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HABILITATION THESIS - ABSTRACT

TITLE: Mathematical methods in water wave problems

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The present habilitation thesis covers the scientific progress we achieved in the years 2007-2020, by exploring different topics in partial differential equations in fluid mechanics, nonlinear water waves, fluid stability and geophysical fluid dynamics. The mathematical methods used to handle various water wave problems are varied and fascinating.

Chapter 2 is devoted to the variational approaches used in the modelling of water waves and flows. The classical mathematical water wave problem involves the Euler equations in a free boundary domain, the fluid incompressibility equation and the appropriate boundary conditions. The high complexity of the full Euler equations, even without taking into account the Earth's rotation and the influence of stratification, led mathematicians and physicists to derive simpler sets of equations convenient to describe the fluid motion in some specific physical regimes. In order to develop a systematic approximation procedure, one needs to characterize the full Euler equations in terms of the sizes of various parameters. The two important parameters that play a crucial role in the theory of water waves are ϵ, which measures the ratio of wave amplitude to undisturbed fluid depth, and δ, which measures the ratio of fluid depth to wavelength. The amplitude parameter ϵ is associated with the nonlinearity of the wave, so that small ϵ implies a nearly-linear wave theory. The shallowness parameter δ measures the deviation of the pressure, in the water below the wave, away from the hydrostatic pressure distribution. The role of δ independent of ϵ is useful in the description of arbitrary amplitude shallow-water waves. Small-amplitude, long-wavelength (or shallow water) waves are approximated by weakly nonlinear long waves models such as the Korteweg-de Vries (KdV) and Boussinesq equations. A series of nonlinear evolution equations (e.g. Green-Naghdi (GN), Camassa-Holm (CH), Degasperis-Procesi (DP), two-component Camassa-Holm (CH2), etc.), which constitute more accurate approximations of Euler's equations than the classical KdV equation, have been studied intensively during the last decades. The derivation of simpler sets of equations that model flow phenomena is done in such a way that the resulting so-called approximate equations are easier to handle than the full equations but still keep some of their important structural features such as a variational or Hamiltonian structure. In the 60-70's, in a series of pioneering papers, Arnold initiates the use of geometric variational methods in describing the equations of an ideal incompressible fluid. The Hamiltonian nature of the Euler equations brings a number of fundamental results in the mathematical theory of the dynamics of an ideal incompressible fluid, especially in the area of hydrodynamic stability. For wave propagation in shallow water, we obtain in the papers [**131, 132, 139, 140**] the CH equation (without or with vorticity), the GN system and the CH2 system, by an interplay of small-parameter expansions and a variational approach in the Lagrangian formalism. By using the same method, we derive in [**144**] a new two-component (N2C) system for propagation of surface shallow-water waves on irrotational flows. This system has a noncanonical Hamiltonian formulation and we can also find an exact solitary-wave solution, which has a different expression from the sech-type solution obtained for the GN system. The Hamiltonian approach to free surface water waves dynamics has been put forward for the first time by Zakharov in 1968. His work for irrotational waves in deep water was extended in 2007-2008 to rotational flows of constant vorticity of finite depth [53, 234, 235]. The two-dimensional two-layer irrotational gravity water flows with a free surface have been shown [76] to possess a Hamiltonian formulation too; for the rotational counterpart (with constant vorticity in each layer) the Hamiltonian approach was developed in [54] for periodic water flows. We show in [**158**] that accounting for Coriolis effects in the equatorial f-plane approximation, for stratified two dimensional periodic water flows with piecewise constant vorticity, does not hinder the Hamiltonian description of the governing equations. We use a variational approach combined with methods from harmonic analysis (Dirichlet-Neumann operators) to pursue our goal.

One fascinating feature of the study of water waves is that their motion can exhibit elementary patterns, such as, fronts, pulses or periodic wave trains. Mathematical understanding of these elementary patterns is essential to gain fundamental insights into the more complex patterns. In Chapter 3, we analyse the travelling wave solutions for some of the important models that appear in the literature - the GN model, the CH2 model, the N2C model, the Zakharov-Itō (ZI) model and the Kaup-Boussinesq (KB) model - for the description of waves in shallow water, propagating on irrotational flows as well as on shear flows. Although the irrotational models (with considerable advantages in their mathematical analysis) of wave motion yield many practical results that tell us about the nature of water waves, in reality there is always some vorticity present in actual wave motion: for wind-driven waves, waves riding upon a sheared current, or waves near a ship or pier. We pay special attention to the case when the vorticity is present but has a constant value. For waves which are long compared with the water depth, the choice of constant vorticity is not just a mathematical simplification but it is also physically reasonable, since, in this case, the non-zero mean vorticity is more important than its specific distribution - see the discussion in [79]. By applying a unified procedure, we derive in **[84, 85]**, the most general ordinary differential equation describing the whole family of travelling wave solutions for each of the above two-component models. The existence and the profile of the travelling waves depend on the values of the constants of integration, and on the existence, the sign and order of multiplicity of the roots of some polynomials of degree 3, 4, 5, 6, depending on the model. Most of the studies devoted to travelling waves are focused on a particular sub-class of solutions: the solitary waves (pulses), which decay quickly at infinity along with all their derivatives. We obtain the equations describing the solitary wave solutions by choosing the constants of integration appropriately. These localized travelling waves, whose shapes do not change as they propagate along with a constant velocity, are less ubiquitous than the periodic wave trains but nevertheless represent observable and beautiful wave patterns. Some of the general equations can be solved analytically to obtain the explicit solutions, but a description of the travelling wave profiles for all the above models can be made by performing a phase-plane analysis **[84, 85]**. A closed curve in the phase-plane yields a periodic travelling wave solution, a homoclinic orbit gives a pulse type solution and a heteroclinic orbit in the phase-plane provides a front type solution. For certain values of the constants, all the above models possess pulses. For the KB system, we find interesting analytical multi-pulse travelling wave solutions. For the ZI system, pulse and anti-pulse solutions are obtained. The two-component Camassa-Holm model, with or without vorticity, possesses front wave solutions too; these front wave solutions decay algebraically in the far field. If we compare the effects of the vorticity on the pulse waves in the CH2 model and $CH2_\omega$ model, we find that the right-going waves propagating in the same direction as the underlying shear flow have a higher amplitude and narrower wavelength and the right-going waves for which the underlying shear flow propagates in the opposite direction are wider, their amplitude decreases.

In Chapter 4 we investigate the motion of water particles under different types of waves which advance across the water. The classical description of the particle paths is obtained within the framework of linear water wave theory. The prevalent view was that the particles within periodic wave-trains travelling across the sea move in closed orbits, elliptic or circular depending on the depth water (see, for example, [80, 165, 179, 192, 219, 220]). Even in the linear water wave theory, the ordinary differential equation system describing the motion of the particles is nonlinear. While in the first approximation of this system all particle paths appear to be closed, Constantin and Villari [71] showed, by using phase-plane considerations, that for small amplitude periodic gravity waves no particles trajectory is actually closed, unless the free surface is flat. A lot of work followed the paper [71], in the framework of linear theory or in the framework of full nonlinear theory, with similar conclusions, see [37, 44, 48, 67, 68, 70, 86, 113, 236]. For small-amplitude water waves, beside the phase-plane analysis, the exact solutions of the nonlinear system describing the particle motion allow a better understanding of the dynamics. We provide in [**133, 134, 135, 136, 137, 138, 141**] analytic solutions of the systems describing the particle paths within different types of small-amplitude progressive water waves, under the effects of gravity and surface tension, and in the presence or not of the background currents and vorticity. These solutions are not closed curves: some particle trajectories are undulating path to the right or to the left, others are loops with forward drift, or with backward drift, some trajectories are peakon-like, others can follow some peculiar shapes. For small-amplitude waves with constant vorticity and small-amplitude irrotational deep-water waves, we also make [**138, 141**] some remarks on the stagnations points, that is, points where the vertical component of the fluid velocity field is zero while the horizontal component equals the speed of the wave profile. The stagnation points are of special interest because they are points where the flow characteristic often change. They could be located on the free surface, in this case the wave is called extreme wave [5, 212, 221, 233], on the bottom or inside the fluid domain [70, 86, 193, 236], [**138, 141**]. The formation of the stagnation points inside the flow may also be connected with a wave-breaking phenomenon in deep water.

Geophysical fluid dynamics is the study of fluid motion where the Earth's rotation plays a significant role - the Coriolis terms are incorporated into the governing equations - and applies to a wide range of oceanic and atmospheric flows [77, 107, 230]. In oceanography, the governing equations appropriate for motion on a sphere are typically simplified by invoking tangent plane-approximations - whereby the Earth's curved surface is locally approximated by a tangent plane. There is a large literature on equatorial wave dynamics. In a first approximation, called f-plane approximation, valid within a band about 2° latitude either side of the Equator, the Coriolis parameter is set to a constant value and the latitudinal variations are not considered. The β-plane approximation, which applies in regions within 5° latitude either side of the Equator, takes further into account that the Coriolis force may vary

from point to point and introduces a linear variation with latitude of the Coriolis parameter. In the first section of Chapter 5, we consider the two-dimensional equatorial water-wave problem with constant vorticity in the f-plane approximation. Within the framework of small-amplitude waves, we derive the dispersion relations and we find the analytic solutions of the nonlinear differential equation system describing the particle paths below such waves [**157**]; the solutions obtained are not closed curves.
Exact solutions of the general governing equations are rather rare and, generally, they describe ideal conditions that do not correspond to the complexities of the observed physical behaviour. However, they are precise and clear in terms of their validity, detail and structure, and they can provide the basis for more direct relevant analyses. The approach pioneered by Gerstner to find explicit exact solution for gravity fluid flows within the Lagrangian framework, was extended to geophysical flows too (see, for example, Constantin [41, 42], the survey [118] and the references therein). In Chapter 5, we also provide exact implicit Gerstner's type solutions to the geophysical edge-wave problem in the f- and β- plane approximations. These solutions, in the Lagrangian framework, describe geophysical edge waves propagating westwards or eastwards over a sloping beach with the shoreline parallel to the Equator and their amplitudes decay exponentially away from the shoreline [**147, 148**].
At an arbitrary latitude, Gerstner-like three-dimensional solutions were first obtained, in the f-plane approximation, by Pollard [211]. In the β-plane approximation, we obtain in [**30**] explicit three-dimensional nonlinear solutions for geophysical waves propagating, both eastward and westward, at an arbitrary latitude, in the presence of a constant underlying background current. In the literature, centripetal forces are typically neglected as they are relatively much smaller than Coriolis forces. The retention of these terms in the appropriate governing equations increases their mathematical complexity but plays a central role in facilitating the admission of a wide-range of depth-invariant underlying currents in their solutions [117]. In [**31**], we consider the β-plane governing equations at an arbitrary latitude, modified to incorporate centripetal forces. We obtain an exact Gerstner-type solution of this problem, which prescribes three-dimensional geophysical wave propagating, both eastward and westward, in a relatively narrow ocean strip at an arbitrary latitude, in the presence of a constant underlying current. The dispersion relation of the obtained waves features contributions from the Coriolis force, the centripetal force and the underlying current. We make a detailed dis-

cussion of the situations encountered in the Northern Hemisphere and the Southern Hemisphere, for admissible following as well as adverse currents of physically plausible magnitude.
In the neighbourhood of the Equator, the motion of the ocean constitutes a very complicated flow system because so many factors are involved (underlying non-uniform currents, stratification, upwelling/downwelling processes, thermoclines, and so on). The Gerstner-type solutions fail to capture strong depth variations of the flows. We are interested in a mathematical approach that enables us to capture strong depth variations of the flow only, the density stratification and the thermocline (the interface separating two adjacent layers of different constant density) play no role in this section - the depth of the thermocline (about 200 m for the Equatorial Undercurrent (EUC)) is short as compared with the average total depth of the ocean (about 4 km for the Pacific). Some exact, steady solutions, representing purely azimuthal flows that do not vary in the azimuthal direction were presented and explored by Constantin and Johnson in [56]; see also the review [170]. In the last section of Chapter 5, based on the articles **[155, 156]**, we investigate a nonlinear three-dimensional model for equatorial flows that are moving slowly in the azimuthal direction, finding exact solutions that capture the most relevant geophysical features: depth-dependent currents, poleward or equatorial surface drift and a vertical mixture of upward and downward motions. The key to the results is a structural feature: two components of the three-dimensional velocity field can be expressed as nonlinear functional of the azimuthal velocity component, and therefore this velocity component defines the flow. We analyse in detail some polynomial (up to the third degree) and exponential azimuthal profiles.

Chapter 6 is devoted to the elegant short-wavelength instability method, a rigorous mathematical approach to the problem of stability for general three-dimensional inviscid incompressible flows, developed independently by Bayly [11], Friedlander & Vishik [98] and Lifschitz & Hameiri [186]. An essential ingredient of the short-wavelength method, related to geometrical optics techniques, is to consider the small-wavelength perturbations in the WKB (Wentzel-Kramers-Brillouin)-form, their evolution in time being governed up to the remainder terms, terms which are incapable of cancelling the growth of the leading-order terms, by a system of partial differential equations formed by the eikonal equation for the wave phase and the transport equation for the wave amplitude of the velocity. Since the eikonal equation and the transport equation involve only the advective derivative along the velocity field of the flow, these partial differential equations can be written as ordinary differential equations along the trajectories of the flow. Thus, sufficient instability conditions and necessary stability conditions for the flow are obtained via the analysis of an ODE system along the trajectories of the flow: the existence of unbounded solutions to this system implies instability, when all solutions of this system are bounded it implies stability of the flow with respect to the class of short-wavelength perturbations; the results have a local nature being localised along the trajectories. The method was successfully applied when the basic flow is described in the Lagrangian framework, starting with Leblanc [181] for the Gerstner solution. In **[143]** we investigate, by the short-wavelength method, the instability of the edge-wave solution along a sloping beach, solution obtained in the Lagrangian framework [35]. We prove that the edge waves with the steepness parameter, defined as the amplitude multiplied by the wavenumber, higher then $\frac{7}{18}\sin\alpha$, α being the sloping angle of the beach, are unstable. The edge-wave solution - originally considered to be a mathematical curiosity [179], but now recognized to play a significant role in near-shore hydrodynamics - is still possible if the water has non-constant density [81, 224, 241]. This remarkable fact is due to the special character of these waves, namely that in a frame of reference moving with the waves, the streamlines are also the lines of constant pressure, and thus, the stratification of the fluid, which makes the density different from streamline to streamline but constant on the same streamline, does not disturb the main structure of the dynamical equations. The short-wavelength method is successful for barotropic incompressible fluids too **[146]**, and not only for non-rotating flows but also for equatorial geophysical flows [51, 103, 119], for equatorial geophysical barotropic flows **[149]**, for geophysical flows at an arbitrary latitude **[30, 31, 150]**, and for general roating flows in the presence or not of nonconservative body forces **[153, 159]**. For other examples in the geophysical context see the survey **[154]** and the references therein. The geophysical Gerstner-type solutions have been shown to be unstable when the wave profiles are steep enough. The critical steepness is very close to $\frac{1}{3}$. In polar regions instability is triggered at a lower steepness threshold than that for equatorial waves. At arbitrary latitude, the waves which travel from east to west are more prone to instability than those which travel from west to east. A constant underlying adverse current favours instability in the sense that the threshold on the steepness for the wave to be unstable is decreased compared to the case without current. Conversely, this threshold is increased by a following current. The presence of underlying depth-dependent currents gives us some local stability results. In the case of the exact steady purely azimuthal flows which model the Equatorial Undercurrent (EUC) and the Antarctic Circumpolar Current (ACC), the short-wavelength stability analysis shows that the wave vector vanishes and for some realistic velocity profiles, the short-wavelength perturbations evolve stably along the streamlines of these flows.

Chapter 7 contains plans and directions for future research.

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