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CONSERVATION LAWS FOR OPTICAL SOLITONS OF LAKSHMANAN-PORSEZIAN-DANIEL MODEL

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Abstract. This paper derives conservation laws for optical solitons of Lakshmanan-Porsezian-Daniel model. The method of multipliers is adopted to locate a few of the conserved densities. The conserved quantities are finally computed for the model using soliton solutions with both Kerr law and power law nonlinearities.

Key words: solitons, conservation laws, Kerr law, power law.

1. INTRODUCTION

The conservation laws of any physical system play a very important role in describing its dynamics. These laws give an insight into the physical meaning of the system. The same is true for optical solitons that are described by some generic models. In this case, the conservation laws for the well known nonlinear Schrödinger's equation have been exhaustively studied. This paper is going to address the conservation laws of a different model that also describes soliton propagation through optical waveguides [1–23]. It is Lakshmanan-Porsezian-Daniel (LPD) equation. This model has been studied earlier and its bright, dark, and singular soliton solutions have been reported [6]. One of the most powerful technique to extract these laws is the double reduction technique using Lie symmetry analysis. This paper will employ the multiplier approach to retrieve conserved densities of the LPD model, which is considered with both Kerr and power laws of optical nonlinearity. The soliton solutions will be subsequently utilized to compute the conserved quantities from the derived conserved densities. The details are enumerated in the next couple of sections.

2. GOVERNING EQUATION

The dimensionless form of the LPD model with higher order dispersion and spatio-temporal dispersion (STD) is [6]:

$$i\,q_t + a\,q_{xx} + b\,q_{xt} + c\,F\left(|q|^2\right)q = \sigma\,q_{xxxx} + \alpha\left(q_x\right)^2 q^* + \beta\left|q_x\right|^2 q + \gamma|q|^2 q_{xx} + \lambda\,q^2 q^*_{xx} + \delta|q|^4 q. \tag{1}$$

Here $q(x,t)$ is the dependent variable that stands for complex-valued wave function. The independent variables x and t represent spatial and temporal co-ordinates, respectively. Also, the group velocity dispersion is given by the coefficient of a and the coefficient of STD is b. The functional F implies the type of nonlinearity under study that must be k-times continuously differentiable and so it has to maintain the following technical condition:

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m);R^2\right).$$

On the right hand side of Eq. (1), σ stands for fourth order dispersion (4OD) while δ is the two-photon absorption parameter. To proceed with Eq. (1), the starting hypothesis is taken to be:

$$q(x,t) = P(x,t)e^{i\phi} \tag{2}$$

where $P(x,t)$ represents the shape of the wave profile, and the phase component $\phi(x,t)$ has the following representation:

$$\phi(x,t) = -\kappa x + \omega t + \theta. \tag{3}$$

Here, κ represents the soliton frequency, and ω is the wave number, while θ is the phase constant. After substituting the hypothesis (2) into Eq. (1) and splitting into real and imaginary parts one arrives at:

$$\begin{gathered}\sigma P_{xxxx} - (a+6\sigma\kappa^2)P_{xx} - bP_{xt} - (b\kappa\omega - \omega - a\kappa^2 - \sigma\kappa^4)P - (\alpha+\gamma+\lambda-\beta)\kappa^2P^3 + \\ \delta P^5 - cF(P^2)P + (\alpha+\beta)PP_x^2 + (\lambda+\gamma)P^2P_{xx} = 0,\end{gathered} \tag{4}$$

and

!Unexpected End of Formula $$(1-b\kappa)P_t - (2a\kappa + 4\sigma\kappa^3 - b\omega)P_x + 2(\alpha+\gamma-\lambda)\kappa P^2P_x + 4\sigma\kappa P_{xxx} = 0, \tag{5}$$

respectively. After assigning the coefficients of linearly independent functions to zero in (5) the following constraints emerge:

$$\sigma = 0 \tag{6}$$

$$\lambda = \alpha + \gamma \tag{7}$$

and the soliton speed is given by

$$v = \frac{b\omega - 2a\kappa}{1-b\kappa} \tag{8}$$

provided

$$b\kappa \neq 1. \tag{9}$$

Therefore, both constraints (6) and (7) modify (4) to

$$aP_{xx} + bP_{xt} + (b\kappa\omega - \omega - a\kappa^2)P + (2\lambda-\beta)\kappa^2P^3 - \delta P^5 + cF(P^2)P - (\alpha+\beta)PP_x^2 - (\lambda+\gamma)P^2P_{xx} = 0. \tag{10}$$

The equation (10) will be used in the following sections to obtain conservation laws for both Kerr law and power law nonlinearities.

3. KERR LAW NONLINEARITY

For Kerr nonlinear medium, the functional is given by

$$F(u) = u, \tag{11}$$

which simplifies the LPD model (1) to

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q = \sigma q_{xxxx} + \alpha\left(q_x\right)^2 q^* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q_{xx}^* + \delta|q|^4 q. \tag{12}$$

In this case, the real part (10) gives

$$aP_{xx} + bP_{xt} + (b\kappa\omega - \omega - a\kappa^2)P + [c + (2\lambda-\beta)\kappa^2]P^3 - \delta P^5 - (\alpha+\beta)PP_x^2 - (\lambda+\gamma)P^2P_{xx} = 0. \tag{13}$$

Its bright one-soliton solution is [6]:

$$q(x,t) = A\text{sec h}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \tag{14}$$

where the width of the soliton is given by

$$B = \sqrt{-\frac{(1-b\kappa)\left(b\kappa\omega-\omega-a\kappa^2\right)}{a(1+b\kappa)-b^2\omega}}, \tag{15}$$

the pulse amplitude is

$$A = \sqrt{-\frac{(1-b\kappa)\left(b\kappa\omega-\omega-a\kappa^2\right)(3\lambda+\beta+\gamma)}{\delta\left\{a(1+b\kappa)-b^2\omega\right\}}}, \tag{16}$$

and the corresponding wave number is:

$$\omega = \frac{4ab^2\delta(1+b\kappa)-(1-b\kappa)^3 Z_1 - b^2(1-b\kappa)Z_2}{4b^4\delta} + \frac{\sqrt{(1-b\kappa)^2[Z_1^2(1-b\kappa)^4 + b^4 Z_2^2 - 2b^2 Z_1(4a\delta-(1-b\kappa)^2 Z_2)]}}{4b^4\delta}. \tag{17}$$

Here we have introduced the notations:

$$Z_1 = (3\lambda+\beta+\gamma)(2\lambda+\beta) \tag{18}$$

$$Z_2 = (3\lambda+\beta+\gamma)\left\{c+(2\lambda-\beta)\kappa^2\right\}. \tag{19}$$

This is possible with the restrictions

$$(1-b\kappa)\left(b\kappa\omega-\omega-a\kappa^2\right)\left\{a(1+b\kappa)-b^2\omega\right\} < 0, \tag{20}$$

$$Z_1^2(1-b\kappa)^4 + b^4 Z_2^2 - 2b^2 Z_1\left\{4a\delta-(1-b\kappa)^2 Z_2\right\} > 0 \tag{21}$$

and

$$b\delta \neq 0. \tag{22}$$

3.1. Conservation Laws

Equation (13) restated is given by

$$\begin{aligned} F(u) &= u - aP_{xx} - bP_{xt} - Pb\kappa\omega + P\omega + Pa\kappa^2 - \\ &- P^3c - 2P^3\kappa^2\lambda + P^3\kappa^2\beta + \delta P^5 + PP_x^2\alpha + PP_x^2\beta + P^2P_{xx}\lambda + P^2P_{xx}\gamma = 0. \end{aligned} \tag{23}$$

For a conserved flow (T^x, T^t) for which $D_x T^x + D_t T^t = 0$ on the solutions of (23), we employ the multiplier approach in which

$$\begin{aligned} F(u) = uE\,[Q(x,t,P,P_x,P_t)(-aP_{xx} - bP_{xt} - Pb\kappa\omega + P\omega + Pa\kappa^2 - P^3c - 2P^3\kappa^2\lambda + \\ + P^3\kappa^2\beta + \delta P^5 + PP_x^2\alpha + PP_x^2\beta + P^2P_{xx}\lambda + P^2P_{xx}\gamma)] \end{aligned} \tag{24}$$

vanishes, where E is the Euler operator. Each multiplier Q leads to a conserved flow. We list below the special cases of the parameters that lead to a multiplier Q and the corresponding conserved density T^t and flux T^x:

(i). $\beta = \lambda + \gamma - \alpha,\ \ Q = P_x$:

$$T_1^t = 1/4\, b\left(P_x^{\,2} - PP_{xx}\right),$$

$$\begin{aligned} T_1^x = [&-1/6\,\delta P^6 - 1/2P_x^{\,2}P^2\gamma + 1/4P^4\kappa^2\lambda - 1/4P^4\kappa^2\gamma + 1/4P^4\kappa^2\alpha - 1/2P_x^{\,2}P^2\lambda + 1/4P^4c + \\ &+ 1/4P_tP_xb + 1/2P_x^{\,2}a - 1/2P^2a\kappa^2 + 1/4PbP_{xt} + 1/2P^2bk\kappa\omega - 1/2P^2\omega]; \end{aligned}$$

(ii). $\beta=\lambda+\gamma-\alpha,\ Q=P_t$:

$$T_2^t=1/6\delta P^6+1/4P^4\kappa^2\gamma-1/4P^4\kappa^2\alpha-1/4P^4\kappa^2\lambda+1/4P^3P_{xx}\lambda+1/4P^3P_{xx}\gamma+1/4P_x{}^2P^2\gamma-1/4P^4c+$$
$$+1/4P_x{}^2P^2\lambda-1/4P_tP_xb-1/4PbP_{xt}+1/2P^2a\kappa^2+1/2P^2\omega-1/2PaP_{xx}-1/2P^2b\kappa\omega$$

$$T_2^x=-[-1/4P_tP_xP^2\lambda-1/4P_tP_xP^2\gamma+1/4P_{xt}P^3\lambda+1/4P_{xt}P^3\gamma+1/4P_t^2b+$$
$$+1/2P_tP_xa-1/4PbP_{tt}-1/2PP_{xt}a]$$

(iii). $\beta=-\alpha$, $\lambda=-\gamma$, $Q=P_t+\dfrac{a}{b}P_x$:

$$T_3^t=1/6\delta P^6-1/4P^4c-1/4P^4\kappa^2\alpha+1/2P^4\kappa^2\gamma-1/4P_tP_xb-1/4P_x{}^2a-1/4PaP_{xx}-$$
$$-1/4PbP_{xt}+1/2P^2\omega-1/2P^2b\kappa\omega+1/2P^2a\kappa^2$$

$$T_3^x=\frac{1}{12b}[2a\delta P^6-3aP^4c+6aP^4\kappa^2\gamma-3aP^4\kappa^2\alpha-3P_t^2b^2-9P_2baP_x-6a^2P_x{}^2+$$
$$+3PabP_{xt}+3Pb^2P_{2,2}-6P^2ab\kappa\omega+6P^2a\omega+6P^2a^2\kappa^2]$$

(iv). $\beta=\alpha-2b$, $\lambda=-\gamma-b$, $Q=P_t+\dfrac{a}{b}P_x+u^2P_x$:

$$T_4^t=1/6\delta P^6-1/8P_{xx}bP^3+1/2P^4\kappa^2\gamma-1/4P^4\kappa^2\alpha-3/8P^2P_x{}^2b-1/4P^4c-$$
$$-1/4P_tP_xb-1/4P_x{}^2a-1/2P^2b\kappa\omega-1/4PaP_{xx}-1/4PbP_{xt}+1/2P^2\omega+1/2P^2a\kappa^2$$

$$T_4^x=\frac{1}{24b}[3bP^8\delta-4bP^6\kappa^2\alpha-12P_x{}^2P^4b^2+4a\delta P^6+8bP^6\kappa^2\gamma-4bP^6c-24aP_x{}^2bP^2+$$
$$+3b^2P^3P_{xt}-15P_tb^2P_xP^2-6aP^4\kappa^2\alpha+6bP^4a\kappa^2+12aP^4\kappa^2\gamma-6aP^4c-6b^2P^4\kappa\omega+$$
$$+6bP^4\omega-18P_tbaP_x-12P^2ab\kappa\omega-6P_t^2b^2-12a^2P_x{}^2+12P^2a\omega+12P^2a^2\kappa^2+$$
$$+6PabP_{xt}+6Pb^2P_{tt}].$$

The conserved quantities are now given by:

$$I_1=\int_{-\infty}^{\infty}T_1^t\,\mathrm{d}x=\frac{b}{4}\int_{-\infty}^{\infty}\left\{\left(P_x\right)^2-P\,P_{xx}\right\}\mathrm{d}x=\frac{b}{2}\int_{-\infty}^{\infty}\left(P_x\right)^2\mathrm{d}x=\frac{b\,A^2\,B}{3},\tag{25}$$

$$I_2=\int_{-\infty}^{\infty}T\,{}^t_2\,\mathrm{d}x=\frac{A^2}{45B}\left\{8\delta A^4+15A^2\left(2\kappa^2\gamma-\kappa^2\alpha-c\right)+45\left(a\kappa^2+\omega-\omega b\kappa\right)\right\},\tag{26}$$

$$I_3=\int_{-\infty}^{\infty}T\,{}^t_3\,\mathrm{d}x=I_2=\frac{A^2}{45B}\left\{8\delta A^4+15A^2\left(2\kappa^2\gamma-\kappa^2\alpha-c\right)+45\left(a\kappa^2+\omega-\omega b\kappa\right)\right\},\tag{27}$$

and

$$I_4=\int_{-\infty}^{\infty}T\,{}^t_4\,\mathrm{d}x=I_3=I_2=\frac{A^2}{45B}\left\{8\delta A^4+15A^2\left(2\kappa^2\gamma-\kappa^2\alpha-c\right)+45\left(a\kappa^2+\omega-\omega b\kappa\right)\right\}.\tag{28}$$

4. POWER LAW NONLINEARITY

For power law nonlinearity the functional F generalizes to [6]

$$F(u)=u^n,\tag{29}$$

where n represents the power law nonlinearity parameter and gives the strength of nonlinearity. For pulse stability, the restriction is $0<n<2$, and in particular $n\neq 2$ for self-focusing singularity to disappear. Now, Eq. (1) is rewritten as

$$i\,q_t+a\,q_{xx}+b\,q_{xt}+c\left|q\right|^{2n}q=\sigma\,q_{xxxx}+\alpha\left(q_x\right)^2q^*+\beta\left|q_x\right|^2q+\gamma\left|q\right|^2q_{xx}+\lambda\,q^2q_{xx}^*+\delta\left|q\right|^4q.\tag{30}$$

The real part equation is

$$a\,P_{xx}+b\,P_{xt}+(b\,\kappa\,\omega-\omega-a\,\kappa^2)P+(2\lambda-\beta)\kappa^2P^3-\delta P^5+cP^{2n+1}-(\alpha+\beta)PP_x^2-(\lambda+\gamma)P^2P_{xx}=0,\tag{31}$$

and its bright soliton solution is [6]

$$q(x,t) = A\,\mathrm{sech}^{\frac{1}{n}}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \tag{32}$$

with the condition

$$\delta = 0. \tag{33}$$

The width of the soliton is

$$B = n\kappa\sqrt{\frac{2\lambda-\beta}{2\lambda+\beta}} \tag{34}$$

for

$$4\lambda^2 > \beta^2. \tag{35}$$

The soliton speed is

$$v = \frac{-2a\kappa[\beta+2\alpha(1-b\kappa)]}{\beta(1-2b\kappa)+2\lambda\left[(1-b\kappa)^2+b^2\kappa^2\right]}, \tag{36}$$

while the wave number is

$$\omega = \frac{-a\kappa^2\left[(1+b\kappa)(\beta-2\lambda)+(1-b\kappa)(\beta+2\lambda)\right]}{\beta(1-2b\kappa)+2\lambda\left[(1-b\kappa)^2+b^2\kappa^2\right]}, \tag{37}$$

provided

$$\beta(1-2b\kappa)+2\lambda\left[(1-b\kappa)^2+b^2\kappa^2\right] \neq 0. \tag{38}$$

The amplitude of the pulse is given as

$$A = \left[-\frac{a\kappa^2(n+1)(\beta-2\lambda)\left\{\beta+4b\kappa\lambda(1-b\kappa)+2\lambda[(1-b\kappa)^2+b^2\kappa^2]\right\}}{c(\beta+2\lambda)\left\{\beta(1-2b\kappa)+2\lambda[(1-b\kappa)^2+b^2\kappa^2]\right\}}\right]^{\frac{1}{2n}}. \tag{39}$$

4.1. Conservation Laws

The conserved density of (31) for $\alpha = -\beta+\lambda+\gamma$ is given by

$$T^t = \frac{b}{4}\left(-P_x^{\,2} + P P_{xx}\right).$$

The conserved flow T^x is a cumbersome function.

Thus, the conserved quantity is given by

$$I = \int_{-\infty}^{\infty} T^t \mathrm{d}x = \frac{b}{4}\int_{-\infty}^{\infty}\left\{\left(P_x\right)^2 - P P_{xx}\right\}\mathrm{d}x = \frac{b}{2}\int_{-\infty}^{\infty}\left(P_x\right)^2\mathrm{d}x = \frac{bA^2B}{2n(n+2)}\,\frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}, \tag{40}$$

where $\Gamma(x)$ is the Euler's gamma function.

5. CONCLUSION

The multiplier approach was employed to derive the conservation laws of the LPD model in optical waveguides. Both Kerr and power law nonlinearities were considered. While for the case of Kerr law nonlinearity essentially two conserved quantities were obtained, the power law case provides one. All of these laws appear with a certain constraints imposed on the parameters of the model. The soliton solutions are utilized to generate the conserved quantity from the corresponding density. The results of this paper are very encouraging to further studies with this generic model. Later, the LPD model will be extended to birefringent fibers, DWDM systems, optical metamaterials, optical couplers, and other relevant physical settings. The conservation laws in such situations will be available and reported.

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