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NONLINEAR CONTROL SYNTHESIS FOR POSITION AND FORCE ELECTROHYDRAULIC SERVOS

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Using as reference point the hydromechanical servomechanism SMHR included in the aileron control chain of the Romanian jet-fighter IAR 99, a nonlinear, backstepping type, position control law was synthesised, which denotes a remarkable improving of tracking performance, quantified by decreasing of system time constant. Another worthy noting result concerns the conversion of the same position type servo, into a force servo, by means of a force control law and a modified valve port.

Key words: control synthesis, backstepping, Lyapunov function, electrohydraulic servo, flight controls.

1. INTRODUCTION

(Electro)hydraulic servoactuators (EHS) are encountered in most industries where heavy objects are manipulated or large forces and torques with high speeds are exerted. The features as their large processing force and stiffness, good positioning and high payload capabilities, and power to weight ratio, make this type of actuation system appropriate for positioning of aircraft control surfaces, high power industrial machinery, position control of military gun turrets and antennas, material handling, construction, mining and agricultural equipment. "Industrial hydraulic technology is firmly entrenched in our global economy. The usage knows no boundary lines" [1].

The demanding performance specifications for such applications are high bandwidth implying fast response time, high accuracy and high fidelity control; such technical challenges have led the researchers to examine how to improve hydraulic control design. There are some factors that limit the applications of EHS, but the final factor limiting these applications is ourselves: human being's understudying and knowledge of these systems. Because of the complexity of EHS analysis and nonlinearities in the systems dynamics, design and control of EHS are still difficult and immature, although various methodologies of the automatic control science were brought to the proof in this field; from classical linearization [2], to artificial intelligence paradigm [3].

A condition *sine qua non* of a systems design is first of all its stability; this condition concerns the basis itself of the system approach, study and design. If the stability of a physical system is primordial, the stability of the system's model is dependent, of course, on this model. The problem is for analyst to find a representative model of the physical system. What means a „representative model" – this is an open debate in the field. From viewpoint of the system analysis, the representative model must obey to a trade-off: the model must be complex enough to describe the physical behaviour of the system and at the same time simple enough to not compromise a qualitative analysis approach.

A way at hand of surpassing these contradictory requirements supposes the covering of the following steps: a) the derivation of an as complex as possible model in accordance with the physical laws and designing constraints which defines the object; b) a certain adjustment of the obtained model in accordance with the qualitative mathematical apparatus used in view of the system synthesis and analysis; c) the providing of designing or optimization indications in the chosen and applied theoretical framework; d) the methodical validation of the provided results by simulation of the complex mathematical model.

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The step b) of this strategy requires some elucidation. On the one hand: the mathematical model, as a thinking first result, which is derived from a physical reality, is more flexible than a methodology (i. e., a mathematical theory), as a thinking second result, which is derived from a metareality – herein, the mathematical thinking. On the other hand: all the mathematical models describing the same object promote in the last analysis a certain firmness and „solidarity", in other words, predict an approximate possible behaviour of the object [3].

In this paper, the *backstepping* approach [4] is employed to successively develop, based on Lyapunov's direct method, position and, then, force tracking controllers for an EHS. A "backstepping" is a recursive procedure, which allows deriving control law, generally for a nonlinear system

$$\dot{x} = F(x) + G(x)u \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^{\boldsymbol{m}}$ is the control vector and $\boldsymbol{F}$ and $\boldsymbol{G}$ are smooth vector fields of appropriate dimensions. Moreover, the method implicitly guarantees stability references tracking and transient performance.

As mentioned in [5], backstepping is a method that can be used on systems of special structure to find an output z having a passivity property [3]: specifically, if a system is passive (plus some other technical requirements), then the feedback $u(t) = -z(t)$ renders the system asymptotically stable. The search for a passivation output is unfortunately nontrivial. So, we can use backstepping for lower-triangular nonlinear systems (for upper-triangular nonlinear systems, we can use a so-named *forwarding* method). Classes of systems, for which this procedure works, are one after the other considered strict-feedback systems and pure-feedback systems [6] and, finally, general nonlinear systems [7].

The key idea of backstepping is simple. At every step of backstepping, a new Control Lyapunov Function (CLF) is constructed by augmentation of the CLF from the previous step by a term which penalises the error between a state variable and its desired value. A major advantage of backstepping is the construction of a Lyapunov function whose derivative can be made negative definite by a variety of control laws rather that by a specific control law.

The paper addresses the problem of the control laws synthesis which provides asymptotic stability and tracking performance for an EHS. The full state information is considered available. The backstepping has not been applied to control of an used in aviation, position tracking, EHS, to the best of the author's knowledge.

2. BACKSTEPPING POSITION CONTROL SYNTHESIS FOR EHS

The differential equations governing the dynamics of the EHS are those given in [3] and are reported having as a reference point the hydromechanical servomechanism SMHR (HM SMHR) included in the aileron control chain of Romanian jet-fighter IAR 99:

$$\dot{x}_1 = x_2, \dot{x}_2 = -k/m\, x_1 - f/m\, x_2 + S/m\, x_3, \dot{x}_3 = \left(-Sx_2 - k_\ell x_3 + c_d w k_\mathrm{v} \sqrt{p_a - x_3}\, u \Big/ \sqrt{\rho}\right) \Big/ k_c\,. \tag{2}$$

The state variables are denoted by: x_1 [cm] – EHS load displacement; x_2 [cm/s] – EHS load velocity; x_3 [daN/cm^2] – load pressure differential; u [V] – control variable. The nominal values of the parameters appearing in equations (2) are: $m = 0.033$ daNs^2/cm – equivalent inertial load of primary control surface reduced at the EHS's rod; $S = 10$ cm^2 – EHS's piston area; $f = 1$ daNs/cm – equivalent viscous friction force coefficient; $k = 100$ daN/cm – equivalent aerodynamic elastic force coefficient; $w = 0.05$ cm – valve-port width; $p_a = 210$ daN/cm^2 – supply pressure; $c_d = 0.63$ – volumetric flow coefficient of the valve port; $k_\ell = 5/210$ cm^5/(daN×s) – internal leakage cylinder's coefficient ; $\rho = 85/(981\times10^5)$ daNs^2/cm^5 – volumetric density of oil; $k_\mathrm{v} = 0.0085/(0.05\times10)$ cm/V – valve displacement/voltage coefficient; $k_c = 30/12\,000$ cm^5/daN – coefficient involving the bulk modulus of the oil used and the EHS's cylinder semivolume. The valve dynamics is evaded in mathematical model (2); a proportionality coefficient k_v

between the control (input voltage to servovalve) and valve displacement was considered. Clearly, system (2) is lower triangular, in strict feedback form, and, therefore, suitable for application of backstepping. Taking the non-adaptive case [8] – the system parameters are assumed to be known – let introduce the notations

$$e_i = x_i - x_{id}\,,\quad i = 1,\ \ldots, 3 \tag{3}$$

where x_{id} stand for the "desired" values of the state variables. So, the control objective is to have the EHS track of a specified x_{1d} position trajectory, in other words, making $e_1 \to 0$.

Proposition 1. *Under assumption of nonsaturating load* $(x_3 < p_a)$, *the control u given by*

$$u = \frac{k_c\sqrt{\rho}}{\rho_3 c_d w k_v \sqrt{p_a - x_3}}\left[-\frac{\rho_2 S}{m}e_2 + \rho_3\left(\frac{S}{k_c}x_2 + \frac{k_\ell}{k_c}x_3 + \dot{x}_{3d}\right) - k_3 e_3\right] \tag{4}$$

$$x_{3d} = kx_1/S + fx_2/S - \rho_1 m e_1/(\rho_2 S) + m\dot{x}_{2d}/S - mk_2 e_2/(\rho_2 S) \tag{5}$$

$$x_{2d} = \dot{x}_{1d} - k_1 e_1 \tag{6}$$

when applied to (2), *guarantees global asymptotic stability of position tracking error* $e_1 = x_1 - x_{1d}$.

Proof: We start by defining the Lyapunov like function

$$V_1 = \rho_1 e_1^2/2\,. \tag{7}$$

The derivative of (7) is given by

$$\dot{V}_1 = \rho_1 e_1(\dot{x}_1 - \dot{x}_{1d}) = \rho_1 e_1(x_2 - \dot{x}_{1d}) = \rho_1 e_1(e_2 + x_{2d} - \dot{x}_{1d})\,.$$

With (6), $\dot{V}_1$ becomes

$$\dot{V}_1 = -k_1\rho_1 e_1^2 + \rho_1 e_1 e_2 \tag{8}$$

where ρ_1, k_1 stand for weighting parameters. Now, in order to go one step ahead, a new Lyapunov like function V_2 is defined as

$$V_2 = V_1 + \rho_2 e_2^2/2\,. \tag{9}$$

By taking the derivative of (9)

$$\dot{V}_2 = \dot{V}_1 + \rho_2 e_2\dot{e}_2 = -k_1\rho_1 e_1^2 + e_2\left[\rho_1 e_1 + \rho_2(\dot{x}_2 - \dot{x}_{2d})\right] =$$

$$= -k_1\rho_1 e_1^2 + e_2\left[\rho_1 e_1 - \rho_2 kx_1/m - \rho_2 fx_2/m + \rho_2 Se_3/m + \rho_2 Sx_{3d}/m - \rho_2\dot{x}_{2d}\right].$$

If x_{3d} is chosen as (5), then $\dot{V}_2$ is simplified to

$$\dot{V}_2 = -k_1\rho_1 e_1^2 - k_2 e_2^2 + \rho_2 Se_2 e_3/m \tag{10}$$

The new weighting parameters ρ_2, k_2 were also introduced. Finally, V_3 is defined to be

$$V_3 = V_2 + \rho_3 e_3^2/2 \tag{11}$$

and taking the derivative of (11), one may write

$$\dot{V}_3 = -k_1\rho_1 e_1^2 - k_2 e_2^2 + e_3\left[\rho_2 Se_2/m + \rho_3\left(-Sx_2/k_c - k_\ell x_3/k_c + c_d w k_v\sqrt{p_a - x_3}\,u\Big/\left(k_c\sqrt{\rho}\right) - \dot{x}_{3d}\right)\right].$$

Now, if the control u is synthesised as (4), then V_3 is indeed obtained as a Lyapunov function for the system defined by (2): the control low given by (4), (5), (6), with given x_{1d} renders the derivative $\dot{V}_3$ negative semidefinite:

$$\dot{V}_3 = -k_1\rho_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{12}$$

and ρ_3, k_3 are the latest weighting parameters introduced. Note that as soon as the values of state variables x_1, x_2, x_3 and desired $x_{1d}, \dot{x}_{1d}, \ddot{x}_{1d}, \dddot{x}_{1d}$ are measured and, respectively, known, the control u can be calculated by using (2), (3), (5) and (6). Substituting (3), (4), (5) and (6) in (2), an autonomous closed loop form, analogous to (1), occurs

$$\dot{\boldsymbol{e}} = \widetilde{\boldsymbol{F}}(\boldsymbol{e}). \tag{1′}$$

Because $\boldsymbol{e} := (e_1, e_2, e_3)$ is the largest invariant set in $E := \{\boldsymbol{e} | \dot{V}_3(\boldsymbol{e}) = 0\}$ then, based on LaSalle's invariance principle ([9], Chapter 2, Theorem 8), we conclude that all tracking errors e_i, $i = 1, ..., 3$, converge to zero asymptotically.

Let note that the mathematical model (2) involves a conjecture: a chosen positive u does not change its sign, in transitory regime, when various positive x_{1d} is claimed to achieve; otherwise, the model must contain the term $\sqrt{p_a - x_3 \mathrm{sgn} u}$ and the backstepping cannot works in the described manner. This technical difficulty can be evaded by introducing a new state variable, the valve position.

3. BACKSTEPPING FORCE CONTROL SYNTHESIS FOR EHS

Let now consider the case of force control. It can be easily verified inspecting the system (2) that the internal states x_1 and x_2, as described by the first two equations in (2), are stable. On the other hand, the output of interest in force control is, of course, the pressure. Then, if it is not necessary to stabilise the states x_1, x_2, a backstepping procedure cannot be applied only about x_3 oneself. In these circumstances, *we adapt the mathematical model to the backstepping methodology:* the system (2) will be completed by adding an equation of first order for the dynamics of the valve position $x_{\mathrm{v}} := x_4$:

$$\begin{gathered} \dot{x}_1 = x_2, \dot{x}_2 = -k/m x_1 - f/m x_2 + S/m x_3 \\ \dot{x}_3 = \left(-S x_2 - k_\ell x_3 + c_d w \sqrt{p_a - x_3}\, x_4 \big/ \sqrt{\rho}\right) \big/ k_c, \dot{x}_4 = -x_4/\tau + k_{\mathrm{v}} u/\tau \end{gathered} \tag{13}$$

where τ – time constant of the (servo)valve.

Proposition 2. *Under assumption of a nonsaturating load* $(x_3 < p_a)$, *the control u given by*

$$u = \tau\left[-\rho_3 c_d w \sqrt{p_a - x_3}\, e_3 \Big/ \left(\sqrt{\rho} k_c\right) + \rho_4 \left(x_4/\tau + \dot{x}_{4d}\right) - k_4 e_4\right] \Big/ \left(\rho_4 k_v\right) \tag{14}$$

$$x_{4d} = k_c \sqrt{\rho} \Big/ \left(c_d w \sqrt{p_a - x_3}\right) \left(S x_2/k_c + k_\ell x_3/k_c + \dot{x}_{3d} - k_3 e_3\right). \tag{15}$$

when applied to (13), *guarantees global asymptotic stability of pressure* (*force*) *tracking error* $e_3 = x_3 - x_{3d}$.

Proof: Consider

$$V_3 = \rho_3 e_3^2/2, \qquad e_3 := x_3 - x_{3d}, \qquad e_4 := x_4 - x_{4d} \tag{16}$$

and differentiating yields

$$\dot{V}_3 = \rho_3 e_3 \dot{e}_3 = \rho_3 e_3 \left(-S x_2/k_c - k_\ell x_3/k_c + c_d w \sqrt{p_a - x_3} \Big/ \left(k_c \sqrt{\rho}\right)(e_4 + x_{4d}) - \dot{x}_{3d}\right).$$

If x_{4d} is chosen as (15), then $\dot{V}_3$ is simplified to

$$\dot{V}_3 = -k_3 \rho_3 e_3^2 + \rho_3 c_d w e_3 e_4 \sqrt{p_a - x_3} \Big/ \left(\sqrt{\rho} k_c\right) \tag{17}$$

where ρ_3 and k_3 are weighting parameters. Now, in order to go one step ahead, V_4 is defined as

$$V_4 = V_3 + \rho_4 e_4^2/2 \tag{18}$$

and taking again the derivative, we have

$$\dot{V}_4 = -k_3\rho_3 e_3^2 + e_4\left[\rho_3 c_d w e_3\sqrt{p_a - x_3}\Big/\left(\sqrt{\rho}k_c\right) + \rho_4\left(-x_4/\tau + k_v u/\tau - \dot{x}_{4d}\right)\right].$$

Now, if the control u is synthesised as (14), then V_4 is obtained as Lyapunov function, because

$$\dot{V}_4 = -k_3\rho_3 e_3^2 - k_4 e_4^2\,. \tag{19}$$

The same referring to LaSalle's principle as in previous Section finish the proof of the Proposition 2.

4. SIMULATION RESULTS AND CONCLUSIONS

As it was pointed out, the objective of backstepping synthesised control is to have the EHS track of specified x_{1d} position or x_{3d} pressure trajectories. Such trajectories can be described in the manner

$$x_{id} = x_{is}\left(1 - e^{-\frac{t}{t_{ir}}}\right), i = 1 \text{ or } 3 \tag{20}$$

which is associated with time response of a first order systems to step input: x_{is} stand for stationary value of the states x_1, respectively x_3 and t_{ir} stand for time constants.

As reference point of the numerical simulations we take the mathematical model of the HM SMHR

$$\dot{x} = x_2\,, \dot{x}_2 = -k/m x_1 - f/m x_2 + S/m x_3\,, \dot{x}_3 = \left[-Sx_2 - k_\ell x_3 + c_d w\sqrt{p_a - x_3}\,\lambda(r - x_1)\big/\sqrt{\rho}\right]\big/k_c \tag{21}$$

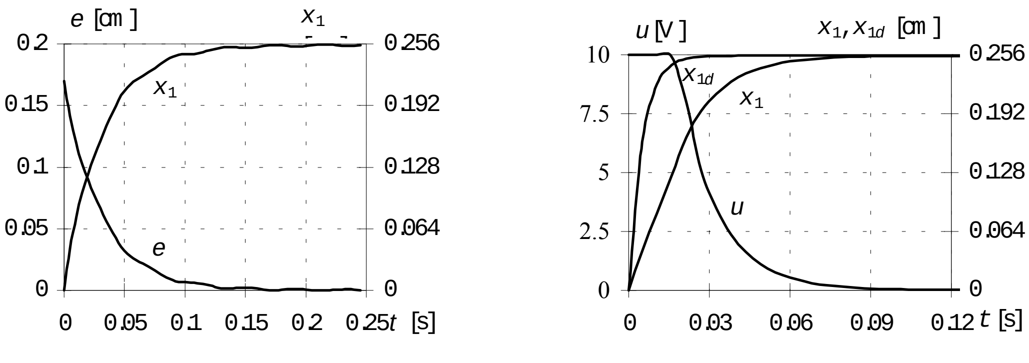
where: $\lambda = 2/3$ − the coefficient of the rigid feedback kinematics; r − the reference input at servo rigid feedback kinematics linkage point [cm].

Simulations have been performed to investigate the performance of the proposed nonlinear controllers, in the two cases defined by the nonlinear control laws (4) and (14). Zero initial conditions − corresponding to the zero equilibrium point − were chosen for Runge-Kutta integration of the systems: (21), with a simple, rigid mechanical position feedback; (2) with control low (4); and (13) with control low (14).

A relatively good tracking of references r is possible with the HM SMHR (21), but only in the absence of perturbations [10]. To have in this servo a concrete term of comparison for the developed in Section 3 position control technique, we choose $r = 0.0085/(0.5\lambda) = 0.0255$ cm. Then, having this reference, the entire valve port is in fact open to flow passing and, consequently, the resulting time constant characterises the best step input tracking with the "passive" system: $\tau \cong 0.035$ s (Fig. 1a, where the error signal $e := \lambda(r - x_1)$ is represented; initial error: $e_0 = 0.017$ cm).

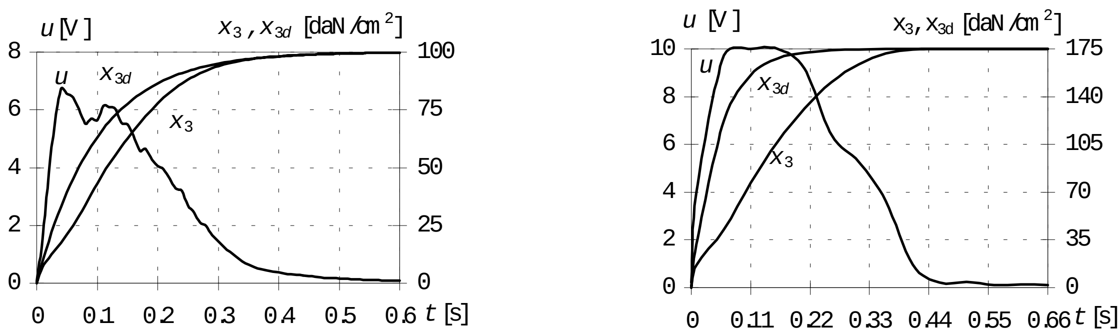
An evidently better tracking of step references is ensured by backstepping position control synthesis. The values of the tuning parameters were: $k_1 = 400$ daN/cm; $k_2 = 4$ daN/cm; $k_3 = 400$ cm^5/(daN×s); $\rho_1 = 400$ daN/cm; $\rho_2 = 0.033$ daN×s^2/cm; $\rho_3 = 1$ cm^5/daN. The desired control objective, in terms of position reference $x_{1s} = 0.0255$ cm and $t_r = 0.01$ s, is accomplished with faster time constant $\tau \cong 0.0255$ s; when t_r was chosen 0.005 s, a better time constant $\tau \cong 0.0217$ s was obtained (Fig. 1b). The transient regime is stable, irrespective of stationary regime values x_{1s} and t_r; however, the designer must be attentive to control saturation. To counteract this effect, special antiwindup strategies can be used [3].

Similar conclusions are valid in the case of backstepping force control (Fig. 2). The values of the tuning parameters were: $k_3 = 400$ s^{-1}; $k_4 = 800$ daN/(cm×s); $\rho_3 = 1$ cm^5/daN; $\rho_4 = 1$ daN/cm. A time constant $\tau = 1/573$ s of the valve was considered. Worthy noting, herein the valve displacement/voltage coefficient $k_v = 0.085/(0.05\times10)$ cm/V means a tenfold necessary valve port area. The desired control objective, in terms of pressure reference $x_{3s} = 100$ daN/cm^2 and $t_r = 0.1$ s (Fig. 2a), and $x_{3s} = 150$ daN/cm^2 and $t_r = 0.1$ s, is accomplished with good time constants: $\tau \cong 0.16$ s and, respectively, $\tau \cong 0.19$ s; when x_{3s} and t_r were chosen 175 daN/cm^2 and, respectively, 0.05 s, an also good time constant $\tau \cong 0.175$ s was obtained (Fig. 2b).



a) b)

Figure 1 – Comparison between the step input tracking.
a) SMHR case: $\tau \cong 0.035$ s; b) EHS case with backstepping position control: $\tau \cong 0.0217$.



a) b)

Figure 2 – Backstepping pressure control: a) $x_{3s} = 100$ daN/cm^2, $t_r = 0.1$ s, $\tau \cong 0.16$ s; b) $x_{3s} = 175$ daN/cm^2, $t_r = 0.05$ s, $\tau \cong 0.175$ s.

It can be seen that the state variables x_1, in the first case, and x_3, in the second case, come very close to the desired x_{1d} and, respectively, x_{3d}, values.

However, one cannot neglect the fact that these results were obtained for the case when all parameters are exactly known. A further work should treat the adaptive case, which supposes off-line and/or on-line identifying of system parameters.

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