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PHASE/SHAPE TRANSITIONS IN NUCLEI

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Recent studies have discussed phase transitional behavior in the evolution of nuclear structure from spherical to deformed shapes and have led to the development of a new class of symmetries, E(5) and X(5), describing analytically nuclei at the critical point of the phase transitions. Empirical realizations of the X(5) symmetry are presented.

1. INTRODUCTION

Central to the understanding of nuclear structure is the existence of a set of paradigms that act as benchmarks describing idealized limits. Without these limits, collective nuclear level schemes would appear incredibly complicated and not amenable to easy understanding. Indeed, these paradigms provide the framework to address one of the two main themes of modern science in general and nuclear physics in particular, namely, how does one account for the astonishing regularity exhibited by complex many-body objects, such as nuclei, that represent the result of the combination of up to hundreds of strongly interacting fermions.

Fundamental to any discussion of nuclear structure and its evolution across the N-Z plane is the concept of magic numbers which provides the simplifying ansatz of an inert core and valence particles. Collectivity is described in terms of harmonic vibrators [1], deformed symmetric rotors [2], and γ-unstable nuclei [3]. The energy spectra corresponding to these benchmarks of nuclear structure are presented schematically in Fig. 1. These three limits of structure have been codified under the umbrella of the algebraic structure U(6) in the framework of the Interacting Boson Approximation (IBA) model [4] in terms of U(5), SU(3), and O(6) dynamical symmetries, respectively.

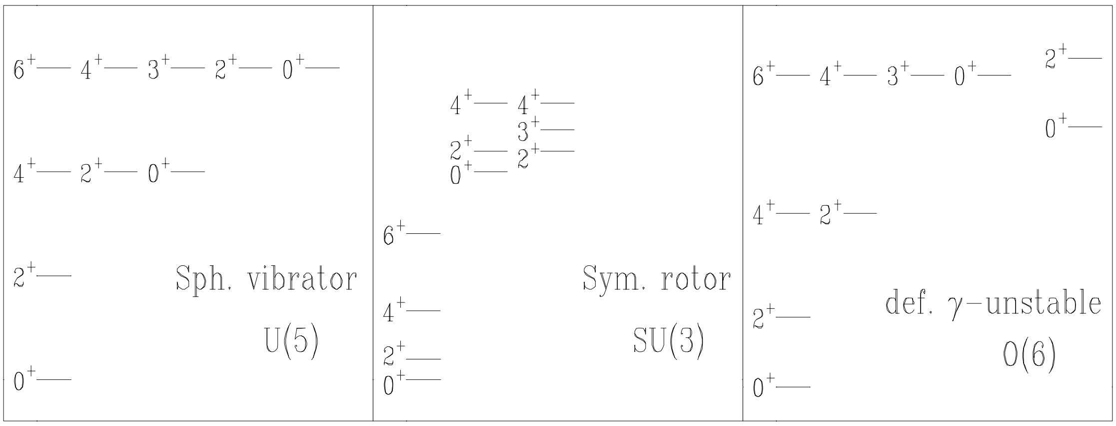

Figure 1. Energy spectra corresponding to a spherical vibrator, axially deformed rotor, and a deformed γ-unstable nucleus

In Figure 2 the symmetry triangle for the IBA [5] is shown. Each vertex represents one of the three symmetries mentioned above, and the legs denote transitional regions in which, over a series of nuclei, the

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structure undergoes a transition from one symmetry to another. Of course many nuclei correspond to internal positions in the triangle.

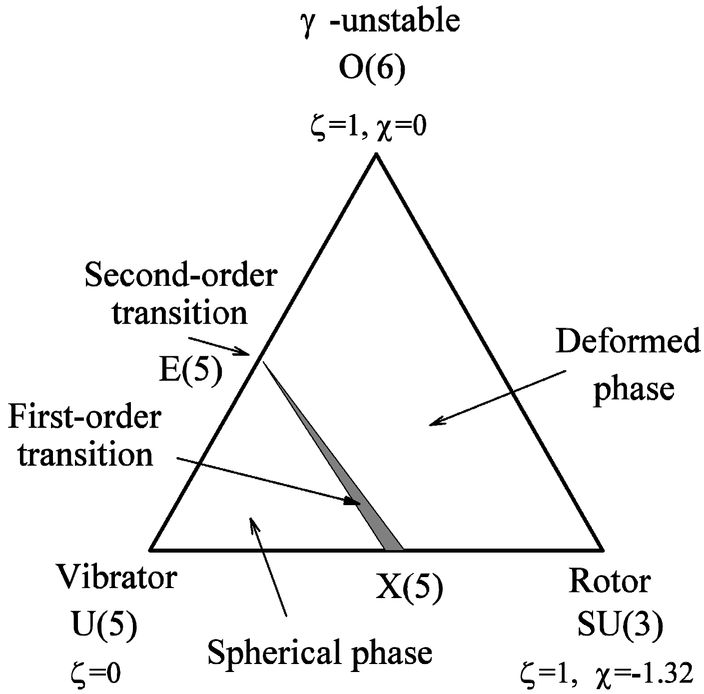

Figure 2. Symmetry triangle for nuclear structure showing the three traditional limits of structure (spherical vibrator, axially symmetric rotor, and deformed γ-unstable corresponding to U(5), SU(3), and O(6), respectively), the two new critical point symmetries, E(5) and X(5), and the phase/shape diagram in the framework of the Interacting Boson Model. See text.

Experimental examples of all three types of symmetries have been found in nuclei. The U(5) nuclei can occur near closed shells with only a few valence nucleons of both types and one of the best example is ^{114}Cd [6]. SU(3) nuclei can occur in the middle of shells and the closest to this symmetry are Yb and Hf with N=104 [7]. The O(6) limit tends to occur when one kind of nucleon comprises particles and the other kind holes and their total number is relatively small. The best realizations of this symmetry are in Pt (especially ^{196}Pt) [8] and Xe-Ba with N<82 (especially ^{124}Xe) [9].

Despite this phenomenological knowledge, it is useful to have a microscopic understanding of where nuclei exhibiting structures close to these symmetries occur. It is well known that the pairing interaction between like nucleons drives the nucleus towards a spherical shape since it forms the $J = 0^+$ coupling of pairs of identical nucleons which have spherically symmetric wave functions. Deformation and collectivity, on the other hand, arise from configuration mixing, which corresponds to a non-uniform distribution of magnetic substate occupation and, hence, of non-spherical shapes. Configuration mixing itself is largely driven by the valence p-n interaction. Hence it is a pairing—p-n competition that tends to drive the structural evolution of nuclei. We can use this idea to estimate the locus of collectivity in nuclei. Since the p-n interaction strength is, roughly, 200-250 keV, and the pairing interaction is 1 MeV, it takes something like five p-n interactions to overcome one pairing interaction. Thus, one expects significant collectivity and the onset of deformation when:

$$P = \frac{N_p N_n}{N_p + N_n} \approx 5 \tag{1.1}$$

where N_p and N_n are the numbers of valence protons and neutrons, respectively, N_pN_n represents the number of p-n interactions and $N_p + N_n$ the number of pairing interactions. Figure 3 illustrates the locus of $P_{crit} \sim 5$ for the principal regions of nuclei where the transition between spherical and quadrupole deformed nuclei takes place. These contours are only guidelines since they ignore sub-shell closures (*e.g.,* at Z = 64) and their evolution with N and Z. Many more such contours could be shown for the new regimes of nuclei far from stability that are beginning to become accessible as beams of exotic nuclei are developed.

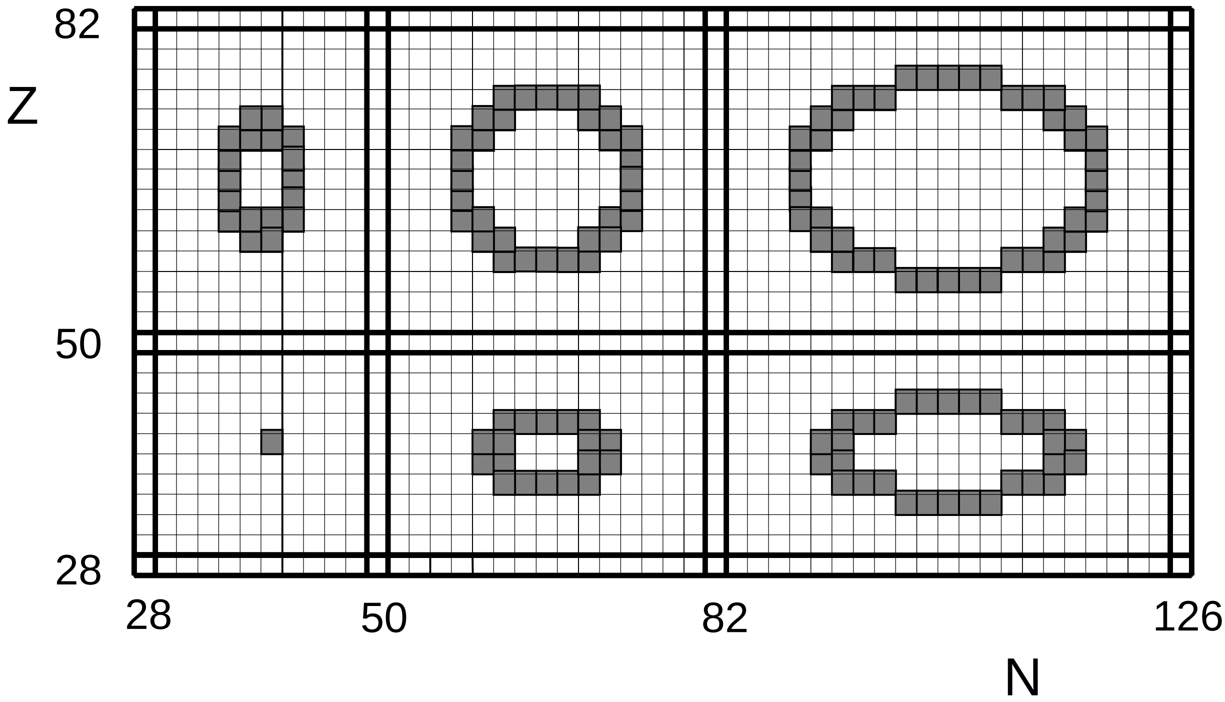

Figure 3. Section of the nuclear chart in the N/Z plane. The shaded area gives the contours of P~5 values which give a guide to the locus of the transition between spherical and quadrupole deformed nuclei, i.e., the locus of potential X(5) critical point nuclei.

The usefulness of the symmetries in Figs. 1, 2 extends beyond the few nuclei that actually realize them since they provide simple benchmarks in terms of which other nuclei can be described, and they serve as convenient limits for regions of structural transition between them. However, the range of structures between the symmetries is large and complex, and, until recently, essentially devoid of inter-symmetry paradigms. Indeed, until recently, the only way to describe transitional nuclei was by numerical diagonalizations of multi-parameter Hamiltonians.

However, this has now changed and, as we will see below, in some cases it is possible to have analytic solutions corresponding to a new class of symmetry, namely critical point symmetry. Such symmetries correspond to the point of phase transition between spherical and deformed shapes. Traditionally, nuclei with such structures are considered to exhibit complex behavior resulting from competing degrees of freedom. However, recently, Iachello found analytic solutions at the critical point of such phase transitions: E(5) for a transition between spherical and deformed γ-soft nuclei [10] and X(5) for a transition between spherical and axially deformed nuclei [11]. His approach is based on analytic solutions of the Schrödinger equation corresponding to a geometric (Bohr) Hamiltonian with a square-well potential.

The new theoretical concept of critical point symmetry was immediately backed up by the discovery of empirical realizations, namely, ^{134}Ba [12] and ^{102}Pd [13] for E(5), and ^{152}Sm [14,15], ^{150}Nd [16], ^{156}Dy [17], ^{154}Gd [18], and ^{104}Mo [19] for X(5). Not all of these nuclei are equally good examples of their respective symmetry nor is the data equally extensive for all. Nuclei such as ^{152}Sm and ^{150}Nd have been very well studied and are quite close to X(5), and the existing data for the others suggest they are good candidates. Further data in these cases would be very useful. These empirical manifestations, in particular X(5), will be briefly summarized here.

2. FIRST ORDER PHASE/SHAPE TRANSITION IN LOW ENERGY NUCLEAR SPECTRA AND THE CRITICAL POINT SYMMETRY X(5)

The study of the functional form corresponding to the total energy of the IBA Hamiltonian has shown [20] that there is a phase/shape transition as a function of the IBA parameters: a first-order phase transition for a U(5)–SU(3) transition and a second-order phase transition for U(5)–O(6) transition. This is best seen with the IBA Hamiltonian in terms of a "control parameter" ζ as follows:

$$H = (1-\zeta)n_d - \frac{\zeta}{4N_B} Q^{\chi} \bullet Q^{\chi} \tag{2.1}$$

where $Q^{\chi} = (s^{\dagger}\tilde{d} + d^{\dagger}s) + \chi(d^{\dagger}\tilde{d})^{(2)}$. This Hamiltonian describes (up to a scaling factor) the entire U(5)-SU(3) transition ($\chi = -\sqrt{7}/2$) by varying only the parameter ζ between 0 [in the U(5) limit] and 1 [in the SU(3) limit].

Figure 4 shows as an example the evolution of the total energy as a function of ζ for the U(5)-SU(3) transition for N_B = 10 bosons. There is a phase/shape coexistence region, which starts where the deformed minimum develops in addition to the spherical one and ends where the spherical minimum disappears and only the deformed minimum remains. In between there is a point where the two minima are equal and the first derivative of E_{min}, $\partial E_{min}/\partial\zeta$, is discontinuous and, consequently, the phase transition is first-order. Phase transitions are, of course, defined only for infinite number of particles and for finite number the discontinuities are smoothed out.

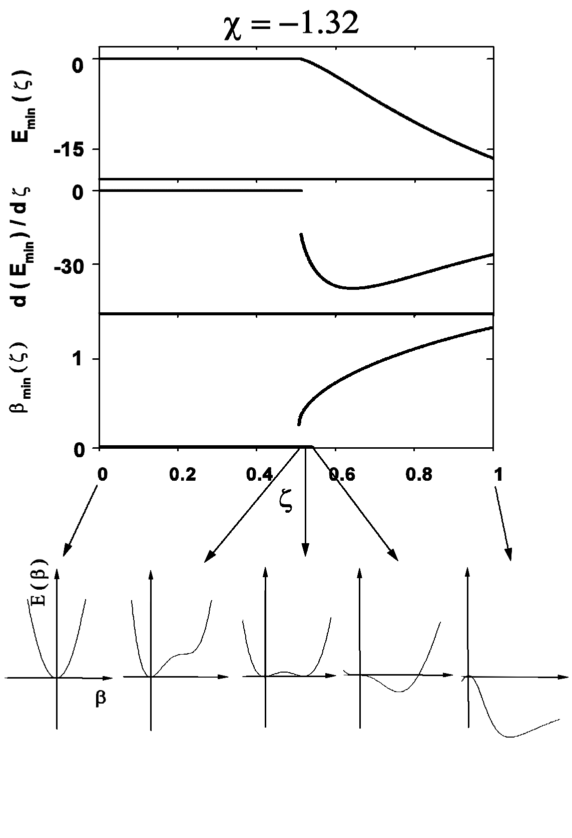

Figure 4. The evolution of total energy for the U(5)-SU(3) leg of the symmetry triangle as a function of the control parameter ζ for the U(5)-SU(3) transition (χ=-1.32) for N_B = 10 bosons (calculations of G. Fernandes).

A well known example of a transition from spherical vibrator [U(5)] to axially deformed [SU(3)] nuclei occurs in the A ~150 mass region, and the IBA model can reproduce very well a large variety of the data. We performed IBA calculations for the entire region N~90 with the simple [21] IBA Hamiltonian in eq.

(2.1) which involves only one parameter, ζ,, which depends only on the neutron number N, for all the isotopic chains [22]. In these calculations, χ was set equal to the value $-\sqrt{7}/2$ corresponding to the bottom leg of the triangle. Figure 5 shows that the evolution of some basic observables in Nd-Dy nuclei is very well reproduced by these calculations. The empirical $R_{4/2}$≡E(4^+)/E(2^+) ratio evolves from ~2.0, characteristic for a spherical vibrator to ~3.33 for a rotor with a sharp rise at neutron number N=90. The energy of the intrinsic excitation 0^+ has a minimum at the phase transition point. The phase/shape transition is mirrored in the calculations and, as noted, the IBA parameter ζ plays the role of a control parameter.

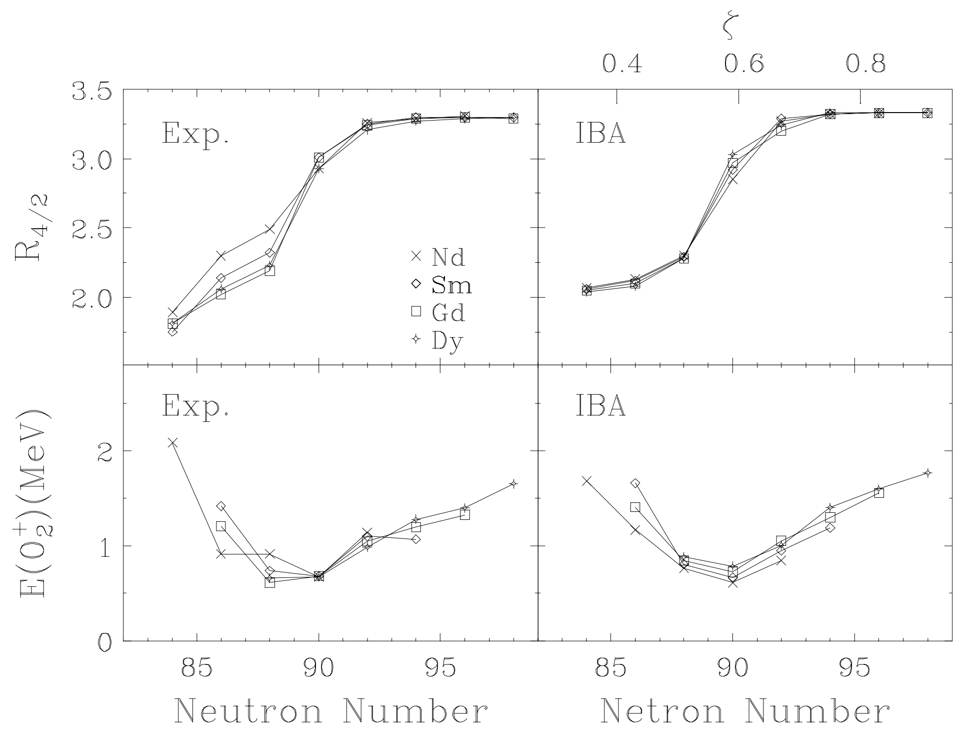

Figure 5. The evolution of empirical values of the $R_{4/2}$≡E(4^+)/E(2^+) ratio and of E(0^+_2) in the Nd-Dy isotopic chains with N>82 compared with the IBA results.

The total energy surface corresponding to the Sm isotopes [23] obtained from the IBA with increasing neutron number changes the location of the deformation minimum, from β = 0 to finite β when the neutron number increases from 88 to 92. Figure 6 shows schematically the evolution of this energy. At the critical value ζ = 0.5, which corresponds to N=90 (^{152}Sm), two phases/shapes coexist [24] and the energy surface has a nearly flat bottom, as envisioned in the X(5) critical point symmetry.

N=90 seems empirically to be exactly the point which corresponds to X(5). Despite the fact that X(5) is a crude approximation to the real problem, the comparison with the data presented in Figure 7 shows that the agreement is impressive. Except for scale, the predictions are parameter free. The experimental energy ratios $R_{4/2}$ = 3.01 and R(0^+) ≡ E(0^+)/E(2^+) = 5.62 are well reproduced by the symmetry where these ratios are 2.91 and 5.67, respectively. Moreover, X(5) predicts a near degeneracy of the 0^+ and 6^+ levels which is observed in ^{152}Sm. The in-band B(E2) values are also very well reproduced and the inter-band B(E2) strengths, despite the fact that they are predicted to be larger than the data, have relative ratios in good agreement with the model [14,15].

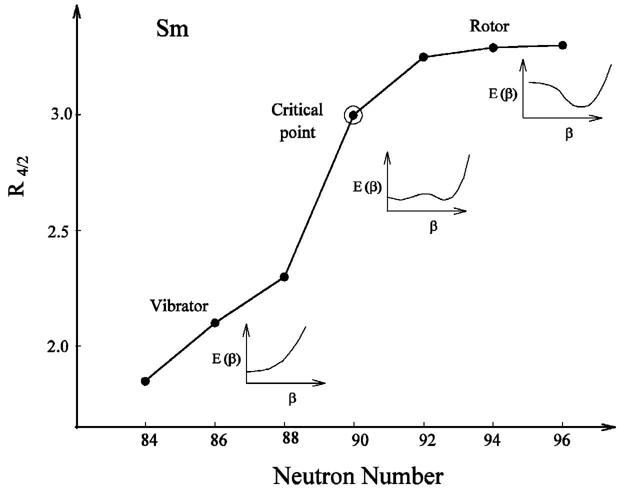

Figure 6. The vibrator-rotor transition in the Sm isotopes with sketches of the total energy as a function of the quadrupole deformation.

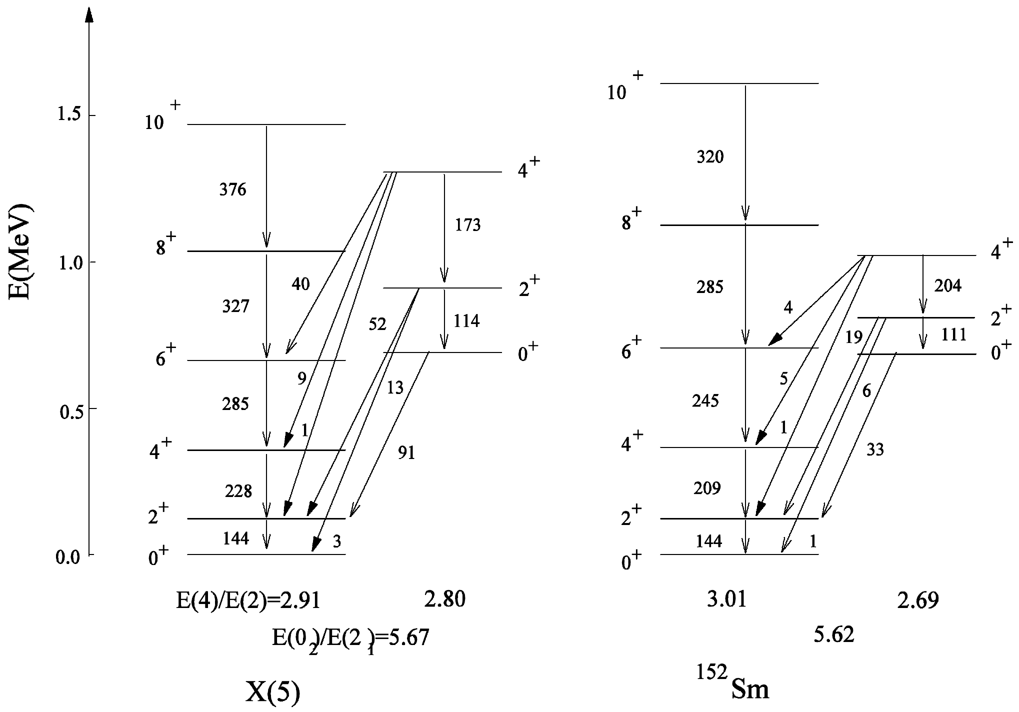

Figure 7. Comparison of predictions of the critical point symmetry X(5) with the data for ^{152}Sm. The B(E2) values in W.u. are given for each transition. Figure based on ref. [14].

Other nuclei in the A~150 transitional region with N=90 have similar properties (see Fig. 8). All these nuclei have $R_{4/2} \sim 3$ and $E(0^+_2)/E(2^+_1) \sim 5$ which make them very good candidates for the X(5) symmetry.

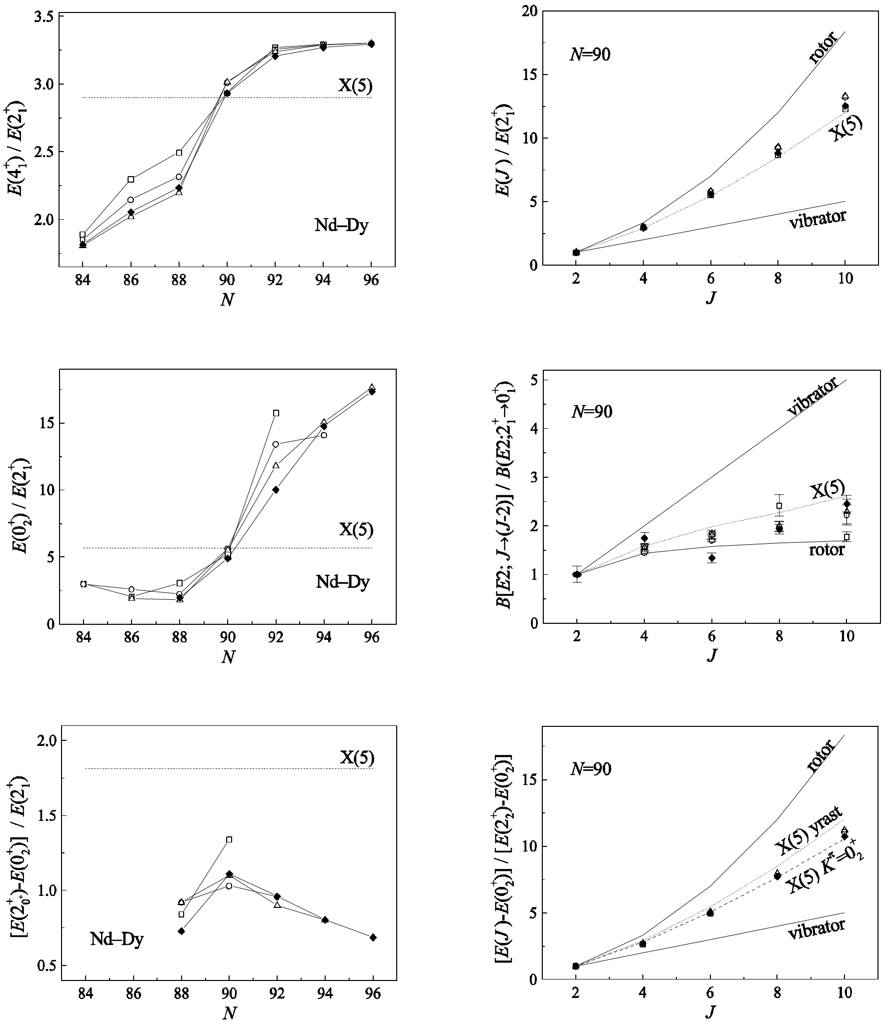

Figure 8. Evolution of different observables across the N=90 region for the Nd, Sm, Gd and Dy isotopic chains compared with the X(5) predictions. Figure based on ref. [17].

The search for other candidates for the X(5) symmetry in the rare earth region is an important issue. We can use the concept illustrated in Fig. 3 to aid in identifying candidates. Figure 9 shows an expanded region of the rare earth nuclei with the P ~ 5 contour. In the box for each nuclide we show the empirical $R_{4/2}$ ratio. Recalling that $R_{4/2} = 2.91$ is the characteristic X(5) value, we see that the P-contour gives the locus of X(5) candidates very well. At Yale, we are currently studying ^{162}Yb and ^{166}Hf which, from Fig. 9, are evidently potential X(5)-like nuclei. ^{162}Yb, in particular, provides an interesting case that also illustrates the power of studying traditional reactions with modern spectroscopic tools. The existing ^{162}Yb level scheme shows a ratio $R(0^+_2)/R(2^+_1) = 3.63$ which is far from the X(5) value of 5.67. There is a second excited 0^+ state, however, with $R(0^+)/R(2^+_1) = 6.03$, much closer to X(5). The lower 0^+ state is empirically based on unpublished β decay data from 1980 [25]. Recent Yale experiments with a Moving Tape Collector/Clover detector system at WNSL showed conclusively that the heretofore assigned first excited 0^+ state does not in fact exist [26]. Hence, ^{162}Yb now takes on added interest as a potential X(5) nucleus. However, the yrast B(E2) values are a bit puzzling. They do not seem to reflect the structure of any single model. We are currently carrying out experiments to re-measure them.

Z

N

78			2.30		2.26	2.44	2.51	2.70	2.68	Pt
76				2.62	2.66	2.74	2.93	3.02	3.09	Os
74			2.68	2.82	2.95	3.07	3.15	3.22	3.24	W
72	2.31	2.56	2.79	2.97	3.11	3.19	3.25	3.27	3.28	Hf
70	2.33	2.63	2.93	3.12	3.23	3.27	3.29	3.31	3.31	Yb
68	2.32	2.74	3.10	3.23	3.28	3.29	3.31	3.31	3.31	Er
66	2.23	2.93	3.21	3.27	3.29	3.30	3.31			Dy
64	2.19	3.02	3.24	3.29	3.30	3.30				Gd
62	2.32	3.00	3.25	3.29	3.30	3.30				Sm
60	2.49	2.93	3.26	3.29	3.32					Nd
58	2.59	2.86	3.15							Ce
56	2.66	2.84	2.99							Ba
Z/N	88	90	92	94	96	98	100	102	104	

Figure 9. Nuclei in the 54<Z<80, 86<N<106 region. The shaded contour shows the locus of P~5 values, i.e., candidates for the critical point symmetry X(5). The numbers in each box are the empirical $R_{4/2}$ ratios. Based on ref. [26].

3. SUMMARY

In this brief discussion, we have tried to outline some of the newest developments in the structure of nuclei undergoing rapid shape transitional behavior.

We have discussed the theoretical description of new symmetries at the critical point of spherical-deformed phase transitions and the experimental evidence for them. The X(5) analytic solutions for the critical point in the spherical to axially deformed phase/shape transition is closely manifested empirically in ^{152}Sm and in other N=90 isotones. We have also discussed a simple ansatz based on a microscopic perspective for identifying possible new regions of critical point behavior, and illustrated this with comments on ^{162}Yb.

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