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PAIRING GAPS AT SCISSION FOR NUCLEAR FISSION

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Abstract. The pairing gaps and the Fermi energies are investigated along the fission process within a state-dependent pairing formalism, based on a Gaussian interaction. The single-particle energies of the active pairing space are calculated within the Woods-Saxon two-center shell model. The fission barrier is obtained in the macroscopic-microscopic approximation along a fission trajectory determined within the least action principle. Large values of the pairing gap were exhibited by the state-dependent pairing model around the outer barrier, leading to a high stability of the nuclear system against particle emission. The pairing gaps at scission exceed several times the values obtained with a constant monopole pairing strength.

Key words: state dependent pairing, ^{234}U nuclear fission, Woods-Saxon two-center shell model.

1. INTRODUCTION

The microscopic approaches used in fission processes should be very well elaborated [1] to allow predictions. A large set of experimental data must be reproduced, as for example, fission barriers, fission mass distributions, total kinetic energies of fission-fragments, and spontaneous lifetimes. Some of these quantities have large experimental uncertainties. The microscopic approximation should start with a mean field able to reproduce all these quantities. This mean field is not well known. Different approximations are still used for the effective interaction or for the parameters of the mean field in the hope of a better agreement with the experimental data. Fission remains one of the most challenging quantum many body problem due to the difficulty of finding an adequate and computationally tractable formulation of the compound nuclear system and the way in which the final fragments are formed. Long time ago, it was stated that the fission is a process that reveals a pronounced collective behavior of a many body system [2]. A main property is the extreme saturation of the nuclear matter; the potential seen by a nucleon inside the nucleus is nearly independent on the positions of the other nucleons but it is controlled by the state of the system as a whole. The boundaries of such a potential are subject of fluctuations or oscillations. The single-particle states are mainly directed by the distortions of the nuclear surface. A quantum mechanics description of a system of independent particles for the collective mode of fission is therefore an appropriate treatment. In any Hartree-Fock or macroscopic-microscopic treatment of the nuclear disintegrations or of the collisions, the dynamical description is made in terms of some constraints or some collective coordinates, associated with degrees of freedom. The generalized coordinates of deformation are forced to vary in time leading to the split of the nuclear system or to the fusion of two nuclei. Thus, these coordinates describe the change in time of the average mean field that determines approximately the behavior of many intrinsic variables. The eigenstates and the eigenfunctions are evaluated for a precise nuclear deformation and the probabilities of occupation of the single-particle states are usually calculated by means of the Bardeen-Cooper-Schrieffer (BCS) theory. At scission, the microscopic models that take into consideration the pairing residual interaction are unable to fix the number of particles in the two fragments. The constraint of particle number conservation in the BCS theory implies the existence of a unique value of the Fermi energy in the precise moment when the nucleus breaks. The total number of nucleons of the system should be equal with the sum of the occupations probabilities of the single-particle states of both fragments. But, in order to have integer numbers of nucleons in each of the two partners, at least two Fermi energies are required, one for each nascent fragment. It is questionable when and how these two Fermi energies are created and how the nucleus shares the nucleons to obtain the final mass numbers. These mass numbers are integers, while the BCS theory gives real numbers for each fragment. In order to describe better how a nucleus is divided into two nuclei characterized by integer numbers of particles, the gaps and the Fermi energies should be investigated at scission within pairing interaction that should depend of the localization of the single particles. A possibility is the use of state-dependent pairing interactions. In this case, a pairing field is defined and the pairing interaction is obtained within the single-particle wave functions obtained from the mean field. In this work, the pairing at scission will be treated with a Gaussian pairing interaction.

2. STATE-DEPENDENT PAIRING FORMALISM

We considered a Gaussian pairing interaction of the type [3]:

$$v(r_{12}) = v_0 \exp\left(-\frac{r_{12}}{r_0}\right), \tag{1}$$

where v_0 and r_0 are constant parameters and $|r_{12} = \vec{r}_1 - \vec{r}_2|$ is the distance between the vectors $\vec{r}_1$ and $\vec{r}_2$. The pairing gap is calculated in the BCS approximation with a state-dependent interaction G_{ij}:

$$\Delta_i = \frac{1}{2}\sum_j \frac{G_{ij}\Delta_j}{\sqrt{(\varepsilon_j-\lambda)^2+\Delta_j^2}}, \tag{2}$$

where i and j denote single-particle states, Δ_i is the pairing gap, ε_j are single-particle energies, and λ is the chemical potential. The summation is made over the single-particle levels of the pairing active space. The constraint of the number of particles N_p should be fulfilled

$$N_p = \sum_i v_i^2, \tag{3}$$

where v_{i} is the BCS occupation amplitude

$$v_i^2 = \frac{1}{2}\left[1-\frac{\varepsilon_i-\lambda}{\sqrt{(\varepsilon_i-\lambda)^2+\Delta_i^2}}\right], \tag{4}$$

with the normalization $u_i^2 = 1 - v_i^2$. The pairing interaction is defined as

$$G_{ij} = -\iint_\infty v(r_{12})\,|\phi_i(\vec{r}_1)|^2\,|\phi_j(\vec{r}_2)|^2 \mathrm{d}\vec{r}_1\mathrm{d}\vec{r}_2. \tag{5}$$

The mean value of the pairing gap of all the nuclear system is

$$\overline{\Delta} = \frac{\sum_i u_i v_i \Delta_i}{\sum_i u_i v_i}. \tag{6}$$

This definition emphasizes the importance of the single-particle levels located in the vicinity of the Fermi energy.

3. RESULTS

In any macroscopic-microscopic treatment of nuclear disintegrations of collisions, the whole nuclear system is characterized by some collective coordinates that determine approximately the behavior of many other intrinsic variables. The basic ingredient in such an analysis is the shape parametrization that depends on several macroscopic degrees of freedom. The generalized coordinates associated to these degrees of freedom vary in time leading to a split of the nuclear system in two separated fragments. The model is valid as long as the time-dependent variations of the generalized coordinates make sense. We will use a parametrization that takes into account the most important degrees of freedom encountered in fission: elongation, necking, mass-asymmetry, and deformations of both fragments.

From a microscopic point of view, the many-body wave function and the single-particle energies are provided by the Woods-Saxon two-center shell model [4]. In this model, the Woods-Saxon potential, the Coulomb interaction and the spin orbit term are diagonalized in a double-center eigenvectors basis, provided by the semi-symmetric two-center oscillator in only one Hermite space. Details concerning these solutions and expressions for the normalization constants are found in [5] and references therein. In this unique Hermite space, the behavior of both fragments can be described simultaneously. The orthogonal wave functions are centered in one of the two regions of the three-dimensional space. Each wave function is analytically continued in both regions. In an intermediate situation of two partially overlapped potentials, each eigenfunction has components in the two subspaces that belong to two fragments. When the elongation is zero, the eigenvectors basis becomes that of a single anisotropic oscillator and the Hermite function is transformed into a Hermite polynomial. When the elongation tends to infinity, a two-oscillator eigenvectors system is obtained naturally in the same Hermite space, centered in the middle of the two fragments. Therefore, the two-center shell model always provides the wave functions associated to the lower energies of the single particle states pertaining to the major quantum number. As a consequence, molecular states formed by two fragments at scission could be precisely described. Due to its ability to treat extreme mass asymmetries, the model was widely used in investigations of fission processes [6–10], alpha-decay mechanisms [11, 12], cluster emission [13, 14], and superheavy elements synthesis [15–19]. The pairing at scission is investigated for the ^{234}U fission. In order to find the scission configuration, a fission path in our multidimensional configuration space should be obtained. By involving the least action principle, such fission trajectory that leads to double fission barriers was already obtained [20].

The fission barrier that agrees to the experimental data is represented in Fig. 1. The pairing gaps and the Fermi energies are calculated along the fission trajectory by using the Gaussian pairing interaction. The results are plotted in Figs. 2 and 3 for neutron and proton, respectively.

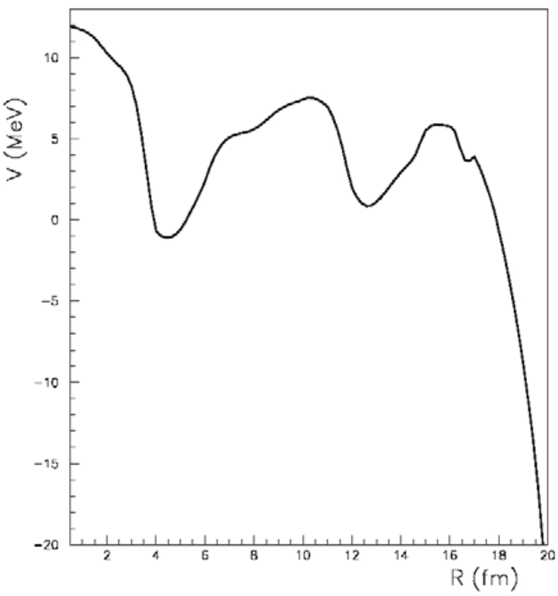

Fig. 1 – Fission barrier V for ^{234}U as function of the elongation R.

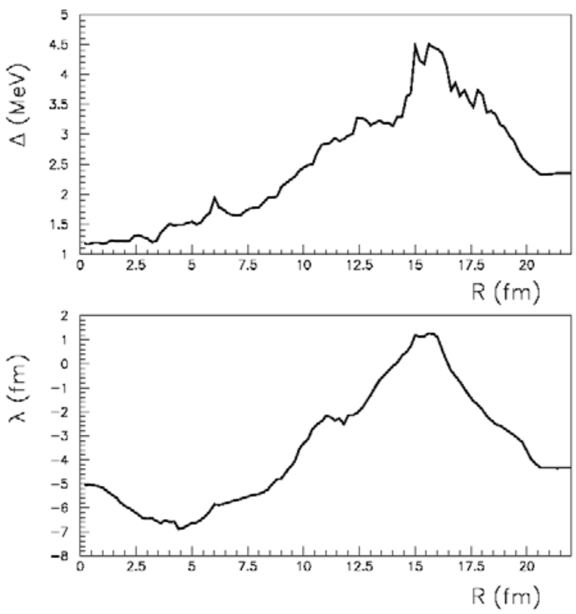

Fig. 2 – Averaged neutron pairing gap Δ (upper panel) and Fermi energy λ (lower panel) as function of the elongation R.

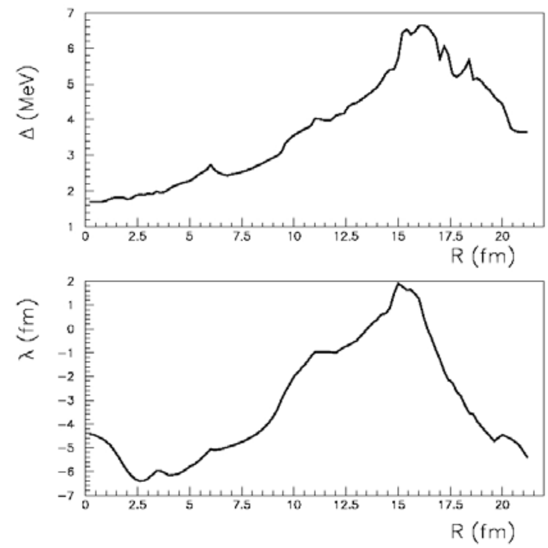

Fig. 3 – Same as Fig. 2, but for proton.

By using a monopole constant pairing interaction, the pairing gap has usually a smooth variation with the deformation, exhibiting some low amplitude variations that are due to rearrangements of the shell structure around the Fermi energy. The pairing gap of a heavy nucleus is in general smaller than that of a light one. Therefore, the gap usually increases to reach the value associated to the final fragment. Surprisingly, by using a state-dependent pairing interaction, the gap in the vicinity of the top of the second barrier up to scission increases to very large values that are at least two times larger than those of the final fragments. Once the scission is produced, the gap reaches a value that corresponds to the fission fragments. Such very large variations of the pairing gap around the scission configuration were also observed in the case of alpha decay treated as a superasymmetric fission process by using density-dependent delta interaction for the pairing field [21].

4. DISCUSSION AND CONCLUSION

The pairing gaps for neutron and proton have values of about 1 MeV in the ground state of the parent nucleus, close to the experimental findings. That is, the parameters used for the Gaussian interaction are realistic. Both values of the pairing gaps increase around the scission configurations. In the asymptotic region of two separated nuclei, the pairing gap decreases to stabilize around a realistic value of 2-3 MeV that characterize the fission fragments. Larger values of the pairing gaps are expected for the fission fragments, because their masses are about half of the mass of the parent nucleus. This behavior is reflected by the empirical rule between the pairing gap and the mass of the nucleus $\Delta \approx 12/A^{1/2}$ MeV. Surprisingly, very large values of the pairing gap are found in the region of the outer barrier up to the exit point from the barrier. The very large values of the pairing gap around the outer barrier, of the order of 4–6 MeV, should lead to an emphasized stability of the nuclear system. It is exactly the region of the rapid descent of the nuclear system during the scission process, where nonadiabatic effects should be dominant. It is expected that the nuclear system acquires a very large dissipation energy that should be manifested mainly by neutron emissions. This excitation energy exceeds usually 20 MeV [22]. However, the prescission number of neutrons detected experimentally is very small [23]. This behavior concerning the small number of prescission neutrons can be explained within the results obtained in the framework of state-dependent pairing formalism model that leads to an enhanced stability of the nascent fragments. In order to emit a scission/prescission nucleon, the nuclear system should break a pair, that is, to overcome the 5 MeV pairing gap energy obtained within this model. A large value of the pairing gap hinders the quasiparticle excitations exactly in the moment when the nucleus splits into two fragments [24, 25]. These properties should explain why a very small number of prescission and scission neutrons are emitted during the fission process. The pairing gap at scission is also an important ingredient for statistical models [26] used in the description of fission observables. A striking behavior can be also mentioned. In the region of the second fission barrier, the Fermi energy approaches zero. That means the single particles have very small binding energies, increasing their emission probability. However, this increase of the emission probability is at the same time hindered by the high values of the pairing gaps, responsible for the emphasized stability of the nuclear system. So, it seems that the nascent fragments in the region of the second barrier are stable despite the fact that they are not very energetically bound if only the single-particle model is considered (the superfluidity phenomenon is not taken into consideration).

In conclusion, a state-dependent pairing model offers a way to understand why a small fraction of prescission neutrons is exhibited by the experimental data, by evidencing that the nascent fragments are strongly bounded in the rapid descent of the outer fission barrier. Therefore, the particle emission is suppressed.

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REFERENCES

1. G.F. BERTSCH, W. LOVELAND, W. NAZAREWICZ, P. TALOU, *Benchmarking nuclear fission theory*, Journal of Physics G, **42**, 077001, 2015.
2. N. BOHR, J. WHEELER, *The Mechanism of Nuclear Fission*, Physical Review, **56**, pp. 426–450, 1939.
3. D.S. DELION, V.V. BARAN, *Systematics of the pairing coherence length*, Romanian Journal of Physics, **60**, pp. 993–1009, 2015.
4. M. MIREA, Time-dependent pairing equations for seniority-one nuclear systems, Physical Review C, **78**, 044618, 2008.
5. M. MIREA, Realistic orbital momentum operators for the superasymmetric two center shell model, Nuclear Physics A, **780**, pp. 13–33, 2006.
6. M. MIREA, *New dynamical pair breaking effect*, Physics Letters B, **680**, pp. 316–320, 2009.
7. M. MIREA, D.S. DELION, A. SANDULESCU, *Microscopic cold fission yields of* ^{252}Cf, Physical Review C, **81**, 044317, 2010.
8. M. MIREA, Energy *partition in low energy fission*, Physical Review C, **83**, 054608, 2011.
9. M. MIREA, Microscopic description of energy partition in fission fragments, Physics Letters B, **717**, pp. 252–256, 2012.
10. M. MIREA, Microscopic description of the odd-even effect in cold fission, Physical Review C, **89**, 034623, 2014.
11. M. MIREA, *Momentum of Inertia for the* ^{240}Pu *Alpha Decay*, Romanian Journal of Physics, **60**, pp. 156–160, 2015.
12. A. SANDULESCU, M. MIREA, D.S. DELION, Microscopic description of the alpha decay of a superheavy element, EPL, **101**, 62001, 2013.
13. M. MIREA, A. SANDULESCU, D.S. DELION, ^{238}Pu *cluster decay in the macroscopic-microscopic approach*, The European Physical Journal A, **48**, pp. 86, 2012.
14. M. MIREA, A. SANDULESCU and D.S. DELION, *Predictions for* ^{232}U *cluster-decays within the macroscopic-microscopic approximation*, Nuclear Physics A, **870–871**, pp. 23–41, 2011.
15. A. SANDULESCU and M. MIREA, *Cold Fission from Isomeric States of Superheavy Nuclei*, The European Physical Journal A, **50**, *110*, 2014.
16. R. BUDACA, A. SANDULESCU, M. MIREA, Quasifission mass distributions in the synthesis of ^{274}Hs with ^{26}Mg and ^{36}S projectiles, Modern Physics Letters A, **30**, 1550129, 2015.
17. D. ARANGHEL, A. SANDULESCU, *Fast fission yields in the synthesis of the* ^{296}Lv *superheavy element at* $E^*=30$ *MeV*, Romanian Journal of Physics, **60**, pp. 1433–1440, 2015.
18. D. ARANGHEL, A. SANDULESCU, *Shell effects in the fragmentation potential for superheavy elements*, Romanian Journal of Physics, **60**, pp. 147–155, 2015.
19. D. ARANGHEL, A. SANDULESCU, Origin of Molecular and Isomeric Minima in the Fragmentation Potential of the ^{296}Lv Superheavy Element, Romanian Reports in Physics, **68**, pp. 160–168, 2016.
20. M. MIREA, L. TASSAN-GOT, *Th and U fission barriers within the Woods-Saxon two center shell model*, Central European Journal of Physics, **9**, pp. 116–122, 2011.
21. M. MIREA, Pairing gaps and Fermi energies at scisison for the ^{296}Lv alpha decay, The European Physical Journal A, **51**, 36, 2015.
22. M. CAAMANO, F. FARGET, O. DELAUNE, K.-H. SCHMIDT, C. SCHMITT, L. AUDOUIN, C.-O. BACRI, J. BENLLIRE, E. CASAREJOS, X. DERKX, B. FERNANDEZ-DOMINGUEZ, L. GAUDEFROY, B. JURADO, A. LEMASSON, D. RAMOS, C. RODRIGUEZ-TAJES, T. ROGER, A. SHRIVASTAVA, *Characterization of the scission point from fission-fragment velocities*, Physical Review C, **92**, 034606, 2015.
23. A. GOOK, F.-J. HAMBSCH, M. VIDALI, Prompt neutron multiplicity in correlation with fragments from spontaneous fission of ^{252}Cf, Physical Review C, **90**, 064611, 2014.
24. R. CAPOTE, N. CARJAN, S. CHIBA, Scission neutrons for U, Pu, Cm, and Cf isotopes: Relative multiplicities calculated in the sudden limit, Physical Review C, **93**, 024609, 2016.
25. M. RIZEA, N. CARJAN, *Analysis of the scission neutrons by a time-dependent approach*, Proceedings of the Romanian Academy A**,** **16**, pp. 176–183, 2015.
26. B. JURADO, K.-H. SCHMIDT, Influence of complete energy sorting on the characteristics of the odd-even effect in fission-fragment element distributions, Journal of Physics G, **42**, 055101, 2015.

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