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DAMPING OPTIMIZATION OF PASSIVE AND SEMI-ACTIVE VEHICLE SUSPENSION BY NUMERICAL SIMULATION

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The Newmark method and *Monte Carlo* simulation are used to obtain the mean square response of SDOF suspension model with nonlinear damping, excited by stationary Gaussian white noise. The aim of the paper is to optimize the system nonlinear damping characteristic, both for passive and semi-active models of vehicle suspension, with respect to the ride comfort criterion (minimum r.m.s. body acceleration). It is shown that the semi-active suspension with sequential dry friction can provide a perceptible comfort improvement in comparison with the optimum linear or nonlinear passive suspensions.

INTRODUCTION

Isolation of passenger and cargo from terrain induced shock and vibration is the important task of the suspension of any ground vehicle. Most suspensions have passive springs and dampers with rather limited vibration isolation performance, both for linear and nonlinear restoring or damping characteristics. Their transmissibility factors show that low damping gives good isolation at high frequency but poor resonance characteristics, whilst higher damping results in good resonance isolation at the expense of high frequency performance.

By using hydraulic or pneumatic power supply and servo actuators controlled by feedback signals, it is possible to produce active suspensions, which are superior to any passive system throughout the frequency range. But they are more complex, more expensive and less reliable than the passive suspensions.

A compromise between passive and active types is semi-active suspension systems consisting of an active damper in parallel with a passive spring. Semi-active control devices have received a great deal of attention in recent years, because they combine the best features of the passive and active ones. They have almost the same environmental robustness, mechanical simplicity and low cost as passive devices and can offer the adaptability of active control systems without requiring the associated large power sources. The damping characteristics are controlled by modulation of fluid-flow orifices, of dry friction forces or of electric or magnetic field applied to electrorheological or magnetorheological fluid dampers. For practical reasons it is important that the feed back signals be relative displacement and the relative velocity across the suspension, since these state variables can be measured directly even for a moving vehicle.

In the case of semi-active suspensions with *sequential damping* [1],[2], the damper force is zero or rather very small as long as the sprung mass is moving away from its static equilibrium position, suddenly increases when the system is returning to the static equilibrium position. The basic idea of this semi-active control strategy is to balance the elastic force by the damping force in order to reduce or even to cancel the forces transmitted through the suspension as long as the spring and elastic forces act in opposite direction (semi-active control strategy based on *balance logic*).

In this paper the *balance logic* is assumed to be achieved with a variable dry friction damper controlled by the modulation of the normal force applied on the friction plates [3].The comfort improvement achieved by using this semi-active control strategy (measured in terms of the r.m.s. vehicle body acceleration), is compared with those provided by the optimum passive linear and quadratic damping. A SDOF vehicle

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suspension model with linear elastic characteristic and stationary Gaussian white noise excitation is assumed. In order to determine the mean square acceleration of the sprung mass, the equation of motion is solved by Newmark method using the numerical simulation of the excitation time histories.

1. SUSPENSION MODEL

The equation of motion of the SDOF vehicle suspension model considered in this paper is given by:

$$m\ddot{x} + \mathrm{d}(x,\dot{x}) + kx = -m\ddot{y}(t) \tag{1.1}$$

where

- $x(t)$ is the relative displacement across the suspension;
- $y(t)$ describes the motion of the system base, imposed by the road profile for a constant riding speed;
- m is the system sprung mass;
- k is the linear spring stiffness;
- $\mathrm{d}(x,\dot{x})$ is the suspension damping characteristic.

If $y(t)$ is modeled as a stationary Gaussian random process with a power spectral density approximated by the analytical expression [4]:

$$S_y(\omega) = S_0\omega^{-4} \quad ,\omega \neq 0 \tag{1.2}$$

then the spectral density of the system excitation $\ddot{y}(t)$ can be approximated by a stationary Gaussian white noise with the auto-correlation function and the spectral density:

$$R(\tau) = 2\pi S_0 \delta(\tau) \quad ; \quad S_{\ddot{y}}(\omega) = S_0 \tag{1.3}$$

where S_0 is a constant dependent on the road roughness and the riding speed.

The damping characteristics of the passive suspensions considered in this paper are linear and quadratic:

$$\mathrm{d}(\dot{x}) = c\dot{x} \tag{1.4}$$

$$\mathrm{d}(\dot{x}) = q\dot{x}|\dot{x}| \tag{1.5}$$

According to the *balance logic*, the damping characteristic of the semi-active suspension with sequential dry friction is;

$$\begin{aligned} &\mathrm{d}(x,\dot{x}) = 2\alpha k|x|\, sign\dot{x} && \text{if} \quad x\dot{x} \le 0 \\ &\mathrm{d}(x,\dot{x}) = 0 && \text{if} \quad x\dot{x} > 0 \end{aligned} \tag{1.6}$$

From (1.1) and (1.6) is easily seen that perfect balance (in the motion sequences when this is physically possible) is obtained for á = 0.5. On the other hand, in order to avoid the damper *"lock up"* (no motion across the suspension) which could occur when $\dot{x} = 0,\ x \neq 0$ it is sufficient that á < 0.5. Nevertheless, this condition in not necessary because of the presence of the inertia force.

Letting:

$$\begin{aligned} &f(x,\dot{x}) = \frac{1}{m}\mathrm{d}(x,\dot{x}) \quad ; \quad z(t) = -\ddot{y}(t) \\ &\omega = \sqrt{\frac{k}{m}} \quad ; \quad \varsigma = \frac{c}{2m\omega} \quad ; \quad \beta = \frac{q}{m\omega^2}a \end{aligned} \tag{1.7}$$

where a is the acceleration unit (1m/s^2), the equation (1.1) can be written as:

$$\ddot{x} + 2\omega\varsigma\dot{x} + \omega^2 x = z(t) \tag{1.8}$$

for linear damping,

$$\ddot{x} + \beta a^{-1}\omega^2 \dot{x}|\dot{x}| + \omega^2 x = z(t) \tag{1.9}$$

for quadratic damping and

$$\ddot{x} + \alpha\omega^2 |x| sign\dot{x} + (1-\alpha)\omega^2 x = z(t) \tag{1.10}$$

for sequential damping.

The damping optimization procedure consists in determination of the parameters ò,â, or ***a*** such that the r.m.s. absolute acceleration of the sprung mass be minimum. This quantity is given by:

$$\sigma_{\ddot{x}_1}{}^2 = E\left[(\ddot{x} - z)^2\right] = E\left[(f(x,\dot{x}) + \omega^2 x)^2\right] \tag{1.11}$$

where $E[\ ]$ is the mathematical expectation operator.

In the analysis of real systems, random processes describing physical phenomena represent families of individual realizations. Each realization (sample function) of the system input leads to a unique solution trajectory if the problem is well posed. The collection of these solution trajectories is an output random process. If the output random process is a second order process than it is just the solution of the stochastic equation of motion in the mean square sense [5].

The numerical simulation methods used to evaluate the statistical properties of the solution random process usually imply a numerical integration method of the deterministic differential equation of motion for numerically simulated trajectories of the system random excitation. The statistical properties of the solution process are then determined by standard estimation procedures [6].

In this paper, the Newmark method and a pseudo-random number generation algorithm are used for numerical integration of equations (1.8)-(1.10) and simulation of the discrete-time trajectories of the white noise excitation $z(t)$.

In order to verify the accuracy of the mean square response evaluation, the approximate solution is compared with the exact solution, known for the linear system (1.8) [7]:

$$\sigma_x{}^2 = \frac{\pi S_0}{2\omega^3\varsigma} \quad ; \quad \sigma_{\dot{x}}{}^2 = \frac{\pi S_0}{2\omega\varsigma} \quad ; \quad \sigma_{\ddot{x}_1}{}^2 = \frac{\pi S_0\omega\left(1+4\varsigma^2\right)}{2\varsigma} \tag{1.12}$$

The minimum value of mean square absolute acceleration is obtained for ò = 0.5 and the corresponding mean square response is:

$$\sigma_x{}^2(0.5) = \frac{\pi S_0}{\omega^3} \quad ; \quad \sigma_{\dot{x}}{}^2(0.5) = \frac{\pi S_0}{\omega} \quad ; \quad \sigma_{\ddot{x}_1}{}^2(0.5) = 2\pi\omega S_0 \tag{1.13}$$

2. NUMERICAL SIMULATION OF WHITE NOISE SAMPLE FUNCTIONS

A discrete time history (sample function) of the stationary Gaussian white noise excitation:

$$z_{n+1} = z(n\Delta t) \quad ; \quad n = 1,2,...,N \tag{2.1}$$

can be derived approximately from a sequence of *pseudo-random numbers* U_n ; $n = 1,2,...N$, uniformly distributed on the unit interval [0,1], [8], [9].

These days most digital computers include a *linear congruential pseudo-random number generator* (*Monte Carlo* simulation). These have the recursive form:

$$X_{n+1} = aX_n + b(\mathrm{mod}\, c) \tag{2.2}$$

where a and c are positive integers and b a nonnegative integer. For an integer initial value or *seed* X_0 , the algorithm (2.2) generates a sequence taking integer values from 0 to c-1, the reminders when the $aX_n + b$ are divided by c. When the coefficients a, b and c are chosen appropriately, the numbers:

$$U_n = X_n / c \tag{2.3}$$

seem to be uniformly distributed on the unit interval [0,1].Since only finitely many numbers occur, the modulus c should be chosen as large as possible. To prevent cycling with a period less than c, the multiplier a should be also taken relatively prime to c.

According to the *Box-Muller method,* if U_1 and U_2 are two independent uniformly distributed random variables on [0,1], then N_1 and N_2 defined by:

$$\begin{aligned} N_1 &= \sqrt{-2\ln U_1}\cos 2\pi U_2 \\ N_2 &= \sqrt{-2\ln U_1}\sin 2\pi U_2 \end{aligned} \tag{2.4}$$

are two independent standard Gaussian random variables. Therefore,

$$\begin{aligned} z_{2k-1} &= \sqrt{2\pi S_0/\Delta t}\sqrt{-2\ln U_{2k-1}}\cos 2\pi U_{2k} \\ z_{2k} &= \sqrt{2\pi S_0/\Delta t}\sqrt{-2\ln U_{2k-1}}\sin 2\pi U_{2k} \end{aligned} \tag{2.5}$$

are independent Gaussian random variables with :

$$E[z_n] = 0 \quad ; \quad E[z_m z_n] = 2\pi S_0 \Delta t \delta_{mn} \tag{2.6}$$

As one can see from (1.3) and (2.6), the discrete random process defined by :

$$z^{\Delta}(t) = z_n \quad \text{for} \quad (n-1)\Delta t \le t \le n\Delta t \tag{2.3}$$

is mean square convergent to the white noise process $z(t)$ when Ä$t \to 0$.

3. NUMERICAL SOLUTION OF THE EQUATION OF MOTION

The Newmark discrete time method in five steps [10] was applied in order to obtain the approximate solution of equation :

$$\ddot{x} + f(x, \dot{x}) + kx = z(t) \tag{3.1}$$

with the initial conditions:

$$x(0) = x_0 \quad ; \quad \dot{x}(0) = \dot{x}_0 \tag{3.2}$$

At the initial time t_0=0, the initial value of the acceleration $\ddot{x}(0) = \ddot{x}_0$ is evaluated from

$$\ddot{x}_0 = z(t_0) - f(x_0, \dot{x}_0) - kx_0 \tag{3.3}$$

The principle of the method consists in the approximation of the discrete values $x_{n+1}, \dot{x}_{n+1}, \ddot{x}_{n+1}$ by using the values obtained at the moment t_n. The steps of the method are:

1. Initialization of $\ddot{x}_{n+1}$ with an arbitrary value $\ddot{x}_{n+1,i}$
2. Evaluation of $\dot{x}_{n+1}$ from:

$$\dot{x}_{n+1} = \dot{x}_n + (\ddot{x}_n + \ddot{x}_{n+1,i}) \cdot \frac{\Delta t}{2} \tag{3.4}$$

3. Approximation of x_{n+1} by:

$$x_{n+1} = x_n + \dot{x}_n \Delta t + (\ddot{x}_n + \ddot{x}_{n+1,i}) \cdot \frac{(\Delta t)^2}{4} \tag{3.5}$$

4. With x_{n+1} and $\dot{x}_{n+1}$ from 2 and 3 is found:

$$\ddot{x}_{n+1,c} = z(t_{n+1}) - f(x_{n+1}, \dot{x}_{n+1}) - kx_{n+1} \tag{3.6}$$

5. The values $\ddot{x}_{n+1,\,c}$ and $\ddot{x}_{n+1,\,i}$ are compared. If the difference is not sufficiently small, $\ddot{x}_{n+1,\,i}$ is replaced by $\ddot{x}_{n+1,\,c}$ and the algorithm is repeated from the step 2. Otherwise, a new iteration can be initiated. Usually, for the initialization of the unknown acceleration value the preceding accepted value is used.

This last step of the numerical method requires a certain *smoothness* of the discrete acceleration values. Therefore, the sign function in (1.10) is approximated by the continuous function tanh ã*x* with a sufficiently large value of ã .

4. NUMERICAL RESULTS

The aim of the numerical analysis is to compare the mean square response of the suspension analytical models (1.8)-(1.0) for optimum linear, quadratic and sequential damping characteristics. The optimum values of the parameters V, b, a are determined such that to minimize the r.m.s. absolute acceleration for different values of the excitation intensity S_0 .

In order to determine a realistic variation range of the excitation intensity S_0, one must consider in (1.13) appropriate r.m.s. values of the sprung mass absolute acceleration $\ddot{x}_1$. The measured values of $s_{\ddot{x}_1}$ for a medium size passenger car (DACIA 1300) were obtained within the range 0.9-2.15 m/s^2 [4]. Assuming that the undamped natural frequency of the car suspension is 1 Hz, then $\omega = 2\pi$.Therefore, in view of (1.13), values of S_0 between 0.01 m^2/s^3 and 0.1 m^2/s^3 are quite reasonable.

The optimum values of the damping coefficients V, b, a , determined by using the numerical solutions of equations (1.8)-(1.10), are almost constant for all values of S_0 within this range:

$$\varsigma_0 = 0.5 \quad ; \quad \beta_0 = 0.9 \quad ; \quad \alpha_0 = 0.8$$

The mean square response values for the three optimum damping characteristics (**L**-linear, **Q**-quadratic and **S**- sequential) are given in table 1.

Figure 1 depicts comparatively the minimum r.m.s. absolute acceleration versus excitation intensity. As one can see from this figure, the semi-active suspension can provide a reduction of aproximately 25% of the r.m.s sprung mass acceleration over the whole excitation intensity range, as compared with both linear and nonlinear passive suspensions,.This reduction, though not so spectacular, is very well perceived in terms of comfort improuvment, as resulted from comparison between r.m.s. body acceleration measurements and the subjectivistic comfort assessment for various damper settings [4]. It should be mentioned that this improuvment is obtained at the expense of a semnificative increase of the suspension relative displacement.Therefore, a certain compromise between comfort and rattle space requirements should be considered in the optimization cost function.

Table 1

	S_0 [m/s^2]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
L	s_x [m]	0.011	0.016	0.019	0.022	0.024	0.027	0.029	0.031	0.033	0.036
	$s_{\dot{x}}$ [m/s]	0.07	0.11	0.12	0.14	0.16	0.17	0.19	0.20	0.21	0.23
	$s_{\ddot{x}_1}$ [m/s^2]	0.61	0.89	1.06	1.24	1.38	1.52	1.66	1.77	1.87	2.05
Q	s_x [m]	0.019	0.024	0.026	0.028	0.030	0.031	0.033	0.035	0.038	0.039
	$s_{\dot{x}}$ [m/s]	0.12	0.15	0.16	0.17	0.18	0.19	0.20	0.22	0.23	0.24
	$s_{\ddot{x}_1}$ [m/s^2]	0.83	1.06	1.21	1.36	1.49	1.63	1.76	1.87	1.98	2.17
S	s_x [m]	0.044	0.058	0.067	0.074	0.080	0.086	0.092	0.097	0.101	0.108
	$s_{\dot{x}}$ [m/s]	0.13	0.17	0.19	0.21	0.23	0.25	0.26	0.27	0.29	0.30
	$s_{\ddot{x}_1}$ [m/s^2]	0.45	0.65	0.78	0.92	1.03	1.14	1.26	1.34	1.43	1.58

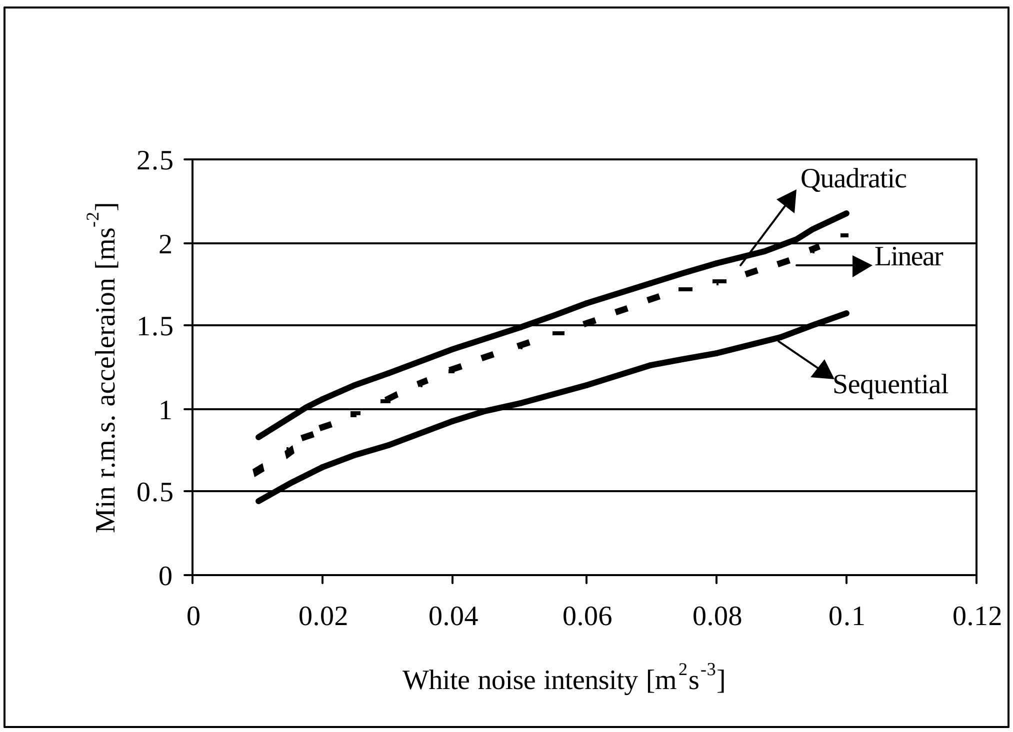

Fig.1

5. CONCLUSIONS

1.The results obtained by the numerical simulation carried out in this paper are in very good agreement with those established by other workers, showing that the acceleration experienced by a system controlled by a semi-active strategy can be perceptibly lower than that of a passive system (either linear or nonlinear).

2.A controlled friction device is superior to any conventional viscous damper (with either laminar or turbulent flow of the hydraulic fluid), since the former is able to generate large damping forces for very low relative velocity, as is required by the balance logic semi-active control strategy.

3.The appeal of a variable friction damper is that it can be utilized to achieve virtually any desired control logic by appropriate adjustment of the force on the friction plates.

4. The work presented here indicates that by using balance logic the r.m.s. sprung mass acceleration can be reduced by nearly 25-30%. Since this reduction is obtained at the expense of a semnificative increase of the r.m.s. relative displacement, the optimization cost function must be expressed in terms of both comfort and rattle space criteria.

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