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UNSTEADY HELICAL FLOWS OF A MAXWELL FLUID

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The exact solutions corresponding to some unsteady helical flows of an incompressible Maxwell fluid, satisfying no-slip boundary conditions, are determined by means of the expansion theorem of Steklov. The similar solutions for a Navier-Stokes fluid appear as a limiting case of these solutions. The steady state solutions are also obtained for $t \longrightarrow \infty$.

1. INTRODUCTION

The simplest constitutive equation for a fluid is the Newtonian one. For the incompressible case it is of the form

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S},\ \ \mathbf{S} = \mu\mathbf{A},\ \ \mathrm{trace}\mathbf{A} = 0, \tag{1}$$

where $\mathbf{T}$ is the stress tensor, p the hydrostatic pressure, $\mathbf{S}$ the extra-stress tensor, $\mathbf{A}$ the first Rivlin-Ericksen tensor and μ the dynamic viscosity. Many common fluids show this "Newtonian" behavior and in fact the whole discipline of classical fluid mechanics is based upon this equation.

However, the constitutive equation (1) does not show any of the normal stress effects or relaxation phenomena and for many fluids, like dilute polymeric fluids, it is for that reason not acceptable. In cases of time dependent flows, for instance due to abrupt changes in the flow geometry, or to time dependencie of boundary conditions, relaxation phenomena should be included. The simplest way to do this is to use an equation of Maxwell type [1]:

$$\mathbf{S} + \lambda\frac{\delta\mathbf{S}}{\delta t} = \mu\mathbf{A}, \tag{2}$$

where λ is the relaxation time and $\delta/\delta t$ denotes a convective derivative. The most popular choice is the upper convected derivative

$$\frac{\delta\mathbf{S}}{\delta t} = \dot{\mathbf{S}} - \mathbf{LS} - \mathbf{SL}^{T}, \tag{3}$$

where $\mathbf{L}$ is the velocity gradient and the dot denotes material time differentiation. The associated "upper convected Maxwell-model" has the advantage that it is consistent with some important microscopical models of polymers and that its predictions of the normal-stress differences are qualitatively acceptable.

Recently, the Maxwell model has received special attention. Thus, some existence and uniqueness results corresponding to different steady flows of a class of fluids including the Maxwell model are obtained in [2, 3]. The existence of a large class of solutions, which arise from spatially periodic perturbations of uniform shear flow, is proved in [4]. The first exact solutions obtained for the flow of a Maxwell fluid seem to be those from [1] and [5]. Other analytical results are obtained in [6-8].

The research reported here is devoted to the study of a helical flow of an incompressible Maxwell fluid between two infinite coaxial circular cylinders. The flow is due to the cylinders, which are assumed to rotate about their axis and slide in the direction of the same axis with prescribed velocities. Finally, the special case of the flow in a cylinder is also considered. The similar solutions corresponding to the Navier-Stokes fluid appear as a limiting case.

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2. HELICAL FLOW BETWEEN CONCENTRIC CYLINDERS

We consider here an unsteady helical flow between two infinite coaxial cylinders located at $r = R_1$ and $r = R_2 \ (> R_1)$ in the cylindrical coordinate system $(r,\ \theta,\ z)$. Such a flow whose physical components of the velocity field are given by [5, 9]

$$v_r = 0, \quad v_\theta = \omega(r,\ t), \quad v_z = u(r,\ t), \tag{4}$$

is called helical because, in general, its streamlines are helices. Since the velocity field is independent of θ and *z*, the extra-stress tensor $\mathbf{S}$ will also be independent of θ and z and the incompressibility condition is automatically satisfied. Moreover, since the fluid was at rest up to the moment $t = 0$

$$\omega(r,\ 0) = u(r,\ 0) = 0 \quad \text{and} \quad \mathbf{S}(r,\ 0) = \mathbf{0}. \tag{5}$$

The flow is produced by the two cylinders which at $t = 0^+$ suddenly begin to rotate about their common axis $(r = 0)$ with the angular velocities Ω_1 and Ω_2 and to slide in the z-direction with the velocities U_1 and U_2. Assuming that the fluid adheres to the walls we have the boundary conditions

$$\begin{aligned} &\omega(R_1,\ t) = R_1\Omega_1, \quad \omega(R_2,\ t) = R_2\Omega_2; \quad t > 0, \\ &u(R_1,\ t) = U_1, \qquad u(R_2,\ t) = U_2; \qquad t > 0. \end{aligned} \tag{6}$$

Substituting (4) into (2) and (3) and taking into account $(5)_3$ we find that $S_{rr} = 0$ and

$$\begin{aligned} &(1+\lambda\partial_t)\tau_1 = \mu(\partial_r\omega - \omega/r), \quad (1+\lambda\partial_t)\tau_2 = \mu\partial_r u, \\ &(1+\lambda\partial_t)\tau_3 = \lambda[(\partial_r\omega - \omega/r)\tau_2 + (\partial_r u)\tau_1], \\ &(1+\lambda\partial_t)\sigma_1 = 2\lambda(\partial_r\omega - \omega/r)\tau_1, \quad (1+\lambda\partial_t)\sigma_2 = 2\lambda(\partial_r u)\tau_2, \end{aligned} \tag{7}$$

where $\tau_1 = S_{r\theta}$, $\tau_2 = S_{rz}$, $\tau_3 = S_{\theta z}$, $\sigma_1 = S_{\theta\theta}$ and $\sigma_2 = S_{zz}$.

The equations of motion, in the absence of body forces, reduce to

$$\partial_r p + \frac{1}{r}\sigma_1 = \rho\frac{\omega^2}{r}, \quad \partial_r\tau_1 + \frac{2}{r}\tau_1 = \rho\,\partial_t\omega, \quad \partial_r\tau_2 + \frac{1}{r}\tau_2 = \rho\partial_t u, \tag{8}$$

where ρ is the density of the fluid, $\partial_\theta p = 0$ due to the rotational symmetry and $\partial_z p = 0$ from the assumption that there is no applied pressure gradient along the axial direction (cf. [10]).

Now, we observe that Eqs. $(7)_{3-5}$ and $(8)_1$ for $\tau_3, \sigma_1, \sigma_2$ and p are not coupled with Eqs. $(7)_{1,2}$ and $(8)_{2,3}$, meaning that one can solve the system of the latter four equations first and then calculate $\tau_3, \sigma_1, \sigma_2$ and p. Eliminating τ_1 and τ_2 between Eqs. $(7)_{1,2}$ and $(8)_{2,3}$ we attain to the next two partial differential equations

$$\begin{aligned} &\lambda\partial_t^2\omega(r,\ t) + \partial_t\omega(r,\ t) = \nu\left(\partial_r^2 + \frac{1}{r}\partial_r - \frac{1}{r^2}\right)\omega(r,\ t), \\ &\lambda\partial_t^2 u(r,\ t) + \partial_t u(r,\ t) = \nu\left(\partial_r^2 + \frac{1}{r}\partial_r\right)u(r,\ t), \end{aligned} \tag{9}$$

where $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. It is also worth emphasizing that these equations are of a higher order than the similar Navier-Stokes equations. In order to obtain exact solutions the additional initial conditions [5]

$$\partial_t\omega(r,\ 0) = \partial_t u(r,\ 0) = 0, \tag{10}$$

have to be satisfied.

2.1. Calculation of the velocity field

Making the change of unknown functions

$$\omega(r,t)=\Omega(r)+v_1(r,t),\quad u(r,t)=U(r)+v_0(r,t), \tag{11}$$

where $\Omega(r)=r\ \ \Omega_2-\frac{\Omega_2-\Omega_1}{R_2^2-R_1^2}\ \ \frac{R_1^2(R_2^2-r^2)}{r}$ and $U(r)=U_2-\frac{U_2-U_1}{\ln(R_2/R_1)}\ \ln(R_2/r)$ we easily get from (9), (6) and $(5)_{1,2}$ the next two problems with initial and boundary conditions

$$\lambda\partial_t^2 v_n(r,t)+\partial_t v_n(r,t)=\mathrm{L}_n v_n(r,t);\quad r\in(R_1,R_2),\quad t>0, \tag{12}$$

$$v_n(r,0)=-V_n(r),\quad \partial_t v_n(r,0)=0;\quad r\in(R_1,R_2)\ , \tag{13}$$

$$v_n(R_1,t)=v_n(R_2,t)=0;\quad t\geq 0\ , \tag{14}$$

where $V_0(r)=U(r),\quad V_1(r)=\Omega(r)$, $\mathrm{L}_n=\partial_r^2+\frac{1}{r}\partial_r-\frac{n}{r^2}$ and $n=0,1$.

In order to solve these problems we shall use, as in [11], the well-known expansion theorem of Steklov. In view of this theorem our solutions $v_n(r,t)$ whose partial derivatives $\partial_r v_n$ and $\partial_r^2 v_n$ have to be piecewise continuous, can be written, for each $t>0$, as Fourier-Bessel series absolutely and uniformly convergent in terms of the eigenfunctions

$$B_n(rr_{nm})=A_n\left[\mathrm{J}_n(rr_{nm})-\frac{\mathrm{J}_n(R_1r_{nm})}{\mathrm{Y}_n(R_1r_{nm})}\mathrm{Y}_n(rr_{nm})\right]\ , \tag{15}$$

of the eigenvalues problems $\mathrm{L}_n v+\lambda v=0,\quad v(R_1)=v(R_2)=0;\quad n=0,1$ i.e.,

$$v_n(r,t)=\sum_{m=1}^{\infty}v_{nm}(t)\mathrm{B}_n(rr_{nm}). \tag{16}$$

Here, $\mathrm{J}_n(\cdot)$ and $\mathrm{Y}_n(\cdot)$ are Bessel functions of order n of the first and second kind, r_{nm} are the positive roots of the transcendental equations $\mathrm{B}_n(R_2r)=0$ and the constants A_n are chosen so that the normalization conditions

$$\int_{R_1}^{R_2} r\ \left[\mathrm{B}_n(rr_{nm})\right]^2\mathrm{d}\,r=1;\quad n=0,\ 1\ , \tag{17}$$

to be satisfied. Now, introducing (16) in (12), multiplying then by $r\mathrm{B}_n\left(rr_{np}\right)$ and integrating we respect to r from R_1 to R_2, we find that

$$\lambda\ddot{v}_{nm}(t)+\dot{v}_{nm}(t)+\nu r_{nm}^2 v_{nm}(t)=0,\quad t>0\ . \tag{18}$$

From (13) it also results

$$v_{nm}(0)=-V_{nm},\quad \dot{v}_{nm}(0)=0\ , \tag{19}$$

where V_{nm} are the finite Hankel transforms of $V_n(r)$, [12].

Solving (18) under initial conditions (19) and having in mind (16) and (11) we get for $\omega(r,t)$ and $u(r,t)$ the expressions:

$$\omega(r,t)=\Omega(r)-\sum_{m=1}^{P}\frac{s_{2m}^{1}\exp(s_{1m}^{1}t)-s_{1m}^{1}\exp(s_{2m}^{1}t)}{s_{2m}^{1}-s_{1m}^{1}}\;\Omega_{1m}\mathrm{B}_1(rr_{1m})-$$
$$-\exp\left(-\frac{t}{2\lambda}\right)\sum_{m=p+1}^{\infty}\left[\cos\left(\frac{\beta_{1m}}{2\lambda}t\right)+\frac{1}{\beta_{1m}}\sin\left(\frac{\beta_{1m}}{2\lambda}t\right)\right]\;\Omega_{1m}\mathrm{B}_1(rr_{1m}), \tag{20}$$

respectively,

$$u(r,t)=U(r)-\sum_{m=1}^{P}\frac{s_{2m}^{0}\exp(s_{1m}^{0}t)-s_{1m}^{0}\exp(s_{2m}^{0}t)}{s_{2m}^{0}-s_{1m}^{0}}\;U_{0m}\mathrm{B}_0(rr_{0m})-$$
$$-\exp\left(-\frac{t}{2\lambda}\right)\sum_{m=p+1}^{\infty}\left[\cos\left(\frac{\beta_{0m}}{2\lambda}t\right)+\frac{1}{\beta_{0m}}\sin\left(\frac{\beta_{0m}}{2\lambda}t\right)\right]\;U_{0m}\mathrm{B}_0(rr_{0m}), \tag{21}$$

where $s_{1m}^{n},s_{2m}^{n}=\dfrac{-1\pm\sqrt{1-4\nu\lambda r_{nm}^{2}}}{2\lambda}$, $\beta_{nm}=\sqrt{4\nu\lambda r_{nm}^{2}-1}$, $r_{np}\le\dfrac{1}{2\sqrt{\nu\lambda}}<r_{nq}$ with $n=0,1$ and $q=p+1$.

2.2. Calculation of the tangential tensions τ_1 and τ_2

The solutions of the ordinary differential equations $(7)_{1,2}$ with the initial conditions $(5)_3$ are

$$\tau_1(r,\ t)=\frac{\mu}{\lambda}\exp\left(-\frac{t}{\lambda}\right)\int_0^t\exp\left(\frac{\tau}{\lambda}\right)\left[\partial_r\omega(r,\ \tau)-\frac{\omega(r,\ \tau)}{r}\right]\;\mathrm{d}\tau,$$
$$\tau_2(r,\ t)=\frac{\mu}{\lambda}\exp\left(-\frac{t}{\lambda}\right)\int_0^t\exp\left(\frac{\tau}{\lambda}\right)\;\partial_r u(r,\ \tau)\;\mathrm{d}\tau. \tag{22}$$

Introducing (20) and (21) in (22) we get

$$\tau_1(r,\ t)=2\mu\left[1-\exp\left(-\frac{t}{\lambda}\right)\right]\;\frac{\Omega_2-\Omega_1}{R_2^2-R_1^2}\;\frac{R_1^2R_2^2}{r^2}+$$
$$+\frac{\mu}{r}\sum_{m=1}^{p}\Omega_{1m}\frac{\exp(s_{2m}^{1}t)-\exp(s_{1m}^{1}t)}{\sqrt{1-4\nu\lambda r_{1m}^{2}}}\;[rr_{1m}\mathrm{B}_1'(rr_{1m})-\mathrm{B}_1(rr_{1m})]- \tag{23}$$
$$-\frac{2\mu}{r}\exp\left(-\frac{t}{2\lambda}\right)\sum_{m=p+1}^{\infty}\frac{\Omega_{1m}}{\beta_{1m}}\sin\left(\frac{\beta_{1m}}{2\lambda}t\right)\;[rr_{1m}\mathrm{B}_1'(rr_{1m})-\mathrm{B}_1(rr_{1m})]$$

and

$$\tau_2(r,\ t)=\mu\left[1-\exp\left(-\frac{t}{\lambda}\right)\right]\;\frac{U_2-U_1}{r\;\ln(R_2/R_1)}+$$
$$+\mu\sum_{m=1}^{p}\frac{\exp(s_{2m}^{0}t)-\exp(s_{1m}^{0}t)}{\sqrt{1-4\nu\lambda r_{0m}^{2}}}\;r_{0m}U_{0m}\mathrm{B}_0'(rr_{0m})- \tag{24}$$
$$-2\mu\exp\left(-\frac{t}{2\lambda}\right)\sum_{m=p+1}^{\infty}\frac{r_{0m}\Omega_{0m}}{\beta_{0m}}\sin\left(\frac{\beta_{0m}}{2\lambda}t\right)\;\mathrm{B}_0'(rr_{0m}).$$

By making $t\to\infty$ in (20),(21), (23) and (24) we get

$$\omega(r) = r\Omega_2 - \frac{\Omega_2 - \Omega_1}{R_2^2 - R_1^2}\ \frac{R_1^2(R_2^2 - r^2)}{r}, \quad u(r) = U_2 - \frac{U_2 - U_1}{\ln(R_2 / R_1)} \ln(R_2 / r) \tag{25}$$

and

$$\tau_1(r) = 2\mu \frac{\Omega_2 - \Omega_1}{R_2^2 - R_1^2}\ \frac{R_1^2 R_2^2}{r^2}, \quad \tau_2(r) = \mu \frac{U_2 - U_1}{r\ \ln(R_2 / R)}, \tag{26}$$

which represent the steady state solutions.

3. HELICAL FLOW THROUGH A CIRCULAR CYLINDER

Taking the limit of Eqs. (15) and (17) when $R_1 \to 0$ and $R_2 \to R$ we find the eigenfunctions $-\sqrt{2}\ \ \mathrm{J}_0(rr_{0m})/[R\mathrm{J}_1(Rr_{0m})]$ and $-\sqrt{2}\ \ \mathrm{J}_1(rr_{1m})/[R\mathrm{J}_2(Rr_{1m})]$ corresponding to the helical flow through an infinite circular cylinder. The boundary conditions (6) must be changed by

$$|\ \omega(0,\ t)| < \infty, \quad \omega(R,\ t) = R\Omega; \quad |\ u(0,\ t)| < \infty, \quad u(R,\ t) = U; \quad t > 0 \tag{27}$$

and the components of the velocity $\omega(r,\ t)$ and $u(r,\ t)$ take the forms

$$\begin{aligned}\omega(r,\ t) = r\Omega - 2\Omega \sum_{m=1}^{p} \frac{s_{2m}^1 \exp(s_{1m}^1 t) - s_{1m}^1 \exp(s_{2m}^1 t)}{s_{2m}^1 - s_{1m}^1}\ \frac{\mathrm{J}_1(rr_{1m})}{r_{1m}\mathrm{J}_2(Rr_{1m})} - \\ -2\Omega \exp\left(-\frac{t}{2\lambda}\right) \sum_{m=p+1}^{\infty}\left[\cos\left(\frac{\beta_{1m}}{2\lambda}t\right) + \frac{1}{\beta_{1m}}\sin\left(\frac{\beta_{1m}}{2\lambda}t\right)\right] \frac{\mathrm{J}_1(rr_{1m})}{r_{1m}\mathrm{J}_2(Rr_{1m})}\end{aligned} \tag{28}$$

(identically with (3.4) of [5] where $J_0(\cdot)$ has to be changed by $J_2(\cdot)$) and

$$\begin{aligned}u(r,\ t) = U - \frac{2U}{R}\sum_{m=1}^{p} \frac{s_{2m}^0 \exp(s_{1m}^0 t) - s_{1m}^0 \exp(s_{2m}^0 t)}{s_{2m}^0 - s_{1m}^0}\ \frac{\mathrm{J}_0(rr_{0m})}{r_{0m}\mathrm{J}_1(Rr_{0m})} - \\ -\frac{2U}{R}\exp\left(-\frac{t}{2\lambda}\right) \sum_{m=p+1}^{\infty}\left[\cos\left(\frac{\beta_{0m}}{2\lambda}t\right) + \frac{1}{\beta_{0m}}\sin\left(\frac{\beta_{0m}}{2\lambda}t\right)\right] \frac{\mathrm{J}_0(rr_{0m})}{r_{0m}\mathrm{J}_1(Rr_{0m})}\ .\end{aligned} \tag{29}$$

The associated tangential tensions

$$\begin{aligned}\tau_1(r,\ t) = -2\mu\Omega \sum_{m=1}^{p} \frac{\exp(s_{2m}^1 t) - \exp(s_{1m}^1 t)}{\sqrt{1 - 4\nu\lambda r_{1m}^2}}\ \frac{\mathrm{J}_2(rr_{1m})}{\mathrm{J}_2(Rr_{1m})} + \\ +4\mu\Omega \exp\left(-\frac{t}{2\lambda}\right) \sum_{m=p+1}^{\infty} \frac{1}{\beta_{1m}} \sin\left(\frac{\beta_{1m}}{2\lambda}t\right) \frac{\mathrm{J}_2(rr_{1m})}{\mathrm{J}_2(Rr_{1m})}\end{aligned} \tag{30}$$

and

$$\begin{aligned}\tau_2(r,\ t) = -\frac{2\mu U}{R} \sum_{m=1}^{p} \frac{\exp(s_{2m}^0 t) - \exp(s_{1m}^0 t)}{\sqrt{1 - 4\nu\lambda r_{0m}^2}}\ \frac{\mathrm{J}_1(rr_{0m})}{\mathrm{J}_1(Rr_{0m})} + \\ +\frac{4\mu U}{R} \exp\left(-\frac{t}{2\lambda}\right) \sum_{m=p+1}^{\infty} \frac{1}{\beta_{0m}} \sin\left(\frac{\beta_{0m}}{2\lambda}t\right) \frac{\mathrm{J}_1(rr_{0m})}{\mathrm{J}_1(Rr_{0m})}\ ,\end{aligned} \tag{31}$$

are also obtained as limiting cases of (23) and (24).

4. LIMITING CASE $\lambda = 0$

Taking the limits of Eqs. (20), (21), (23), (24), (28), (29), (30) and (31) as $\lambda \to 0$, we obtain

$$\omega(r,t) = \Omega(r) - \sum_{m=1}^{\infty} \Omega_{1m} \quad \mathrm{B}_1(rr_{1m})\exp(-\nu r_{1m}^2 t), \tag{32}$$

$$u(r,t) = U(r) - \sum_{m=1}^{\infty} U_{0m} \quad \mathrm{B}_0(rr_{0m})\exp(-\nu r_{0m}^2 t), \tag{33}$$

$$\tau_1(r,t) = 2\mu \frac{\Omega_2 - \Omega_1}{R_2^2 - R_1^2} \quad \frac{R_1^2 R_2^2}{r} - \frac{\mu}{r}\sum_{m=1}^{\infty} \Omega_{1m} \quad \left[rr_{1m}\mathrm{B}_1'(rr_{1m}) - \mathrm{B}_1(rr_{1m}) \right]\exp(-\nu r_{1m}^2 t), \tag{34}$$

$$\tau_2(r,t) = \mu \frac{U_2 - U_1}{r \quad \ln(R_2 / R_1)} - \mu \sum_{m=1}^{\infty} r_{0m} U_{0m} \quad \mathrm{B}_0'(rr_{0m})\exp(-\nu r_{0m}^2 t), \tag{35}$$

$$\omega(r,\ t) = r\Omega - 2\Omega \sum_{m=1}^{\infty} \frac{\mathrm{J}_1(rr_{1m})}{r_{1m}\mathrm{J}_2(Rr_{1m})} \exp(-\nu r_{1m}^2 t), \tag{36}$$

$$u(r,\ t) = U - \frac{2U}{r} \sum_{m=1}^{\infty} \frac{\mathrm{J}_0(rr_{0m})}{r_{0m}\mathrm{J}_1(Rr_{0m})} \exp(-\nu r_{0m}^2 t), \tag{37}$$

$$\tau_1(r,\ t) = 2\mu\Omega \sum_{m=1}^{\infty} \frac{\mathrm{J}_2(rr_{1m})}{\mathrm{J}_2(Rr_{1m})} \exp(-\nu r_{1m}^2 t), \tag{38}$$

respectively,

$$\tau_2(r,\ t) = \frac{2\mu U}{R} \sum_{m=1}^{\infty} \frac{\mathrm{J}_1(rr_{0m})}{\mathrm{J}_1(Rr_{0m})} \exp(-\nu r_{0m}^2 t), \tag{39}$$

which are the similar solutions corresponding to a Navier-Stokes fluid.

Finally, by making $t \to \infty$ in anyone of the above expressions we get the steady state solutions. They are the same for both types of fluid.

5. CONCLUSIONS

In this paper we have established the exact solutions corresponding to a helical flow of a Maxwell fluid between two infinite coaxial circular cylinders. By letting $R_1 \to 0$ and $R_2 \to R$ in these solutions we attain to the similar solutions corresponding to a helical flow through an infinite circular cylinder. All these solutions, given by (20), (21), (23), (24), respectively, (28), (29), (30) and (31), contain sine and cosine terms. That indicates that by contrast with the Newtonian fluid, whose solutions (32) – (39) do not contain such terms, oscillations are set up in the fluid. The amplitudes of these oscillations decay exponentially in time, the damping being proportional to $\exp(-t/2\lambda)$ or $\exp(-t/\lambda)$.

Direct computations show that $\omega(r,\ t)$, $u(r,\ t)$, $\tau_1(r,\ t)$ and $\tau_2(r,\ t)$ satisfy both the associate partial differential equations and all imposed initial and boundary conditions, the differentiation term by term in r and t being clearly permissible. In the special case, when the relaxation time $\lambda \to 0$, our solutions reduce to those corresponding to a Newtonian fluid. The steady state solutions are also obtained as a limiting case for $t \to \infty$. They are the same for both types of fluid.

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