

## ON THE AUTOMOTIVE SEMI-ACTIVE SUSPENSIONS

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An application of semi-active systems to automotive suspension is considered. The study is based on the fact that the vibratory behavior of the system with motion-dependent suspension force is considerably different than that exhibited by damped systems with constant-magnitude suspension force. In the semi-active case, the damping force can be controlled and the suspension force can be estimated during the motion. But we know that this problem is ill-posed. A simultaneous optimization technique is applied to transform the ill-posed problem into a well posed one. It is suggested a solution that is based on wavelets.

*Key words:* semi-active suspension, energy dissipation, simultaneous optimization, ill-posed problem, Haar wavelets.

### 1. INTRODUCTION

Suspension systems, by involving large forces and velocities, can be sources of disturbing vibrations that may influence the performance characteristics of a vehicle. A semi-active absorber is to properly adjust the absorber parameters so that it absorbs the vibratory energy within the frequency interval of interest. The vibration absorbers have a history of almost a century, and have proven to be useful to suppress vibrations in hundreds of diverse applications (Frahm 1911 [1]). Because of their low energy requirement and cost, semi-active suspensions can provide useful vibration suppression solutions for a large tonal and broadhand applications.

The importance of energy dissipation in suspensions systems is recognized most in automotive suspensions, where ride comfort and vehicle handling are encountered (Valasek and Kortum [2], Jalili [3], Sireteanu *et al.* [7, 8]). The concept of semi-active suspension and semi-active vibration control in connection with the power consumed was introduced by Karnopp [4-6]. There some classes of semi-active devices.

The first class is the variable orifice viscous dampers, which dissipate energy. The flow control is achieved using electro- and magneto-rheological fluids and is available as industrial products (Sireteanu *et al.* [8]). The second class is variable force transformers (variable lever arm on which the force acts), which conserve energy between suspension and spring storage. By moving the point of force application, the force transfer ratio change. The third class of semi-active elements exhibits a variable stiffness, which dissipates energy. For the variable free length of a hydro-pneumatic spring, if the valve is shut, one volume is connected only and the spring is stiff. When the valve is open, both volumes are connected and the spring is soft.

Using these semi-active devices the properties of vehicle suspensions can be controlled according to the scheme on fig. 1.1, where  $m_s$  represents sprung mass and  $m_u$  unsprung mass ([2, 3]). There are two main categories of disturbances on a vehicle: the road and load disturbances. Road disturbances are characterized by large magnitude in low frequencies (hills) and small magnitude in high frequency (road roughness).

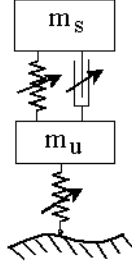


Fig. 1.1  
Semi-active vehicle suspension.

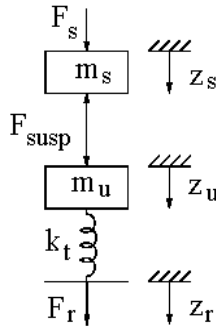


Fig. 1.2  
The quarter-car model.

Load disturbances are given by variations of loads induced by accelerating, braking and cornering. A good suspension needs to be “soft” to insulate against road disturbances, and “hard” to insulate against load disturbances. The design of a suspension must make a compromise between stability and comfort. A damper having a large damping constant provides good stability, keeping the tires in contact with the road, but will transfer much of the road input to the passenger, causing an uncomfortable ride. Conversely, a damper with a low damping constant will disturb the travelers less, but it lowers the stability of the vehicle. There are three main categories of suspensions: passive, active and semi-active.

A passive suspension has not sources of energy, The active suspension incorporates extra energy sources to refine the compromise.

The idea of a semi-active suspension is to replace active force generators with continually adjustable elements which can vary/or shift the rate of energy dissipation in response to an instantaneous condition of motion. The suspension force can be controlled through active means in response to sensory feedback (Ferry [13]). But this desiderate needs the problem to be well posed. In this paper a simultaneous optimization technique is applied for adaptive control of a quarter car model as shown in fig. 1.2. The unknown functions are expressed as a combination of Haar wavelets. Despite its simplicity, the quarter-car model covers the basic properties of suspensions for a vehicle.

## 2.THE QUARTER-CAR MODEL

The suspension consists of a damper of unknown function  $c_s(t)$  in parallel with a spring of unknown function  $k_s(t)$ . The tire is modeled as a linear spring with given constant  $k_t$ . The external disturbances are the load  $F_s(t)$ , and the road displacement  $z_r(t)$ . Consider that in the interval  $0 \leq t \leq T$ , where  $T$  is a sufficient large end time, we can take in each moment measurements of the acceleration of the sprung mass  $\ddot{z}_s$ , and of the relative velocity between the sprung and unsprung masses  $v_{rel} = \dot{z}_s - \dot{z}_u$ .

The evolution equations of the system are given by

$$m_s \ddot{z}_s(t) = F_s(t) - F_{susp}(t), \quad m_u \ddot{z}_u(t) = F_{susp}(t) + F_r(t), \quad (2.1)$$

$$F_r(t) = k_t(z_r - z_u), \quad (2.2)$$

where  $z_s, z_u, z_r$  are the relative displacements of the sprung, unsprung masses and respectively of the road,  $\ddot{z}_s$  and  $\ddot{z}_u$  are accelerations of the sprung and unsprung masses,  $F_{susp}$  represents any possible suspension force given by the sum of a damping force  $F_d$  and an elastic force  $F_e$

$$F_{susp}(t) = F_d(t) + F_e(t). \quad (2.3)$$

The semi-active friction is mathematically similar to semi-active dampers based on other energy dissipation mechanisms. But the physics of friction dampers is different. This difference may be exploit in

the design of control law. The suspension force  $F_{susp}$  is modeled to vary continuously between a soft suspension (to insulate against road disturbances  $z_r$ ), and a hard suspension (to insulate against load disturbances  $F_s$ ). So, the demand damper force  $F_d$  may be written as

$$\text{if } F_d(\dot{z}_s - \dot{z}_u) > 0, F_d(t) = c_s(t)(\dot{z}_s - \dot{z}_u), \quad (2.4)$$

$$\text{if } F_d(\dot{z}_s - \dot{z}_u) \leq 0, F_d = 0. \quad (2.5)$$

In control strategy the elastic force may be given as

$$F_e = k_s(t)(z_s - z_u). \quad (2.6)$$

The initial conditions for functions  $c_s(t)$  and  $k_s(t)$  are given for a soft suspension, i.e.

$$k_s(0) = k_s^s, c_s(0) = c_s^s. \quad (2.7)$$

Because the system (2.1), (2.2) is controllable and observable almost everywhere over the functions  $c_s(t)$  and  $k_s(t)$ , we must define these functions. One of the way is to write  $c_s(t)$  and  $k_s(t)$ , as a combination of Haar wavelets (Bernard [9], Poterasu and Zăgan [12]). Consider a family of Haar wavelets  $\Psi_{jk}$ ,  $j, k \in Z$ ,

$$\Psi_{jk}(t) = 2^{j/2} \Psi(2^j t - k), \quad (2.8)$$

where  $j$  is a resolution index and  $k$  a translation index, and

$$\Psi(t) = \begin{cases} -1, & \text{for } 0 \leq t \leq \frac{1}{2} \\ 1, & \text{for } \frac{1}{2} \leq t \leq 1 \end{cases} \quad (2.9)$$

The functions  $c_s(t)$  and  $k_s(t)$  may be chosen as

$$c_s(t) = \sum_{j, k \in Z} \sigma_{jk} \Psi_{jk}, k_s(t) = \sum_{j, k \in Z} \eta_{jk} \Psi_{jk}, \quad (2.10)$$

where  $\sigma_{jk}$  and  $\eta_{jk}$ ,  $j, k \in Z$ , are unknown parameters.

The problem we consider in this article is to determine the unknowns parameters  $\sigma_{jk}$  and  $\eta_{jk}$ ,  $j, k \in Z$ , from an optimization procedure that consists in the minimization of a two-objective function.

The first objective is to introduce a control sequence  $F_{susp}(t)$ , for determining the unknown parameters from measured quantities  $(\ddot{z}_s^{mes}, \dot{z}_s^{mes} - \dot{z}_u^{mes})$ , at some moments of time. The second objective represents the performance of the system. There are three basic performance requirements we consider here: the comfort represented by  $\ddot{z}_s$ , the suspension deflection represented by  $z_u - z_s$ , and the tire force represented by  $z_s - z_r$ .

To decide a good suspension, these three performance criteria could be summed by their root mean square values in response to road and load inputs.

### 3. THE SIMULTANEOUS OPTIMIZATION PROBLEM

When carrying out the optimization we first have to define the objectives. Let consider a discrete grille  $D = (t_1, \dots, t_j, \dots, t_n = T)$  of the interval  $0 < t \leq T$ . The motion begins at  $t_0 = 0$ . The unknown parameters  $\sigma_{jk}$  and  $\eta_{jk}$ ,  $j, k \in Z$ , of the functions  $c_s(t)$  and  $k_s(t)$  defined by (2.10) are solutions of the problem

$$\text{Minimize } \Psi(t) = \Psi_1(t) + \Psi_2(t), \quad t \in D. \quad (3.1)$$

The first objective function  $\Psi_1(t)$  represents the condition of fitting the measured quantities  $(\dot{z}_s^{mes}, \dot{z}_s^{mes} - \dot{z}_u^{mes})$  with the calculated quantities  $(\dot{z}_s, \dot{z}_s - \dot{z}_u)$  at each time of  $D$ ,

$$\Psi_1(t) = \alpha_1 (\dot{z}_s^{mes} - \dot{z}_s)^2 + \alpha_2 [(\dot{z}_s^{mes} - \dot{z}_u^{mes})^2 - (\dot{z}_s - \dot{z}_u)^2], \quad t \in D, \quad (3.2)$$

where  $\alpha_i, i=1,2,3$ , are weighting factors.

The second objective function  $\Psi_2(t)$  represents the performance function

$$\Psi_2(t) = E[\gamma_1 \ddot{z}_s^2 + \gamma_2 (z_s - z_r)^2 + \gamma_3 (z_u - z_s)^2], \quad t \in D, \quad (3.3)$$

where  $E$  denotes the expectations necessary because of the random road disturbances  $z_r$ , and  $\gamma_i, i=1,2,3$ , are weighting factors.

The problem of minimization the quadratic matching functional (3.1) (3.3) is ill-posed because the functional is positive, but not positive definite.

An ill-posed problem consists in solving an equation  $Ax = B$  where  $A$  is a non-invertible operator. A general way to handle ill-posed problems is regularization. Regularization consists in rewriting the inversion problem as a minimization of a quadratic matching functional  $x = \min_{x'} \|Ax' - B\|^2$ , as we done by (3.1) (3.2). But the last one is also ill-posed because this functional is positive but not positive definite. The regularization method consists again in penalizing the matching functional with additional term that is strictly positive definite, for example  $x = \min_{x'} \|Ax' - B\|^2 + I \|x'\|^2$ . The resulting functional is then positive definite by construction, and the minimization problem has a unique solution. Regularization is thus a systematic way to transform an ill-posed problem into a well posed one.

In our case we penalize the matching functional  $\Psi(t)$  with additional terms that is strictly positive definite

$$\text{Minimize } \tilde{\Psi}(t) = \Psi(t) + I F_{susp}(\dot{\theta}) [\dot{z}_s(t) - \dot{z}_u(t)] + \mathbf{m}[F_m - |F_{susp}(\dot{\theta})|], \quad t \in D, \quad (3.4)$$

Where  $F_{susp}(t)[\dot{z}_s(t) - \dot{z}_u(t)] \geq 0$ , is a passivity constraint for the control sequence  $F_{susp}(t)$ , and  $0 \leq F_m - |F_{susp}(\dot{\theta})|$  is a condition required by the vehicle to tolerate only bounded suspension forces,  $F_m$  being the maximal allowed force (Valasek and Kortum [2], Jalili [3]). The resulting functional  $\tilde{\Psi}(t)$  is then positive definite and the minimization problem has a unique solution.

The idea of a semi-active damper is to transform the required force to certain settings of the damping rate  $c_s$  such that the damping force to be nearest to the desired value. The function  $c_s(t)$  is set for the interval  $[c_s^s, c_s^h]$  as

$$c_s = \begin{cases} c_s^h, & \text{if } c_s^h < c_s \text{ (hardsuspension),} \\ c_s, & \text{if } c_s^s < c_s < c_s^h, \\ c_s^s, & \text{if } c_s < c_s^s \text{ (soft suspension).} \end{cases} \quad (3.5)$$

With the same reason, the function  $k_s(t)$  is set for the interval  $[k_s^s, k_s^h]$  verifying the conditions

$$k_s = \begin{cases} k_s^h, & \text{if } k_s^h < k_s \text{ (hardsuspension),} \\ k_s, & \text{if } k_s^s < k_s < k_s^h, \\ k_s^s, & \text{if } k_s < k_s^s \text{ (soft suspension).} \end{cases} \quad (3.6)$$

The quantities  $(\dot{z}_s, \dot{z}_s - \dot{z}_u)$  at the moment  $t_0 = 0$  are calculated by the problem (2.1), (2.2) and (2.7). The new estimates for  $c_s(t_0)$  and  $k_s(t_0)$  are determined from (3.1) (3.6). For the next step  $t = t_1$  the

quantities  $(\ddot{z}_s, \dot{z}_s - \dot{z}_u)$  are calculated by the problem (2.1), (2.2) by using the last estimated values for  $c_s(t_0)$  and  $k_s(t_0)$ . This algorithm continues in this way on the entire interval of time  $0 < t \leq T$ .

To account for the measured data, each set of theoretical results is multiplied by  $(1 + r_i)$ , where  $r_i$  are random numbers uniformly distributed in a given interval  $[-\varepsilon, \varepsilon]$ , with  $\varepsilon = 0.01$ . The resulting quantities are considered “measured data  $(\ddot{z}_s^{mes}, \dot{z}_s^{mes} - \dot{z}_u^{mes})$ ”.

#### 4. RESULTS AND DISCUSSION

The damper is characterized by unknown function  $c_s(t)$ . Therefore, the damping force may vary as a result of passive or semi-active means. In the passive case, the force is made to depend on the relative slip displacement through the structure, but in semi-active case, it depends on the motion and can be controlled. To illustrate the theory, we select the following parameters for the quarter-car model:  $m_s = 250\text{kg}$ ,  $m_u = 35\text{kg}$ ,  $k_t = 150\text{kN/m}$ ,  $k_s^h = 120\text{kN/m}$ ,  $k_s^s = 12\text{kN/m}$ ,  $c_s^h = 40\text{kNs/m}$ ,  $c_s^s = 4\text{kNs/m}$ . Two cases of road profiles are considered. The first case represents a short bump road signal and it is represented in fig. 4.1, and the second case is represented in fig. 4.2. The amplitudes  $A$  of the sprung mass and the unsprung mass displacements and the road tire force  $F_r$  are illustrated in fig. 4.3 for the road profile given in fig. 4.1. The unit weighting factors  $\alpha_1, \alpha_2, \mu$  and  $\lambda$  are considered. We compare the passive case with the sky-hook suspension case ( $\gamma_1 = 1, \gamma_2 = \gamma_3 = 0$ ), the ground hook case ( $\gamma_2 = 1, \gamma_1 = \gamma_3 = 0$ ) and the semi-active case ( $\gamma_1 = \gamma_2 = \gamma_3 = 1/3$ ).

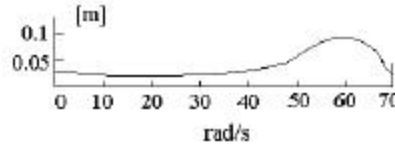


Fig. 4.1  
The road signal.

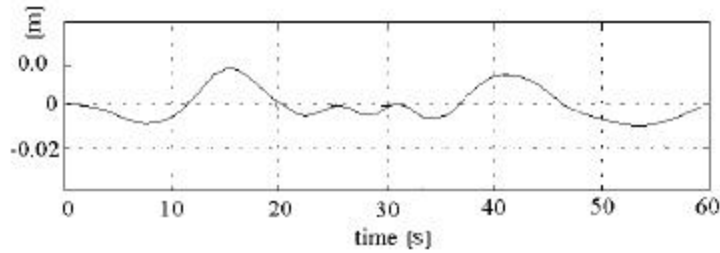


Fig. 4.2  
The profile of road.

It is observable that the semi-active suspension works as a combination of fictitious cases: sky-hook and ground-hook controls. Fig. 4.4 shows the velocity and displacement histories of the unsprung mass in the case of the profile road displayed in fig. 4.2. Fig. 4.5 shows the velocity and displacement time histories in the semi-active case, while fig. 4.6 shows the velocity and displacement time histories in the semi-active case. We observe that is a considerable improvement in the semi-active case over the passive case. The relative displacement between the unsprung and the sprung masses in the semi-active case are represented in fig. 4.7.

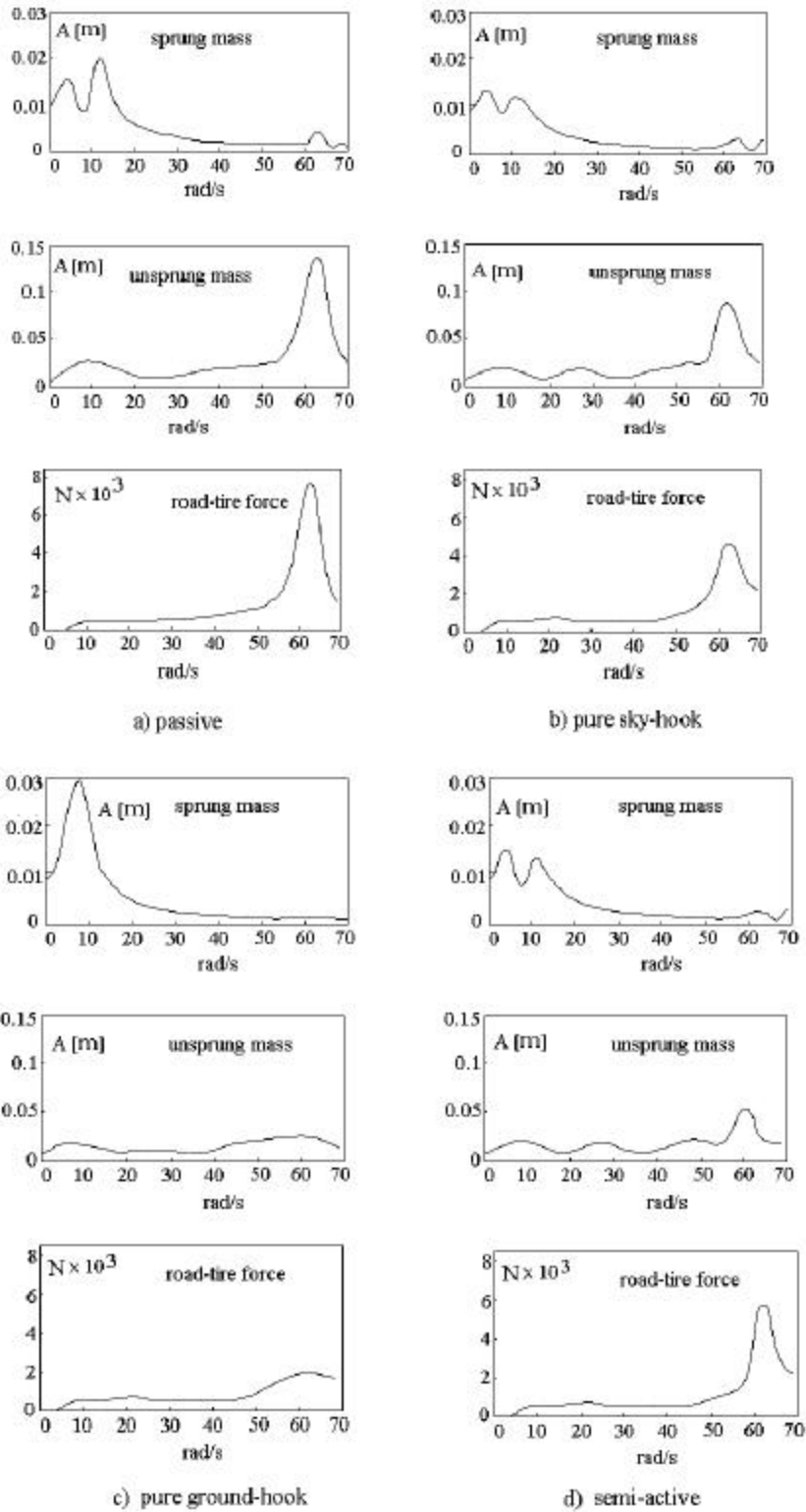


Fig. 4.3.

Comparison of the response (displacement amplitude and the road-tire force) a) passive suspension, b) pure sky-hook, c) pure ground-hook and d) semiactive suspension to a short signal in rad/s.

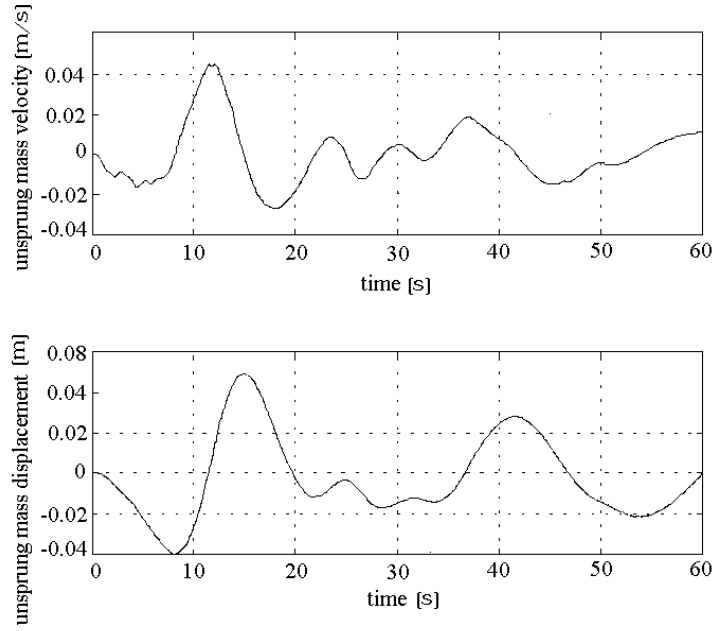


Fig. 4.4

Unsprung mass velocity and displacement in the passive case.

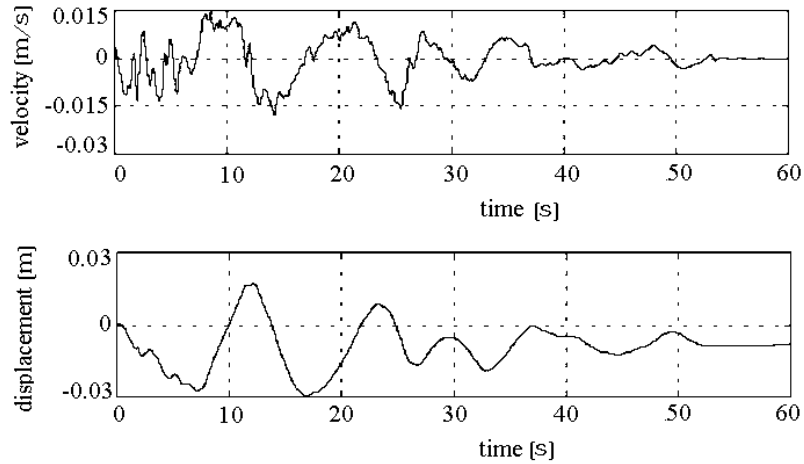


Fig. 4.5.

Unsprung mass velocity and displacement in the semi-active case.

It is well known that the efficiency of the formalism depends on the sensibility upon the measurements errors. To analyze this, we introduce a measurement noise by considering  $\varepsilon = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}$ . The relative errors  $e_1$ ,  $e_2$  and  $\tilde{e}$  for  $\Psi_1(t)$ ,  $\Psi_2(t)$  and  $\tilde{\Psi}(t)$  evaluations are computed. Expressions of these indicators are as follows

$$e_1 = \frac{D_{1mes}}{D_1} - 1, \quad e_2 = \frac{D_{2mes}}{D_2} - 1, \quad \tilde{e} = \frac{\tilde{D}_{mes}}{\tilde{D}} - 1, \quad (4.1)$$

$$D_1 = \int_0^T [\Psi_1(t)]^2 dt, \quad D_2 = \int_0^T [\Psi_2(t)]^2 dt, \quad \tilde{D} = \int_0^T [\tilde{\Psi}(t)]^2 dt. \quad (4.2)$$

The results are displayed in table 1.

Table 1. Results for evaluations of error indicators.

	$\varepsilon = 10^{-4}$	$\varepsilon = 10^{-3}$	$\varepsilon = 10^{-2}$	$\varepsilon = 10^{-1}$
$e_1$	$6.44 \times 10^{-5}$	$1.13 \times 10^{-4}$	$1.55 \times 10^{-3}$	$8.56 \times 10^{-3}$
$e_2$	$5.09 \times 10^{-6}$	$1.19 \times 10^{-4}$	$1.15 \times 10^{-3}$	$0.44 \times 10^{-2}$
$\tilde{e}$	$8.21 \times 10^{-7}$	$1.66 \times 10^{-4}$	$1.87 \times 10^{-3}$	$1.43 \times 10^{-2}$

We mention the difference between friction and viscous semi-active suspension elements. We need damping forces even though the relative velocity between the car mass and the wheel mass (stroke velocity) is zero. This happens when a *skyhook damping* control is employed, which commands the damper force to be proportional to the absolute car velocity rather than the relative stroke velocity. The force can be made zero in a friction element by setting the normal force equal zero. The damping force in a viscous element cannot be zero due to the fact that a force exists as long as the fluid has some viscosity and the orifice has a finite opening (Ferry [13]).

In conclusion, the results exhibit a good accuracy, especially for small-perturbed data. The error indicators are found to vary linearly with  $\varepsilon$ . The functional  $\Psi_2(t)$  behaves better than  $\Psi_1(t)$ , and the final functional  $\tilde{\Psi}(t)$  behave well with respect to measurement noise. This is probably a consequence of the strong assumption made on the last functional to be penalized with additional terms to be strictly positive definite. In conclusion, we must mention that highly accurate measurements are required to obtain good predictions for the solutions. Large perturbations in these values can lead to erroneous estimates. The efficiency of the line research is an important issue since the problem of semi-active forces plays a vital role in many mechanical systems, despite of the fact that it is ill-posed.

The functions  $c_s(t)$  and  $k_s(t)$  are expressed as a combination of Haar wavelets, and the unknown parameters are determined by using a simultaneous optimization technique. This approach enables finding parameters of control, and also allows finding a satisfactory compromise among the performance criteria despite the possibility that they may conflict with each other. The optimal compromise can be evaluated by giving different values to the weighting factors. Comparing the performance, the Haar wavelets are better in this case to other approximations because they are simple to be applied, are stable and the solutions are detected through a relative small cost time at each iteration.

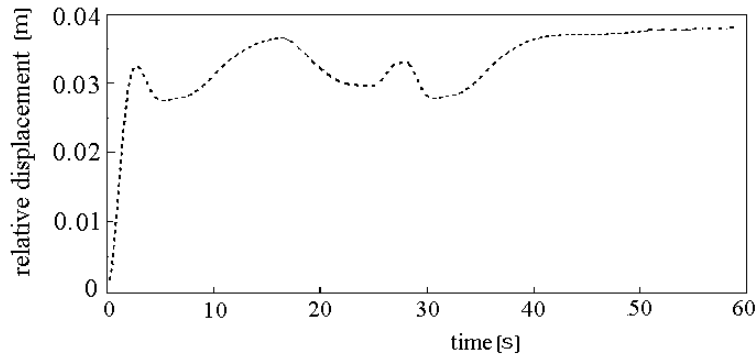


Fig. 4.6.  
Relative displacement between the sprung and unsprung masses in the semi-active case.



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