar{\gamma}+\bar{\varepsilon})W_{1,\beta\beta}-(\bar{\gamma}+\bar{\zeta}-\bar{\varepsilon})W_{2,\alpha\beta}=0\,, \tag{3.13}$$

$$\delta[W_{2,\tau}+V_1W_{2,\alpha}-(\bar{\gamma}+\bar{\varepsilon})W_{2,\alpha\alpha}+4\bar{\alpha}W_2]+V_2W_{2,\beta}-(2\bar{\gamma}+\bar{\zeta})W_{2,\beta\beta}-(\bar{\gamma}+\bar{\zeta}-\bar{\varepsilon})W_{1,\alpha\beta}=0\,, \tag{3.14}$$

$$V_{2,\alpha}-V_{1,\beta}=0\,,\ W_{2,\alpha}-W_{1,\beta}=0\,,\quad \delta V_{1,\alpha}+V_{2,\beta}=0\,,\quad \delta(Y_{,\tau}+V_1Y_{,\alpha})=V_2\ \text{ at }\ \beta=Y\,, \tag{3.15}$$

$$P=0\ \text{ at }\ \beta=Y\,,\ \delta V_1m/H=V_2\ \text{ at }\ \beta=d\,,\ W_1=W_2=0\ \text{ at }\ \beta=d\,. \tag{3.16}$$

We assume that the unknowns can be expressed as power series in terms of δ

$$V_i=\sum_{k=0}^{\infty}V_i^{(k)}(\alpha,\beta,\tau)\delta^k\ ,\ \ i=1,2\,,\ W_i=\sum_{k=0}^{\infty}W_i^{(k)}(\alpha,\beta,\tau)\delta^k\ ,\ \ i=1,2\,,$$

$$P=\sum_{k=0}^{\infty}P^{(k)}(\alpha,\beta,\tau)\delta^k\ ,\quad Y=\sum_{k=0}^{\infty}Y^{(k)}(\alpha,\tau)\delta^k\,. \tag{3.17}$$

The series (3.17) are inserted in (3.11)-(3.16) and the resulting coefficients of like powers of δ are equated. Consider that

$$M_i^{(k)}=\frac{\partial^k}{\partial\alpha^k}\log f_i^{(k)}(\alpha,\beta,\tau)\ ,\quad k=1,2,...N\,,\quad i=1,2...6\,, \tag{3.18}$$

where $M=(V_1,V_2,W_1,W_2,P,Y)$ and

$$f_i^{(1)}(\alpha,\beta,\tau)=1+\exp\theta_{1i}\,,\quad f_i^{(2)}(\alpha,\beta,\tau)=1+\exp\theta_{1i}+\exp\theta_{2i}+\exp(\theta_{1i}+\theta_{2i})\,,$$

$$\ldots\ldots\ldots\ldots$$

$$f_i^{(N)}(\alpha,\beta,\tau)=1+\sum_{j=1}^{N}\exp\theta_{ji}+\sum_{j\neq l=1}^{N}\exp(\theta_{ji}+\theta_{li})+\sum_{j\neq l\neq r=1}^{N}\exp(\theta_{ji}+\theta_{li}+\theta_{ri})+....\,, \tag{3.19}$$

$$\theta_{ki}=a_{ki}\alpha+b_{ki}\beta-\omega_{ki}\tau+\varsigma_{ki}\ ,\ \ k=1,2,...N\,,\ \ i=1,2...6\,,$$

and a_{ki}, b_{ki} the nondimensional wave numbers, ω_{ki} the nondimensional frequencies and ς_{ki} the nondimensional phases. The parameters in this formulation a_{ki}, b_{ki}, ω_{ki} and ς_{ki}, $k=1,2,...N$, $i=1,2...6$ are computable from (3.18),(3.19) and the like powers of δ equations. The numerical determination of these parameters are discussed in the next section. We find that asymptotically the solutions become

$$M_i^{(k)} = A_{ik}\,\mathrm{sech}^2(a_{ki}\alpha + b_{ki}\beta - \omega_{ki}t - 2\Delta_{ki}) ,$$
$$k=1,2,...N ,\quad i=1,2...6 \quad \text{at} \quad t\to\pm\infty , \tag{3.20}$$

where the constants A, Δ can be easily calculated with respect to a_{ki}, b_{ki}, ω_{ki} and ς_{ki}. The functions $M_i^{(k)}$ are periodic with the period $2\Delta_{ki}$. These solutions represent a linear superposition of solitons, a row of solitons, spaced $2\Delta_{ki}$ apart.

4. APPLICATION OF GENETIC ALGORITHM TO PARAMETERS DETERMINATION

Next step is to use (3.18),(3.19) and the like powers of δ equations to determine $23\times$N parameters

$$p=\{a_{ki}, b_{ki}, \omega_{ki}, \varsigma_{ki}\} ,\ k=1,2,...N ,\ i=1,2...6 , \tag{4.1}$$

The wave numbers, frequencies and constant phases are also vectors

$$a_{ki}=(a_{11},a_{12},a_{13},.....a_{N6}),\ \ b_{ki}=(b_{11},b_{12},b_{13},....b_{N5}),$$
$$\omega_{ki}=(\omega_{11},\omega_{12},\omega_{13},....\omega_{N6}),\ \ \varsigma_{ki}=(\varsigma_{11},\varsigma_{12},\varsigma_{13},....\varsigma_{N6}). \tag{4.2}$$

The resulting system is a system of 36 equations to determine a number of $23\times$N unknowns. In this paper a new method is proposed to determine the model parameters (Goldberg 1989 [8]). It is assumed the parameters p are discretized into discrete values with the step width $\Delta p=\{\Delta a_{ki}, \Delta b_{ki}, \Delta\omega_{ki}, \Delta\varsigma_{ki}\}$. The set of parameters for arbitrary values $p=\{a_{ki,m}, b_{ki,n}, \omega_{ki,q}, \varsigma_{ki,s}\}$ can be expressed as 6N numbers

$$N_{ikmnqs}=(m-1)N_{ik}Q_{ik}S_{ik}+(n-1)Q_{ik}S_{ik}+(q-1)S_{ik}+s ,$$

where M_{ki}, N_{ki}, Q_{ki} and S_{ki} denote the total number of discretized values for each parameter p. These numbers represent an individual in a population and for the discretized parameters indicate a specific solution (Tanaka, Nakamura, 1994 [9], Chiroiu et al., 1999 [13]). An individual is expressed as a row of the integer number with N_{gen} = 6N genes. To compute the fitness F we write (3.18)-(3.19) in the form $L_k^{(m)}=\pi_k^{(m)}$, $m=0,1,2$, $k=1,2,...12$, and note the square sum of differences $L_k^{(m)}-\pi_k^{(m)}$ by $\Im$

$$\Im=\sum_{j=0}^{2}\sum_{k=1}^{12}(L_k^{(j)}-\pi_k^{(j)})^2 . \tag{4.3}$$

We define fitness as follows $F=\Im_0/\Im$, with $\Im_0=\sum_{j=0}^{2}\sum_{k=1}^{12}(\pi_k^{(j)})^2$. As the convergence criterion of iterative computations we use the expression Z to be maximum $Z=\dfrac{1}{2}\log_{10}\dfrac{\Im_0}{\Im}\to$ max. Numerical simulation is carried out for $\lambda=1.055\times10^{-3}$ Kg/ms, and $\mu=1.205\times10^{-3}$ Kg/ms. The micropolar coefficients of viscosity have values $\alpha=\zeta=\varepsilon=1.035\times10^{-3}$ mKg/s. We consider $m=\tan\theta$ with $m\in$ [0.2, 0.8]. The value m = 0.8 represents an upper limit on the slopes for which the shallow fluid theory furnish a good approximation. The number r^2 must satisfy the condition $4r^2\le 0.7m$, , which is important for existence of waves. If the resistance is too large, the waves cannot form. This condition is obtained numerically. We take $r^2\in$ [0.035, 0.14]. The value r^2 = 0.14 was chosen as the greatest value for the resistance since it satisfies the condition (4.8) for $m = 0.8$. The intervals for the model parameters are evaluated from the condition that the total mass of fluid per wavelength is constant and the same in all approximations. In order to illustrate the results three cases are considered (N = 4):

$$\text{- case 1} \quad \theta = 45^0 \;\; (m = 1)\,, \;\; r^2 = 0.17\,,$$

$$\text{- case 2} \quad \theta = 31^0 \;\; (m = 0.6)\,, \;\; r^2 = 0.1,$$

$$\text{- case 3} \quad \theta = 22^0 \;\; (m = 0.4)\,, \;\; r^2 = 0.06\,.$$

In all cases we have assumed that the number of populations is 25, ratio of reproduction is 1, number of multi-point crossovers is 1, probability of mutation is 0.2 and maximum number of generations is 250.

The linear summation of the solution $Y(\alpha,\tau)$ for $\tau \to \infty$ is given in fig. 2-4 ($\tau \to \infty$ means in the numerical simulation the time interval after that the solutions have a permanent profile in time). In all cases the fluid velocity is greater in the region of the crests than in the shallower regions, but nowhere will the fluid velocity be as great as the wave speed. For example, in the case 1 the average fluid velocity is about 3.05 m/s while the wave velocity is about 4.1 m/s. From numerical simulations we conclude that the remaining solutions have a similar evolution with respect to α: they increase and decrease in the same manner as Y. The micro-rotation components and the vertical component of the fluid velocity are greater in the crest regions than in the shallower regions. The model parameters were obtained after 149 iterations in the case 1, 167 in the case 2 and 187 in the case 3.

In conclusion, the solutions we have obtained in this paper describe the phenomenon called "roll-waves" for fluid flow along a wide inclined channel. This phenomenon appears in hydraulic applications like run-off channels and open aqueducts. When a liquid flows turbulently downwards on an inclined open channel, the wave profile represented as sums of solitons moves downstream as a progressing wave at a constant speed and without distortion, and such that the velocities of the fluid particles are everywhere less than the wave velocity. Comparing only the performance, the genetic algorithm is superior to other conjugate gradient methods because it is simple to be applied, is stable and the correct solutions are detected through a relative small number of iterations, without requiring the stopping criterion for them. We need more computer memory in order to store the data, but in view of today's computer capabilities we do not consider this as a real disadvantage.

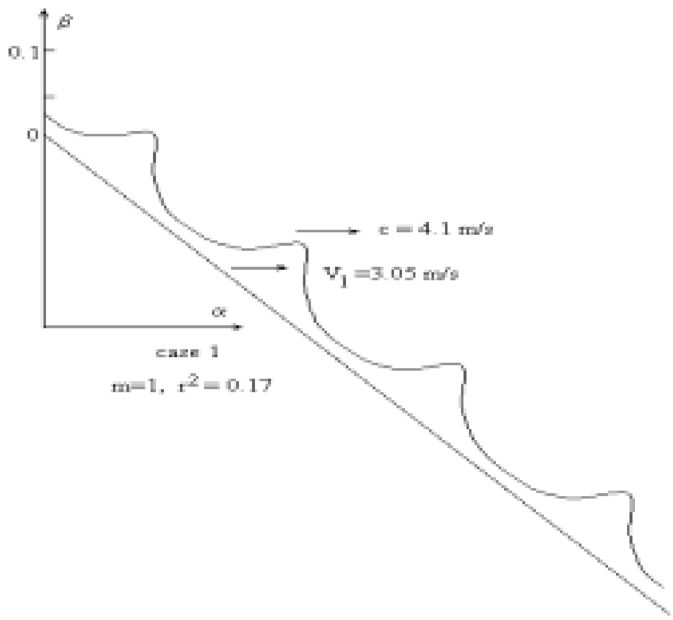

Fig. 2 The profile of the wave $Y(\alpha,\tau)$ in the case 1

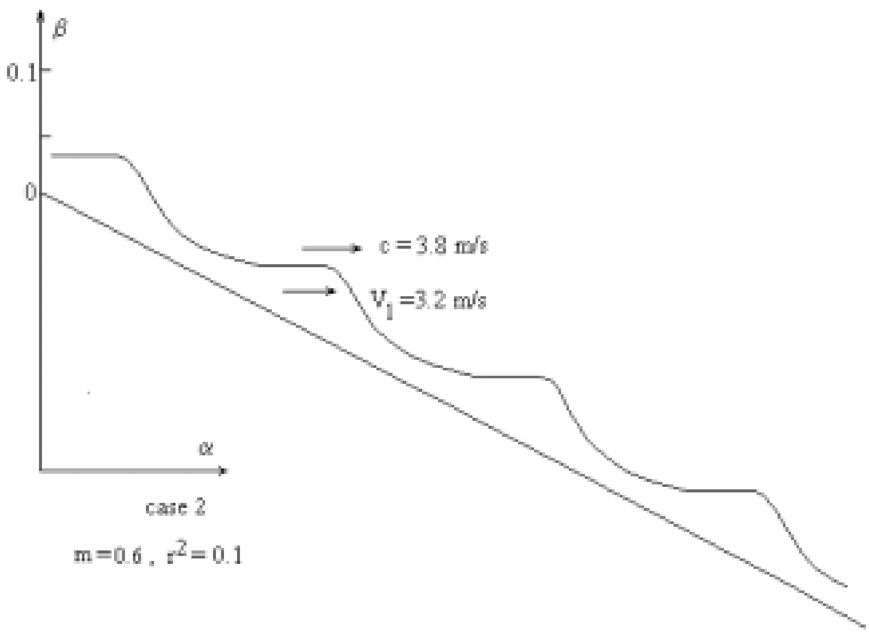

Fig. 3 The profile of the wave $Y(\alpha, \tau)$ in the case 2

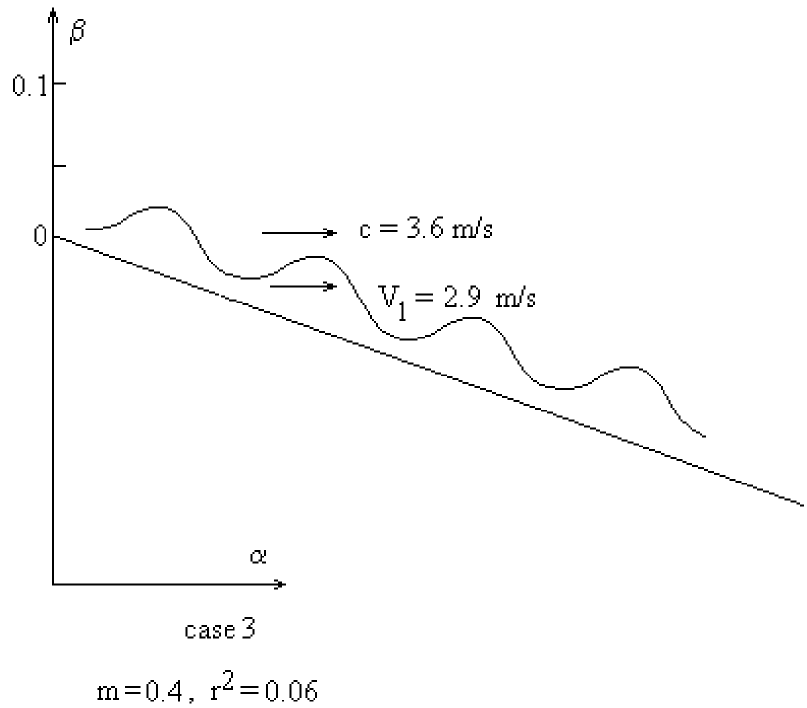

Fig. 4 The profile of the wave $Y(\alpha, \tau)$ in the case 3

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