PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 4, Number 1/2003, pp. 00-00

TRANSVERSAL FREE VIBRATION OF REISSNER PLATES

Mărgărit BAUBEC, Aron IAROVICI

Institute of Solid Mechanics, Romanian Academy
Corresponding author: Mărgărit BAUBEC (mars2000@k.ro)

A dynamic version of the linear Reissner theory, for the bending of plates, is deduced. This theory is adequate to study the transverse vibrations of plates with moderate thickness. The effect of shear forces is considered, so that three boundary conditions along the edge of the plate can be fulfilled. The influence of the inertia is also estimated and discussed. Finally, we applied the theory in order to find a few natural frequencies of a built in edge circular plate. By comparing with the classical theory of Kirchhoff, one establishes the relevance of the above-mentioned quantities.

Key words: Free vibration; Reissner theory; Three boundary conditions.

1. INTRODUCTION

Let us consider a homogeneous and isotropic plate, of an arbitrary shape. We take the cartesian frame *Oxyz* so that *Ox* and *Oy* are in the middle plane of the plate and *Oz* is normal at the plate. The coordinate z is varying between [-h/2, h/2], h being the thickness. The density ρ is supposed to be constant and the body forces vanish.

Standard notations for linear elasticity and theory of plates are used (Timoshenko and Woinowski [4] or Teodorescu [5]). The Lamé moduli are λ, μ and the technical constants, widely used in engineering sciences, are denoted with E and ν. The points, over some quantities, represent derivatives with respect to time. With the symbol Δ, we noted the two-dimensional harmonic operator, in cartesian or polar coordinates. The functions appearing in the paper are considered smooth enough so that all the calculus would have sense.

The Latin indices range over the values 1, 2, 3, while the Greek indices are restricted to the values 1, 2. As usual, when deriving a theory of bending, we suppose for now that the plate is compressed by the transversal load $q = q(x, y, t)$ on the face $z = -h/2$, and it is free of stresses on the face $z = + h/2$.

The summation convention is used throughout.

The relations between shear stresses and shear forces, commonly used in a theory of Reissner type, are:

$$\sigma_{\alpha 3} = \frac{3Q_\alpha}{2h}\left[1-\left(\frac{2z}{h}\right)^2\right]. \tag{1.1}$$

where $Q_1 = Q_1(x, y, t)$, $Q_2 = Q_2(x, y, t)$ are shear forces and $\sigma_{\alpha 3}$ are shear stresses.

One must satisfy two equations for the moments around the *OX* and *OY* axes and one equation for the forces in the direction of *OZ* axis:

$$M_{\alpha\beta,\alpha} - Q_\beta = \int_{-h/2}^{+h/2} \rho z \ddot{u}_\beta dz, \qquad Q_{\alpha,\alpha} + q = \int_{-h/2}^{+h/2} \rho \ddot{u}_3 dz. \tag{1.2}$$

These equations are sufficiently accurate approximations of the three-dimensional equations of motions.

Recommended by Radu VOINEA
Member of the Romanian Academy

In the equations (1.2), $M_{\alpha\beta}$ are bending and shear moments:

$$M_{\alpha\beta} = \int_{-h/2}^{+h/2} z\sigma_{\alpha\beta} dz$$

The displacements along the axes Ox, Oy, Oz are noted with u_1, u_2, u_3, respectively. The three-dimensional equation of motion:

$$\sigma_{j3,j} = \rho \ddot{u}_3 . \tag{1.3}$$

is exactly fulfilled in the theory. This relation will be considered as a differential equation in z for σ_{33}.

Excepting the equation:

$$\varepsilon_{33} = \frac{1}{E}[\sigma_{33} - \nu(\sigma_{11} + \sigma_{22})] .$$

one satisfies all the three-dimensional constitutive relations.

The stress conditions, on the faces of the plate, are also fulfilled:

$$\sigma_{31|z=\pm h/2} = \sigma_{32|z=\pm h/2} = \sigma_{33|z=+h/2} = 0 , \quad \sigma_{33|z=-h/2} = -q . \tag{1.4}$$

As we shall see later, the theory admits three boundary conditions on the edge of the plate.

The number of initial conditions depends on the variant of the theory, with or without the inertia.

Integrating the equation (1.3) with respect to z and using the conditions $(1.4)_{3,4}$, one obtains:

$$\sigma_{33} = \frac{3q}{2h} \int_{h/2}^{z} \left[1 - \left(\frac{2z}{h} \right)^2 \right] dz + \int_{h/2}^{z} \rho \ddot{u}_3 dz - \frac{3}{2h} \int_{h/2}^{z} \left[1 - \left(\frac{2z}{h} \right)^2 \right] dz \int_{-h/2}^{+h/2} \rho \ddot{u}_3 dz . \tag{1.5}$$

In the static variant of the theory, one defines an average transversal displacement u_{3m}:

$$u_{3m} = \frac{3}{2h} \int_{-h/2}^{+h/2} \left[1 - \left(\frac{2z}{h} \right)^2 \right] u_3 dz .$$

which can be used instead of the real displacement, for a complete determination of the theory.

In the dynamic version, presented here, we need to make a supposition about the variation of the displacement u_3 along the thickness. The simplest choice, acceptable in the plate theories, is:

$$u_3 = u_3(x, y, t) . \tag{1.6}$$

In the static case, equation (1.6) leads to the equality between u_3 and the average displacement. Using (1.5) and (1.6), we find:

$$\int_{-h/2}^{+h/2} z\sigma_{33} dz = \frac{qh^2}{10} - \frac{\rho h^3 \ddot{u}_3}{60} . \tag{1.7}$$

Let us define now the rotations:

$$\varphi_\alpha = \frac{12}{h^3} \int_{-h/2}^{+h/2} z u_\alpha dz . \tag{1.8}$$

In the Reissner type theory, these two quantities replace $u_{3,1}$ and $u_{3,2}$, used in the Kirchhoff theory.

Integrating the constitutive equation for σ_{13} and σ_{23}, with respect to z, neglecting the extension of the middle plane and using the relations (1.1), the displacements u_1 and u_2 results:

$$u_\alpha = -z\, u_{3,\alpha} + \frac{Q_1 z}{2\mu h}\left[3 - \left(\frac{2z}{h}\right)^2\right]. \tag{1.9}$$

For the rotations, one obtains:

$$\varphi_\alpha = -u_{3,\alpha} + \frac{6}{5\mu h} Q_\alpha . \tag{1.10}$$

From the constitutive equations for σ_{11}, σ_{22} and σ_{12} and using (1.10), one calculates the bending and shear moments:

$$\begin{aligned}
M_{11} &= -D(u_{3,11} + \nu u_{3,22}) + \frac{h^2}{5} Q_{1,1} - \frac{\nu\, h^2}{10(1\text{-}\nu)} q + \frac{11\nu\rho h^3 \ddot{u}_3}{60(1\text{-}\nu)}, \\
M_{22} &= -D(u_{3,22} + \nu u_{3,11}) + \frac{h^2}{5} Q_{2,2} - \frac{\nu\, h^2}{10(1\text{-}\nu)} q + \frac{11\nu\rho h^3 \ddot{u}_3}{60(1\text{-}\nu)}, \\
M_{21} &= -D(1-\nu) u_{3,12} + \frac{h^2}{10}(Q_{1,2} + Q_{2,1}) .
\end{aligned} \tag{1.11}$$

where:

$$D = \frac{Eh^3}{12(1-\nu^2)} .$$

is the transversal rigidity of the plate.

Replacing (1.11) in the equations (1.2), after rearranging the terms, we get:

$$\begin{aligned}
&\frac{h^2}{10}\Delta Q_\alpha - Q_\alpha - D\Delta u_{3,\alpha} - \frac{h^2}{10(1-\nu)} q_{,\alpha} = \frac{\rho\, h^2}{10\mu}\ddot{Q}_\alpha - \left[\frac{1}{12} + \frac{6+5\nu}{60(1-\nu)}\right]\rho h^3 \ddot{u}_{3,\alpha}, \\
&Q_{\beta,\beta} + q = \rho h \ddot{u}_3 .
\end{aligned} \tag{1.12}$$

Neglecting the terms depending on the time, one obtains the known static equations from Reissner [1], [2]. The effect of the inertia (the terms $\rho\ \ddot{u}_1$ and $\rho\ \ddot{u}_2$ in the equations of moments) appears in the equations (1.12) through the terms:

$$\frac{\rho\, h^2}{10\mu}\ddot{Q}_\alpha \quad \text{and} \quad -\frac{\rho\, h^3}{12}\ddot{u}_{3,\alpha} .$$

If this effect is neglected, these terms must be omitted.

In order to uncouple the unknown functions, from the equations of motion, we search the solution in the form:

$$Q_1 = P_{,1} + K_{,2}, \quad Q_2 = P_{,2} - K_{,1} . \tag{1.13}$$

$P = P(x, y, t)$, $K = K(x, y, t)$ are two potential functions, which have to be determined.

Replacing the relations (1.13) in the equations (1.12) and separating the derivatives with respect to x and y, one obtains some representations for the solution.

When we neglect the inertia terms, it results:

$$\begin{aligned}
Q_1 &= -D\Delta u_{3,1} - \frac{h^2}{10}\frac{2-\nu}{1-\nu} q_{,1} - C_1 \rho h^3 \ddot{u}_{3,1} + K_{,2}, \\
Q_2 &= -D\Delta u_{3,2} - \frac{h^2}{10}\frac{2-\nu}{1-\nu} q_{,2} - C_1 \rho h^3 \ddot{u}_{3,2} - K_{,1} .
\end{aligned} \tag{1.14}$$

where u_3 and K satisfy:

$$\frac{h^2}{10}\Delta K - K = 0\,,$$

$$D\Delta\Delta u_3 + C_1\rho h^3 \Delta\ddot{u}_3 + \rho h\ddot{u}_3 = q - \frac{h^2}{10}\frac{2-\nu}{1-\nu}\Delta q\,. \tag{1.15}$$

Under the assumption (1.13), the solutions of the system (1.14) - (1.15) satisfy the equations of motion. One sees that this representation admits three boundary conditions and two initial conditions.

When we take into consideration the inertia, we shall obtain a more complicated representation:

$$\frac{\rho h^2}{10\mu}\ddot{P} + P = -D\Delta u_3 - \frac{h^2}{10}\frac{2-\nu}{1-\nu}q + C_2\rho h^3\ddot{u}_3\,,$$

$$\Delta P - \rho h\ddot{u}_3 + q = 0\,, \tag{1.16}$$

$$\frac{h^2}{10}\Delta K - K - \frac{\rho h^2}{10\mu}\ddot{K} = 0\,.$$

The solutions of this system also fulfill the equations of motion. From the equations 1.16)$_{1,2}$, one finds that the displacement u_3 satisfies:

$$D\Delta\Delta u_3 + \rho h\ddot{u}_3 + \frac{\rho^2 h^3}{10\mu}\ddddot{u}_3 - C_2\rho h^3\Delta\ddot{u}_3 = q + \frac{\rho h^2}{10\mu}\ddot{q} - \frac{h^2}{10}\frac{2-\nu}{1-\nu}\Delta q\,. \tag{1.17}$$

The constants C_1 and C_2, used in the formulas (1.14) - (1.17), have the values:

$$C_1 = -\frac{12-\nu}{60(1-\nu)}\,, \quad C_2 = \frac{1}{12} + \frac{12-\nu}{60(1-\nu)}\,. \tag{1.18}$$

Whenever the equation (1.17) and (1.16)$_1$ involve (1.16)$_2$, it is more convenient to use (1.17) instead of the coupled equation (1.16)$_2$. The displacement u_3 results from the equation (1.17), then from (1.16)$_1$ - (1.16)$_3$ one obtains the potentials P and K, respectively.

We shall see that there are some cases when the equations (1.16)$_1$ and (1.17) do not involve the equation (1.16)$_2$.

To solve the systems (1.16) or (1.16)$_{1,3}$ - (1.17), we need three boundary conditions and six initial conditions. The increased number of initial conditions, in comparison with the case without inertia, is deriving from considering the terms $\rho\,\ddot{u}_1$ and $\rho\,\ddot{u}_2$ in the equations for the moments.

For studying the free vibration without damping, with or without inertia, we have to separate the time and the spatial coordinates, by taking:

$$q = 0\,,\quad u_3 = U_3(x,y)e^{i\omega t}\,,\quad P = P_0(x,y)e^{i\omega t}\,,\quad K = K_0(x,y)e^{i\omega t}\,,\quad i = \sqrt{-1}\,,\ \omega\in R\,. \tag{1.19}$$

Similar expressions have to be considered for all the functions appearing in the theory. In calculus one takes the real or the imaginary part, depending on the solution form, which we choose. Replacing expressions like (1.19), in the previous equations, both the systems (1.15) and (1.16)$_{1,3}$ - (1.17) reduce at the following system:

$$D\Delta\Delta U_3 + \rho h^3\omega^2 C_3\Delta U_3 - \rho h\omega^2 C_4 U_3 = 0\,,$$

$$C_4 P_0 = -D\Delta U_3 - \rho h^3\omega^2 C_3 U_3\,, \tag{1.20}$$

$$\frac{h^2}{10}\Delta K_0 - C_4 K_0 = 0\,.$$

It takes place also:

$$Q_1 = (P_{0,1} + K_{0,2})e^{i\omega t}\,, \qquad Q_2 = (P_{0,2} - K_{0,1})e^{i\omega t}\,.$$

When (1.20) proceeds from (1.15), the coefficients C_3 and C_4 have the values:

$$C_3 = -C_1 , \qquad C_4 = 1 . \tag{1.21}$$

If (1.20) is derived from the equations $(1.16)_{1,3}$ - (1.17), one takes:

$$C_3 = C_2 , \quad C_4 = 1 - \frac{1}{10}\frac{\rho h^2 \omega^2}{\mu} , \quad C_4 \neq 0 . \tag{1.22}$$

When ω has a value, so that the coefficient C_4 becomes null, the equation $(1.20)_1$ is deriving from $(1.20)_2$ and we need another equation to complete the system. Therefore, we must consider the equation $(1.16)_2$ (instead of $(1.20)_1$) and using (1.19), we get:

$$\Delta P_0 + \rho h \omega^2 U_3 = 0 . \tag{1.23}$$

The equations (1.20) or $(1.20)_{2,3}$ - (1.23) (when $C_4 = 0$), admit three boundary conditions.

With or without inertia, for a given plate having specific boundary conditions, we can try now to find its natural frequencies, namely those values ω, for which the equations system - together with the boundary conditions - admits more than null solutions.

2. NATURAL FREQUENCIES FOR A CIRCULAR PLATE

Let us consider a built in edge, circular plate, of radius R and thickness h. Polar coordinates (r, θ) are adequate in order to study circular plates. The equations established before – (1.20) or $(1.20)_{2,3}$ - (1.23) – remain valid, with the operator Δ written in polar coordinates. The boundary conditions are:

$$U_3(R,\theta) = 0 , \quad \varphi_n(R,\theta,t) = 0 , \quad \varphi_t(R,\theta,t) = 0 ,$$
$$\forall \theta \in [0, 2\pi] , \ \forall t \geq 0 . \tag{2.1}$$

The rotations φ_n and φ_t are defined around the tangent $-\vec{e}$, respective the outward normal $\vec{n}$, at the exterior circle of radius R. They are linked on to the cartesian rotations, through the formulas:

$$\varphi_1 \vec{i} + \varphi_2 \vec{j} = \varphi_n \vec{n} + \varphi_t \vec{e} ,$$
$$\vec{n} = \cos\theta \vec{i} + \sin\theta \vec{j} , \qquad \vec{e} = -\sin\theta \vec{i} + \cos\theta \vec{j} . \tag{2.2}$$

Let us introduce the notations:

$$v_s = \sqrt{\mu/\rho} , \quad a_s = h\omega / v_s .$$

where v_s is the speed of transversal waves in the plate and a_s is a non-dimensional quantity.

For $C_4 \neq 0$, the following formulas should be used to calculate the rotations:

$$\varphi_n(R,\theta,t) = \Phi_n(R,\theta) e^{i\omega t} ,$$
$$\varphi_t(R,\theta,t) = \Phi_t(R,\theta) e^{i\omega t} ,$$
$$\Phi_n(r,\theta) = -\left(1 + \frac{6C_3}{5C_4} a_s^2\right) U_{3,r} - \frac{h^2}{5(1-\nu)C_4} \Delta U_{3,r} + \frac{6}{5\mu h}\frac{1}{r} K_{0,\theta} ,$$
$$\Phi_t(r,\theta) = -\left(1 + \frac{6C_3}{5C_4} a_s^2\right) \frac{1}{r} U_{3,\theta} - \frac{h^2}{5(1-\nu)C_4}\frac{1}{r} \Delta U_{3,\theta} - \frac{6}{5\mu h} K_{0,r} .$$

Separating the variables, for equations like (1.20), sufficiently general solutions exist, in polar coordinates, so that all the boundary conditions can be fulfilled. Using these solutions, together with the boundary conditions (2.1), we found transcendent equations for a_s, in the both presented variants of Reissner type theory.

The values a_s correspond to the following shape of vibration:

$$U_3 = F_m(r)\cos m\theta\,,\quad K = G_m(r)\sin m\theta\,,\qquad m \geq 0\,. \tag{2.3}$$

where the functions $F_m(r)$ and $G_m(r)$, determined in the frame of a specific theory, depend on the Bessel functions of first type and order "m", J_m and I_m.

In the theory developed here, when $C_4 > 0$, we have:

$$F_m(r) = a_m I_m(l_1 x) + A_m J_m(l_2 x)\,,\qquad G_m(r) = \alpha_m I_m(dx)\,.$$

with a_m, A_m and α_m undetermined coefficients.

We introduced the notations:

$$x = \frac{r}{R},\quad d = \frac{R}{h}\sqrt{10|C_4|}\,,$$

$$f_1 = \sqrt{9(1-\nu)^2 C_3^2 a_s^4 + 6(1-\nu)C_4 a_s^2} - 3(1-\nu)C_3 a_s^2\,,$$

$$f_2 = \sqrt{9(1-\nu)^2 C_3^2 a_s^4 + 6(1-\nu)C_4 a_s^2} + 3(1-\nu)C_3 a_s^2\,,$$

$$l_1 = \frac{R}{h}\sqrt{|f_1|}\,,\quad l_2 = \frac{R}{h}\sqrt{f_2}\,.$$

In the Kirchhoff theory, the transcendent equation, corresponding to $(2.3)_1$, is:

$$I_m(f_w)J_{m+1}(f_w) + J_m(f_w)I_{m+1}(f_w) = 0\,. \tag{2.4}$$

with:

$$f_w = \frac{R}{h}\sqrt{a_s\sqrt{6(1-\nu)}}\,.$$

In the developed theory, for $C_4 > 0$, instead of the equation (2.4), we found a more complicated equation:

$$\left(m\frac{h}{R} + \sqrt{10C_4}R_m(d)\right)\times\left[\sqrt{f_2}\left(\frac{f_2}{5(1-\nu)} - C_4 - \frac{6C_3}{5}a_s^2\right)J_{m+1}(l_2) - \right.$$
$$\left. -\sqrt{f_1}\left(\frac{f_1}{5(1-\nu)} + C_4 + \frac{6C_3}{5}a_s^2\right)J_m(l_2)R_m(l_1)\right] - \frac{mh}{R}\sqrt{10C_4}\,\frac{f_1+f_2}{5(1-\nu)}J_m(l_2)R_m(d) = 0\,. \tag{2.5}$$

where:

$$R_m(y) = \frac{I_{m+1}(y)}{I_m(y)}\,,\quad y > 0$$

In the inertia case, we have also the possibility $C_4 \leq 0$.

For $C_4 < 0$, $m \geq 1$, the equation for the eigenvalues becomes:

$$\left(m\frac{h}{R}J_m(d) - \sqrt{10|C_4|}J_{m+1}(d)\right)\times\left[\sqrt{f_2}\left(\frac{f_2}{5(1-\nu)} - C_4 - \frac{6C_3}{5}a_s^2\right)J_{m+1}(l_2)J_m(l_1) + \right.$$
$$\left. +\sqrt{|f_1|}\left(\frac{f_1}{5(1-\nu)} + C_4 + \frac{6C_3}{5}a_s^2\right)J_m(l_2)J_{m+1}(l_1)\right] + \frac{mh}{R}\sqrt{10|C_4|}\,\frac{f_1+f_2}{5(1-\nu)}J_m(l_1)J_m(l_2)J_{m+1}(d) = 0\,. \tag{2.6}$$

For $C_4 < 0$, $m = 0$, instead the equation (2.6), we have:

$$\sqrt{f_2}\left(\frac{f_2}{5(1-\nu)} - C_4 - \frac{6C_3}{5}a_s^2\right)J_1(l_2)J_0(l_1) + \sqrt{|f_1|}\left(\frac{f_1}{5(1-\nu)} + C_4 + \frac{6C_3}{5}a_s^2\right)J_0(l_2)J_1(l_1) = 0$$

The case $C_4 = 0$ requires a special analysis, using the equations $(1.20)_{2,3}$ - (1.23).

Further on, in the frame of Kirchhoff's and Reissner type theories (with and without inertia), a_s is calculated for different thickness and orders of frequency.

We have to notice that for the frequencies presented in the tables, the inequality $C_4 > 0$ occurs. For finding the solutions, we have to use the system (1.20) and the Boggio's theorem, applied on the equation $(1.20)_1$.

One introduces the notations:

$a_R = a_s$ – for the theory without inertia,
$a_{RI} = a_s$ – for the theory with inertia,
$a_K = a_s$ – for Kirchhoff's theory,

$\varepsilon_1 = 100 \cdot (a_K - a_R)/a_R$,
$\varepsilon_2 = 100 \cdot (a_{RI} - a_R)/a_R$.

For different m and h/R, we put in the following tables, few values for a_R and ε_1, ε_2.

The natural frequencies were calculated using our FORTRAN programs.

Table 1 ($m = 0$),

a_R (h/R = 0,125)	ε_1 (%)	ε_2 (%)	a_R (h/R = 0,0125)	ε_1 (%)	ε_2 (%)
0.075040	3.7945	-0.3860	0.000778	0.0386	-0.0045
0.271260	11.7832	-1.4168	0.003028	0.1243	-0.0216
0.553592	22.7166	-2.3966	0.006776	0.2527	-0.0513
0.888116	35.7965	-3.0495	0.012009	0.4237	-0.0931
1.252007	50.4169	-3.3731	0.018713	0.6371	-0.1466
1.631283	66.1844	-3.4431	0.026869	0.8927	-0.2111

Table 2 ($m = 4$),

a_R (h/R = 0,125)	ε_1 (%)	ε_2 (%)	a_R (h/R = 0,0125)	ε_1 (%)	ε_2 (%)
0.446287	19.0147	-2.1631	0.005300	0.2138	-0.0402
0.805171	32.6690	-2.9845	0.010641	0.3869	-0.0827
1.184761	47.7005	-3.3729	0.017394	0.6011	-0.1365
1.575219	63.7944	-3.4667	0.025582	0.8570	-0.2013
1.969928	80.7141	-3.3738	0.035193	1.1546	-0.2764
2.365125	98.2881	-3.1759	0.046207	1.4937	-0.3611

Table 3 ($m = 7$).

a_R (h/R = 0,125)	ε_1 (%)	ε_2 (%)	a_R (h/R = 0,0125)	ε_1 (%)	ε_2 (%)
0.804413	32.7450	-3.0504	0.010635	0.4000	-0.0834
1.236515	49.7561	-3.4403	0.018400	0.6430	-0.1451
1.656701	67.1508	-3.4794	0.027438	0.9230	-0.2164
2.070415	85.0371	-3.3374	0.037840	1.2432	-0.2974
2.478536	103.364	-3.1071	0.049609	1.6040	-0.3873
2.881502	122.063	-2.8473	0.062730	2.0053	-0.4853

The value $m = 0$ corresponds to axial-symmetric vibration. In this case, we have a null potential K, the boundary condition for the rotation φ_t being identically satisfied.

3. CONCLUSIONS

Comparing between the Kirchhoff theory and the theory without inertia, we can see the influence of the shear forces and the thickness, on the obtained eigenvalues. When we compare the two variants of the Reissner type theory, we simply measure the effect of inertia, namely the presence of the dynamic terms $\rho \ddot{u}_\alpha$ in the first two equations of motion.

For moderate thickness (h/R = 0,125), the influence of the shear forces is increasing fast with the order of the frequency and with the number 'm'. In the presented cases, the inertia influence remains small in comparison with the influence of the shear forces, for both thin and moderate thickness plates.

We can see that these few first frequencies, calculated in the Reissner type theory (with or without inertia) are always less than the corresponding values calculated in the Kirchhoff theory. Therefore, for practical calculus of plates having moderate thickness, especially when superior frequencies are necessary, it is preferable to use the more precise theory developed here, instead of Kirchhoff theory, in order to avoid an unexpected early resonance, when the frequency of the external excitation increases in time.

REFERENCES

1. REISSNER, E., *The effect of transverse shear deformation on the bending of elastic plates*, J. Appl. Mech., **12**, pp. 69-77, 1945.
2. REISSNER, E., *On the theory of bending of elastic plates,* J. Math. & Phys., **23**, pp. 184 – 191, 1944.
3. VOYIADJIS, G. Z., BALUCH, M. H., *Refined theory for flexural motions of isotropic elastic plates*, J. of Sound and Vibrations, **76**, 1, pp. 57-64, 1981.
4. TIMOSHENKO, S., WOINOWSKY, K.S., *The theory of plates and shells,* (in Romanian), Bucharest, Edit. Tehn., 1968.
5. TEODORESCU, P. P., *The Dynamics of Linear Elastic Bodies*, (in Romanian), Bucharest, Edit. Acad. R.S.R., 1972.
6. MINDLIN, R.D., *Influence of rotatory inertia and shear on flexural motion of isotropic elastic plates*, J. Appl. Mech., **18**, 31, pp. 31-38, 1951.

Received April, 26 , 2002