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ANGULAR MOMENTA AND MOMENTUM OF INERTIA WITHIN THE WOODS-SAXON TWO-CENTER SHELL MODEL

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Abstrac. In order to estimate the influence of the so-called recoil term that appears in the framework of the Bohr-Mottelson rotor model, the angular momenta of the constituent particles are required for different deformations of a nuclear system. The variation of the intrinsic spins is calculated for the emission of an alpha particle in the framework of the Woods-Saxon two-center shell model. A simple nuclear shape parametrization of two intersected spheres is used.

Key words: Woods-Saxon two-center shell model, angular momenta, alpha decay.

1. INTRODUCTION

In the axial symmetric particle-rotor model of Bohr-Mottelson, the total Hamiltonian is composed from two parts: a Hamiltonian for the eigenvalues of the intrinsic motion and a rotational part. This rotational part includes a constant of motion energy related to the orbital velocity of the nucleus as a whole, a recoil term, and a Coriolis coupling. The recoil term is usually neglected in calculations of nuclear structure. Due to the fact that the angular momentum of a particle j acts only on the intrinsic wave functions, it is considered that this term can be absorbed in the Hamiltonian that gives the eigenstates of the nuclear system. Therefore, the influence of this term can be ignored by a proper choice of the potential parameters. As pointed out in Ref. [1], the recoil term is inversely proportional to the total momentum of inertia, hence exhibiting a strong dependence on the deformation. Unfortunately, as mentioned in Ref. [2], they are no way to absorb an operator that is strongly deformation dependent in the framework of the Woods-Saxon or of the Nilsson single particle models, without modifying its parameters accordingly. Therefore, the parameters of the mean field should be deformation dependents.

In the following, the behaviours of the j^2 (intrinsic angular momentum) operator and of the total momentum of inertia are investigated in order to estimate its influence in nuclear structure calculations. For this purpose, we will use a deformation space that starts from a spherical nucleus to reach a configuration of two tangent nuclei at an extreme mass asymmetry compatible to alpha decay. Only two degrees of freedom are required in this context: the elongation of the nuclear system and its mass asymmetry. The Woods-Saxon two-center shell model [3] is used to provide the wave functions of the intrinsic motion. Usually, the alpha decay is considered preformed on the nuclear surface [4, 5].

In our treatment, we will consider the alpha decay as a superasymmetric fission process, involving all stages of the disintegration from the ground state of the parent up to the scission point, as in Refs. [6–10]. Due to its ability to treat precisely touching configurations, the two-center shell model was widely used in calculations of disintegration processes in a wide range of mass asymmetries [11–15].

2. INTRINSIC ANGULAR MOMENTA

The diagonal matrix elements of the single particle spin

$$j^2 = j_x^2 + j_y^2 + j_z^2, \tag{1}$$

are calculated between the wave functions given by the Woods-Saxon two-center shell model. Here, j_x and j_y are components of the spin perpendicular on the axis of symmetry, while j_z is the component along the axial symmetry axis. In the case of the j_z, the eigenvalues are simply

$$j_z|\varphi_n\rangle = \Omega|\varphi_n\rangle, \tag{2}$$

where Ω is a quantum number that denotes the spin projection. To determine the matrix elements of j_x and j_y the usual definition involving ladder operators is used

$$j_x = \frac{j_+ + j_-}{2}; \quad j_y = \frac{j_+ - j_-}{2i}. \tag{3}$$

The following expression emerges

$$\begin{aligned}\langle\varphi_m|j_x^2 + j_y^2|\varphi_n\rangle &= \frac{1}{4}\sum_k \langle\varphi_m|j_+ + j_-|\varphi_k\rangle\langle\varphi_k|j_+ + j_-|\varphi_n\rangle - \\ &-\frac{1}{4}\sum_k \langle\varphi_m|j_+ - j_-|\varphi_k\rangle\langle\varphi_k|j_+ - j_-|\varphi_n\rangle = \\ &= \frac{1}{2}\sum_k \left(\langle\varphi_m|j_+|\varphi_k\rangle\langle\varphi_k|j_-|\varphi_n\rangle + \langle\varphi_m|j_-|\varphi_k\rangle\langle\varphi_k|j_+|\varphi_n\rangle\right),\end{aligned} \tag{4}$$

where the analytic formulae for the matrix elements of j_+ and j_- within the Woods-Saxon two-center formalism are given in Ref. [16].

For spherical systems, the eigenvalues of j^2 should be

$$j^2|\varphi_n\rangle = j(j+1)|\varphi_n\rangle. \tag{5}$$

3. RESULTS

The eigenvectors and the eigenstates of the intrinsic Hamiltonian are obtained within the Woods Saxon two-center shell model. In this model, an average field of Woods-Saxon type is determined by the nuclear shapes. This average field is completed with spin-orbit and Coulomb interactions. All these contributions are diagonalized in a double-center oscillator basis.

A very simple nuclear shape parametrization given by two intersected spheres is considered to describe the α-decay of ^{210}Po, involving only two degrees of freedom associated to the elongation and to the mass-asymmetry. The initial parent nucleus is considered spherical, with a radius R_0. The generalized coordinate for the elongation is considered as the distance between the centers of the fragments R. The mass asymmetry is given by the radius of the emitted fragment R_2. So, a sphere that represents the emitted fragment is extracted from the initial nucleus by keeping unchanged the volume of nuclear system. The radius R_2 is modified gradually as function of the elongation R, starting from R_0 for R=0 and reaching a value compatible to the α-particle for a configuration given by two tangent spheres.

The single particle energies as function of the elongation R are displayed in Figs. 1 and 2 for neutron and proton levels schemes.

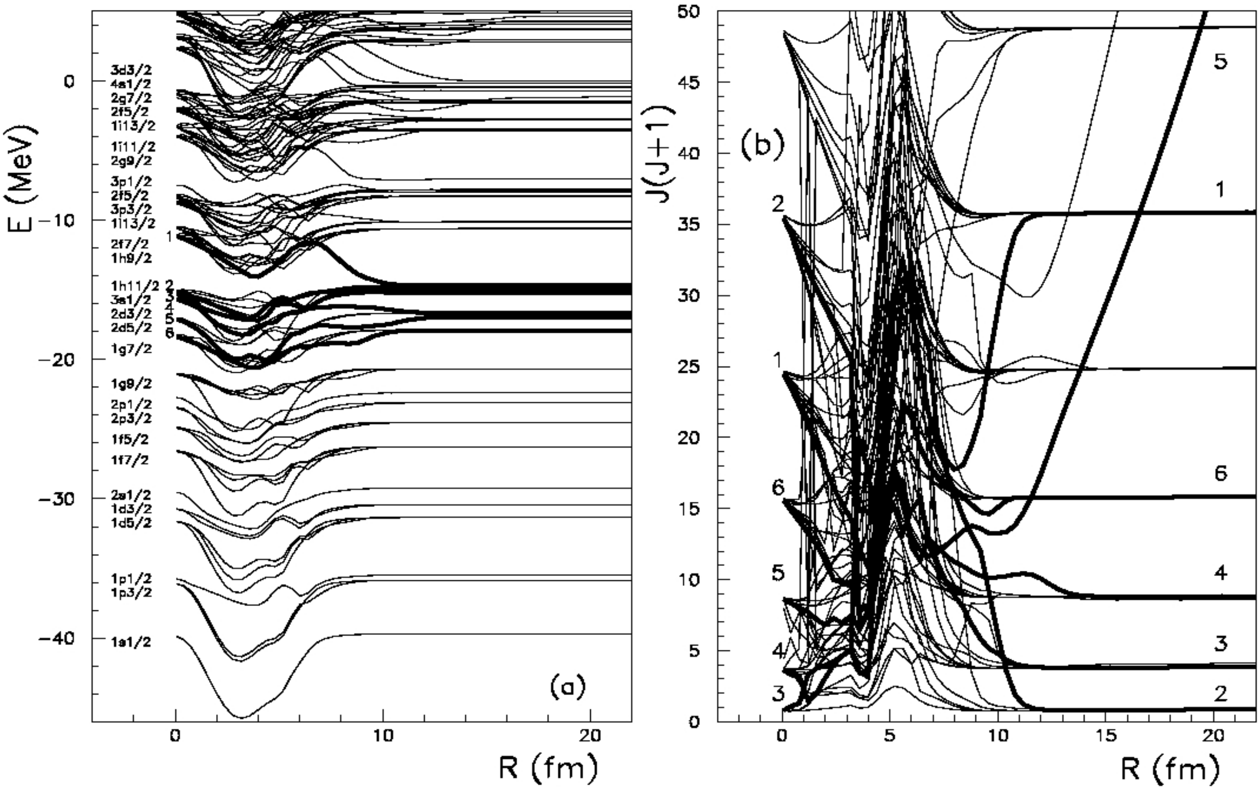

Fig. 1 – a) Neutron single particle energies as function of the elongation R; b) square of total angular momentum of a particle in $\hbar^2$ units as function of R. We used the notation $j(j+1)$ that resorts for spherical shapes.

The recoil term is defined as

$$R = \frac{\hbar^2 \left(j^2 - j_z^2 \right)}{2J}. \tag{6}$$

The contribution given by j_z^2 is a constant of motion in the case of the axial symmetric rotor. It is always smaller than the contribution given by j^2. Therefore, we will investigate the squared angular momentum of a particle j^2. In Fig. 1 the single particle energies and the values of the total angular momentum j^2 are displayed as function of the elongation R in the neutron case. The states of the spherical ^{212}Po are labelled on the left with the spectroscopic notations. At an elongation R of about 10 fm, the alpha particle is emitted and we retrieve a superposition of the levels schemes of the daughter and of the alpha particle. The level of the alpha particle emerges from the orbital $2d_{5/2}$ and reaches the state $1s_{1/2}$ in its own potential well. The daughter final state $\Omega = 1/2$ of the orbital $2d_{5/2}$ should be filled by superior levels belonging to the $\Omega = 1/2$ space. Therefore the superior $\Omega = 1/2$ levels should replace the level of the alpha particle in the daughter fragment. On the figure, it can be observed that a single particle level emerging from $1h_{9/2}$ (numbered with the digit 1) replaces the final state $\Omega = 1/2$ in the orbital $1h_{11/2}$. Additional replacements are produced up to a complete filling of all orbitals of the daughter nucleus level scheme is obtained. Several levels in the $\Omega = 1/2$ space are plotted with a thick line in Fig 1 (a) and are numbered in a descending energy order. The level numbered 1 emerges from $1h_{9/2}$ and arrives finally on the orbital $1h_{11/2}$. The level numbered 2 emerges from 1 $h_{11/2}$ and arrives on $3s_{1/2}$. The level 3 emerges from $3s_{1/2}$ and reaches the $2d_{3/2}$ orbital. The level 4 emerges from $2d_{3/2}$ and arrives on $2d_{5/2}$. The level 5 emerges from $2d_{5/2}$ and arrives on the orbital $1s_{1/2}$ of the alpha particle. The level 6 starts from $1g_{7/2}$ and arrives on $1g_{7/2}$. These behaviours are reflected by the rearrangements of the square angular momenta displayed in Fig. 1b. The level 1 starts with a value $9/2(9/2 + 1)$ and arrives asymptotically to a value close to $11/2(11/2 + 1)$. The single exception is the level 5 that tends asymptotically to a very large value. This value is however compatible to the alpha particle state.

The angular momentum is computed in the center of mass of the nuclear system. Therefore, after separation of the two particles, the angular momenta should increase due to the increasing distance between the center of the alpha particle and the center of mass of the dinuclear system. The angular momentum increases moderately for the daughter that is much heavier than the alpha particle. It should also be mentioned that during the rearrangement of the single particle levels, a lot of avoided level crossing can be produced. Two levels with the same good quantum number associated to the same symmetry of the system cannot intersect and exhibit avoided levels crossings regions. In the avoided level crossing regions, the properties of the single particle states can be exchanged. A very pronounced avoided level crossing is produced, for example, between the levels 2 and 3 at $R = 4$ fm, as exhibited in Fig. 1a. Exactly in the same region, the values of the angular momenta of the levels 2 and 3 are interchanged as it can be seen on the plot in Fig. 1b. The level 3 enters in the avoiding level crossing region with a value of the square of the angular momentum of about 4 $\hbar^2$ and resorts with a value of about 20 $\hbar^2$.

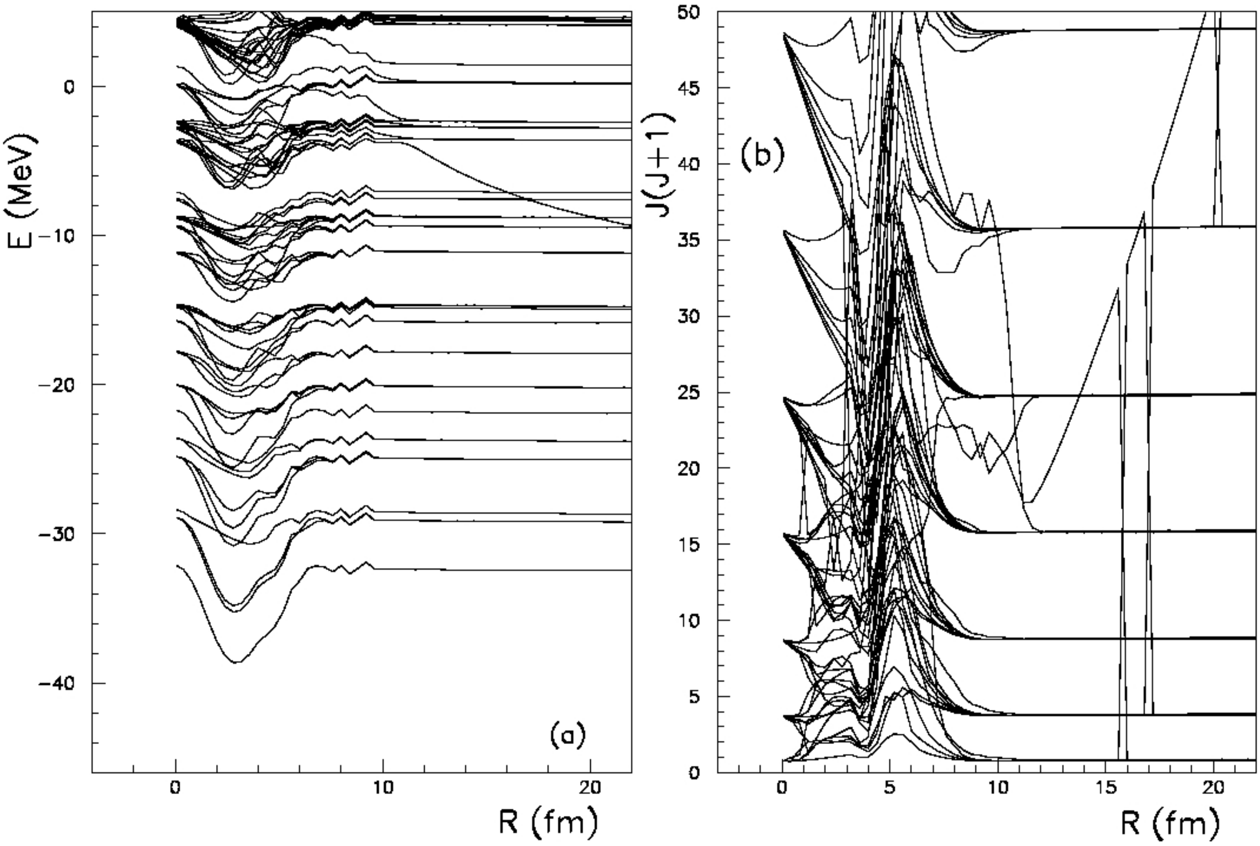

Fig. 2 – a) Proton single particle energies as function of the elongation R;
b) square of total angular momentum of a particle in $\hbar^2$ units as function of R.

A similar behavior is exhibited by the proton level scheme and the angular momenta plotted in Fig. 2. The alpha levels decrease in energy after the scission because of the reduction of the Coulomb polarization when the distance between the nuclear fragments increases. The daughter levels decrease with a much smaller slope. True levels crossings are produced between the single particle orbitals of the daughter nucleus and that of the alpha particle. The squared angular momentum of the proton belonging to the alpha particle increases also after the separation of the two nuclei as can be noticed on the plot in Fig. 2b. An interchange of angular momentum values is produced in the region of the true crossings. This is a peculiar effect due to the mathematical way in which the eigenvalues solutions are obtained after the diagonalization. The eigenstates are always obtained in an ascending or a descending order, no matter if the particle belongs to the daughter nucleus or to the alpha particle.

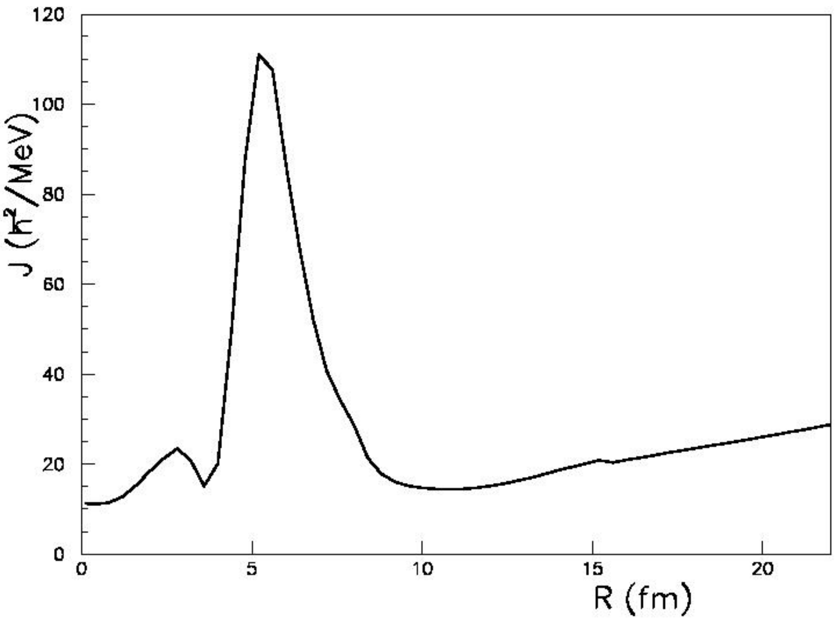

Fig. 3 – Total momentum of inertia of the nuclear system *J* as function of the elongation *R*.

The total momentum of inertia *J* is calculated in the framework of the cranking model. The momentum of inertia is represented in Fig. 3 as function of the distance between the centers of the fragments *R*. For nearly spherical configurations, *J* has small values. For *R* in the vicinity of 5 fm, the *J* increases to very large values. Close to scission, $R = 10$ fm, the momentum of inertia descents to values compatible to those of two tangent nuclei. After the separation, *J* has a positive slope as function of *R*. A similar behavior was observed for the alpha decay of Pu [17]. This dependence is characteristic for two bodies. For configurations characterized by the elongation *R* comprised between 5 fm and 10 fm, both the matrix element of the denominator of the recoil term ($j^2 – j_z^2$) and the momentum of inertia *J* increase. Usually the momentum of inertia increases with deformation, as evidenced for calculations concerning the fission [18]. Both effects are cancelling in the expression of the recoil term (6).

4. CONCLUSIONS

It can be concluded that the angular momenta increase to very large values during the rearrangement of the nuclear particle. But close to scission, the angular moments have a descending behavior, reaching asymptotically their final values, compatible to those of a two-body system. For the alpha particle emision treated as superasymmetric fission process, the total momentum of inertia also increases in the overlap region. As mentioned in Ref. [2], the modification of the momentum of inertia as function of the deformation should introduce a variation of the mean field potential. But, this modification of the moment of inertia is somewhat reduced by a similar behaviour of the matrix elements of j^2. These two cancelling effects should allow us to consider that in a first approximation the recoil term can be absorbed in the mean field potential. After the scission, the recoil term becomes smaller due to the increasing slope of the momentum of inertia. Also, very large values of the momentum of inertia in the overlap regions should hinder the emission of prescission particles [19].

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